Sustainable Inventory Management Model for High-Volume Material with Limited Storage Space under Stochastic Demand and Supply

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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ABSTRACT

Inventory management and control has become an important management function, which is vital in ensuring the efficiency and profitability of a company's operations. Hence, several research studies attempted to develop models to be used to minimise the quantities of excess inventory, in order to reduce their associated costs without compromising both operational efficiency and customers' needs. The Economic Order Quantity (EOQ) model is one of the most used of these models; however, this model has a number of limiting assumptions, which led to the development of a number of extensions for this model to increase its applicability to the modern-day business environment. Therefore, in this research study, a sustainable inventory management model is developed based on the EOQ concept to optimise the ordering and storage of large-volume inventory, which deteriorates over time, with limited storage space, such as steel, under stochastic demand, supply and backorders. Two control systems were developed and tested in this research study in order to select the most robust system: an open-loop system, based on direct control through which five different time series for each stochastic variable were generated, before an attempt to optimise the average profit was conducted; and a closed-loop system, which uses a neural network, depicting the different business and economic conditions associated with the steel manufacturing industry, to generate the optimal control parameters for each week across the entire planning horizon. A sensitivity analysis proved that the closed-loop neural network control system was more accurate in depicting real-life business conditions, and more robust in optimising the inventory management process for a large-volume, deteriorating item. Moreover, due to its advantages over other techniques, a meta-heuristic Particle Swarm Optimisation (PSO) algorithm was used to solve this model. This model is implemented throughout the research in the case of a steel manufacturing factory under different operational and extreme economic scenarios. As a result of the case study, the developed model proved its robustness and accuracy in managing the inventory of such a unique industry.

KEYWORDS: Economic Order Quantity (EOQ), Inventory Management, Large-volume Material, Limited Storage, Deteriorating Item, Steel Industry, Closed-loop System, Open-loop System, Neural Network, Particle Swarm Optimisation (PSO), Sensitivity Analysis

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NOTATION

t	Time expressed in weeks
$t \in \{1,2,\dots,T\}$	The index for time referring to a period of t weeks ahead
$H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$	Heaviside step function
In each period, the state of a steel manufacturing factory is characterised by several variables that are changing over time:	
$m(t)$	Amount of available funds (£)
$raw(t)$	Level of raw materials available in storage
$buy(t)$	Quantity of raw materials to purchase at time t (units)
$ord(t)$	Amount of production ordered so it will be available at time t. During a specific period $t \in \{1,2,\dots t_{lead}\}$ this value is predefined based on the final products' arrival during first leading time periods (units)
$prod(t)$	Level of the final product available in storage (units)
$inv(t)$	Amount of invested money (£)
$sell(t)$	Level of sold final product (units)
$up(t)$	Amount of money in up credit (£)
$down(t)$	Amount of money in down credit (£)
$price(t)$	Current selling price that the factory sets for final product (£)
$demand(t)$	Demand for the final product as a function of the selling price, market demand and market price
$sent(t)$	Raw materials sent to production lines
Moreover, during each planning period, the factory's management needs to make control decisions, which will affect business performance in the next periods:	
$u_{inv}(t+1)$	Percentage of available funds to invest at time $t+1$ (%)
$u_{buy}(t+1)$	Percentage of money, from the available funds, to spend on purchasing raw materials at time $t+1$ (%)
$u_{order}(t+1)$	Percentage of raw materials to send to production lines at time t (%)
$u_{price}(t+1)$	Percentage of maximum price for which we will sell the final product at time $t+1$ (%)
The next set of notations describe the stochastic parameters used:	
Related to raw material ordering (demand)	
$P_{df}(t)$	Probability of delivery failure of the raw materials
$C_{df}(t)$	Extra charge incurred by the steel manufacturing factory per unit of raw material in case of delivery failure (£)
$\overline{C_{raw}}(t)$	Fixed ordering cost of raw materials (£)
$\widetilde{C_{raw}}(t)$	Basic cost of one unit of materials (£)
Related to storage	
$S_{raw}(t)$	Raw materials storage costs per period (£)
$S_{prod}(t)$	Production storage costs per period (£)
Related to backorders	
$shortage(t)$	Possible raw material shortage for week t

BO_{loss}	Backorder loss
Related to the final product	
$C_{prod}(t)$	Selling price of the final product (£)
$D_{prod}(t)$	Final product's demand (£)
Furthermore, the next set of notations describe the fixed variables used, grouped by their economic essence and the part of business chain in which they are applied:	
Related to raw material ordering (demand)	
$raw(0)$	Initial raw material level at the last available time before the planning horizon, i.e. at week zero
d_{am}^{raw}	Discount size, which is the amount of raw materials ordered that have to be placed to warrant a discount in cost (units)
d_{val}^{raw}	Value of the discount, in %, which will be subtracted from the purchase price and applied to the part of the order that exceeded discount size (%)
min_{ord}^{raw}	Minimum order quantity (units)
d_{fix}^{raw}	Small order discount – if a company buys less than this value, the fixed cost will be reduced by quadratic dependence (units)
t_{df}^{raw}	Number of planning periods in which the company will have to pay extra costs for raw materials in case of delivery failure (units)
Related to storage	
c_{inv}^{raw}	Value of each item of raw materials already in inventory
c_{inv}^{prod}	Value of each item of the final product already in inventory
max_{stor}^{raw}	Maximum storage capacity (units)
c_{stor}^{raw}	Cost of additional storage needed in case the company exceeds the maximum limit (£)
$frac_{det}^{raw}$	Fraction of raw materials that will deteriorate during each period
$frac_{det}^{prod}$	Fraction of the final product that will deteriorate during each period
Related to the final product (supply)	
$prod(0)$	Initial level of the final products in inventory
max^{prod}	The maximum number of units that can be produced daily (units)
max_{over}^{prod}	The maximum number of units that can be produced overtime (units)
p_{def}^{prod}	Probability of moderate defect
p_{cdef}^{prod}	Probability of major defect
Financial variables	
$m(0)$	Initial funds available (£)
c^{prod}	Cost of production per unit (£)
c_{over}^{prod}	Overtime cost of production per unit (£)
c_{def}^{prod}	Cost incurred to fix a moderate defect (£)
c_{fix}	Fixed costs per period, such as salaries and taxes (£)
i_{bank}	Overdraft rate per period (%)
i_{tax}	Tax rate (%)
i_{up}	Interest rate for up credit money (%)
i_{down}	Interest rate for down credit money (%)

$frac_{up}$	Percentage of up credit to repay per period (%)
$frac_{down}$	Percentage of down credit to repay per period (%)
c_{loss}	Additional penalty for realising losses. If company ends a period in loss, then, in some cases, additional penalty is applied $(loss(T) \cdot c_{loss})$ to reflect how much profit can be sacrificed to reduce the expected losses by one dollar. (£)
c_{max}^{prod}	Maximum selling price for the final product (£)
Related to raw material in storage	
$V_{raw}(m)$	Volume of raw materials stored for m weeks
Related to the net profit	
$Total\ worth(t)$	All the assets of the factory at time t
$Net\ profit$	Net profit to be maximised
Other parameters	
M	Number of samples in the Monte Carlo method (units)
T	Number of planning periods (units)
t_{lead}	Lead time (weeks)
π	Inflation rate per period (%)
el_c^{prod}	Demand elasticity per price
M_{raw}	Maximum observed maturity for raw products
M_{prod}	Maximum observed final products
$P_{raw(m)}$	Proportion of raw materials stored during m weeks
$P_{prod}(m)$	Proportion of final products stored during m weeks
D_{prod}	Deterioration of final products
D_{raw}	Deterioration of raw materials
$Loss$	Loss function
$Ploss$	Probability loss function
PD	Probability deterioration function
Related to EOQ assumptions	
D	Constant demand
p	Unit cost
L	Lead time
S	Order size
r	Cost of order holding
H	Expense of holding a unit in inventory for a whole period
Q	Order quantity
$Q*$	Optimal order quantity
Related to ANNs	
W_i	Weights
$f(\cdot)$	Function
X_i	Input signals
B_i	Bias
U	Vector of control variables
$S(x) = \dfrac{1}{e^{-x}+1}$	Sigmoid function
$relu(x) = \max(x, 0)$	Rectified Linear Unit function

I	Vector of inputs
Related to PSO	
pbest	Previous best position
gbest	Global best position
N_P	Total number of particles
f	Fitness function
P	Position of the particle
V	Velocity of the particle
ω	Inertia weight
r_i	Uniformly distributed random variables within the range of [0, 1]
c_i	Acceleration coefficients
Related to data processing	
$\mu(t)$	Mean values
$\sigma(t)$	Standard deviations
$l_b(t)$	Lower bound
$u_b(t)$	Upper bound
$\alpha(t) = \frac{\mu(t) \cdot \left(\sigma^2(t) + \mu^2(t) - \mu(t)\right)}{\sigma^2(t)}$	Matlab parameter
$\beta(t) = \frac{\left(1 - \mu(t)\right) \cdot \left(\sigma^2(t) + \mu^2(t) - \mu(t)\right)}{\sigma^2(t)}$	Matlab parameter

LIST OF ACRONYMS

EOQ	Economic Order Quantity
EPQ	Economic Production Quantity
MILP	Mixed-Integer Linear Programming
JIT	Just-in-Time
ELRS	Economic Lot Release Size
MRP	Materials Requirement Planning
UQM	Uniform Quotation Mode
DQM	Differentiated Quotation Mode
EDLP	Every-Day-Low-Price strategy
MTS	Make-to-Stock
MTO	Make-to-Order
FIFO	First-In-First-Out
LIFO	Last-In-First-Out
PSO	Particle Swarm Optimisation
T	Order Quantity
TRC	Total Annual Cost of the Retailer
P	Models with a price-dependent demand rate
TI	Models with a time-dependent demand rate
LT	Models with a lead-time-dependent demand rate
S	Models with a space-dependent demand rate
PR	Models with a promotion-dependent demand rate
A	Models with a advertising-dependent demand rate
PQ	Models with a product-quality-dependent demand rate
SQ	Models with a service-quality-dependent demand rate
RP	Random Probability
WTP	Willingness-to-pay
MNL	Multi-nominal-Logit
NAF	Normalised Advantage Functions
PG	Policy Gradient Algorithms
DDPG	Deep Deterministic Policy Gradient
WIP	Work-In-Progress
ERP	Enterprise Resource Planning
UOQ	Uniform Order Quantity
UQM	Uniform Quotation Mode
DQM	Differentiated Quotation Mode
RTI	Returnable Transport Items
TAC	Total Average Cost
VMI	Vendor Managed Inventory
MINL	Mixed-Integer Non-Linear
SLR	Systematic Literature Review
ELRS	Economic Lot Release Size
VED	Vital Essential Desirable
MRP	Materials Requirement Planning
T-S	Total Setup Lot Sizing

POQ	Periodic Order Quantity
PMP	Pontryagin’s Maximum Principle
GRG	Generalised Reduced Gradient
ANN	Artificial Neural Network
GUI	Graphic User Interface
NLIP	Non-Linear Integer-Programming

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1 Introduction

This chapter provides an overview of the research conducted and an introduction to the overall study. In Section 1.2, the research background is outlined together with the major issues related to this research study. This is followed by highlighting the research problem and identifying the research gaps in Section 1.3. The research aims and objectives, as well as the research questions, are presented in Section 1.4. In addition, Section 1.5 provides a summary of the approach and methodology adopted in this research study, while Section 1.6 highlights the contribution of this research study to the body of knowledge. Finally, Section 1.7 outlines the organisation of this thesis, and Section 1.8 provides a summary and conclusion for this chapter.

1.1 Research Background

Inventory is the quantity of an item or resource utilised by a company (Chase, Jacobs and Aquilano, 2007); in other words, inventory can be defined as “the stored accumulation of material resources in a transformation system” (Pycraft et al., 2010, p.424). The role of inventory is considered extremely significant for the survival and growth of an organisation, as the mismanagement of levels of inventory may result in excess stock, which increases inventory costs, while, on the other hand, a shortage of inventory can result in operational inefficiency and customer dissatisfaction. Moreover, inventory is also important for the activities of production and maintenance of the plant and the machinery, along with other operational requirements. In many situations, increasing inventory results in restricting the money that can be used to invest in some other productive means. Therefore, continuous management and control of levels of inventory has become an important management function in ensuring the efficiency and profitability of the firm’s operations (Sallemi, 1997). As a result, managing the logistics aspect of inventory has gained enormous attention in the last few eras, from both managers and researchers, due to the excessive costs associated with holding additional inventory in warehouses; consequently, the goal of corporate management is to store only the inventory that is essential to satisfy customer requirements (Gourdin, 2001).

According to Wild (2017), the term “inventory control” is widely used to organise the procedure of inventory management, so that customers can acquire the products when needed. The activities of purchasing, manufacturing, storing and distributing are mainly based on the marketing and sales function of the company; therefore, inventory control manages the supply of finished goods, spare parts, raw material, obsolete parts and other necessary supplies. In addition, Jaber et al. (2009) argued that the functions of logistics, customer services and production are largely dependent on the efficiency of inventory control. Several research studies attempted to develop models that can be used to minimise the quantities of excess inventory in order to reduce associated costs without compromising both operational efficiency and customers' needs. In that regard, operation management research largely focused on inventory control and its valid approaches in different industries. Clodfelter (2010) argued that efficient management of the inventory system benefits the organisations in: 1) providing satisfactory service provision to customers throughout the financial year; 2) reducing the level of investment needed on work through proper planning and allocation of inventories; 3) gaining discount on trade purchases; 4) ensuring the purchase and storage of material that matches the required product specifications; and 5) effectively managing the production schedules and new order procedures. In addition, Saxena (2003) argued that the costs incurred due to the shortage or excess of inventory need to be minimised, so that inventory control remains effective in managing proper inventory turnover. Moreover, cost reduction in inventory control has a major emphasis on reduction of the product's cost to facilitate customer satisfaction and increase sales.

Nevertheless, effectively managing inventory levels presents its own set of challenges. Due to the fact that organisations face conflicting targets of attempting to improve customer satisfaction by avoiding under-stocking which can cause deficiency orders, lost deals, sales bottlenecks and despondent customers and minimising the expenses associated with the production of final products, companies have to carefully manage their levels of inventory to achieve equilibrium between these two conflicting targets and ensure the optimal trade-off between them. As a result, inventory control is one of the variables which can make or break an organisation's business, and the entire significance of inventory control can be outlined by the typical saying that inventories are the burial ground of a business (Mathur, 1994). Moreover, Ouyang, Chang and Teng (2005) identified that the genuine dilemma regarding inventory control lies in deductively deciding the most ideal inventory measure, and not in decreasing its size only. Furthermore, Zinn and Charnes (2005) highlighted that the principal goal of inventory control is to give a high stream of good quality, significant and imperative material that empower retailers and suppliers to provide non-stop and opportune support to the end customers.

The next issue in inventory control is material administration. There are four essential exercises associated with materials: envisioning the necessity of material, containing and sourcing of materials, situation of materials in association, and the status of materials which need persistent observing. Federgruen and Zipkin (1984) stated that the elements of materials incorporate procurement, warehousing, generation planning, inventory control of crude materials, and transportation. Similarly, according to Bowersox et al. (2002), the next vital issue of inventory control is procurement. In order for organisations to operate, producers, distributers and retailers must purchase materials from outside suppliers. From the executives' point of view, inventory procurement fundamentally puts accentuation on connection among purchasers and merchants at key dimensions with higher association of both. Another imperative parameter in inventory control is demand forecasting. Demand forecasts depend on the currently booked requests, deals history, showcasing activities and data gathered from customers over time.

1.2 Statement of the Problem

To minimise the inventory cost, including ordering, handling and storage costs, many models have been developed to determine the optimal inventory levels. The earliest of these models, and the one that is considered as the basis for all subsequent models, is the Economic Order Quantity (EOQ) model, which was developed by Harris in 1913. This model aims at determining the optimal size of the order, which minimises the frequency of the orders and achieves the maximum possible cost savings. According to Lucey (1992) and Schroeder (2000), the EOQ is defined as the optimal order quantity that balances the minimum inventory holding cost with the cost of reordering. However, the EOQ model is based on certain specific assumptions that somehow limit its applicability in modern-day supply chain problems. The major assumptions of the EOQ model are related to constraints on demand, time, availability and costs, being one of its main limitations the assumption of a constant, linear and deterministic demand rate over the entire planning horizon. Although, according to Silver et al. (1998), the EOQ model can still be applied in the case of small variations in demand over a constant interval of time, major changes in the demand rates over less time require the model to be modified or extended to hold for these variations. In addition, EOQ models assume constant and known lead time, which are other important constraints for receiving orders, compromising their accuracy in the presence of variable lead times. Furthermore, several cost assumptions which are associated with the EOQ model are not applicable in the current business environment. On one hand, the unit cost should stay uniform over the planning horizon, and the cost of inventory purchasing is considered fixed. Thus, changes in the exchange rates and the economic conditions of the country significantly affect this price constraint. On the other, the annual cost of holding the inventory is known and independent of the measure of the quantity ordered. Finally, the cost of the firm's ordering is considered to be self-regulating with the size of the order quantity.

Despite these limiting assumptions, the EOQ model's strength and simplicity make it function admirably in practice (Drake and Ptak, 1988), being accepted in century-long research, establishing the basis for the inventory control procedures at later stages of development. Nearly thousands of research studies were directly or indirectly based on the EOQ model, using its assumptions to form decision-making models for different situations. In fact, only by relaxing one or more of the EOQ's model assumptions it can be adapted to the current business scenario and help companies to reduce their inventory costs. Several research studies were devoted to examine which of the EOQ assumptions violate as well as how to modify the model accordingly to obtain better results in terms of inventory costs. In (Lev and Weiss, 1990), the relaxation of the infinite horizon and static costs to develop a model that incorporates both finite and infinite horizons with changes in cost was proposed, whereas Cheng (1990) integrated the product pricing and order sizing to maximise profits given the storage space and inventory investment constraints. Carlson, Miltenburg and Rousseau (1996) extended the EOQ model to include quantity discounts when all costs are incurred on different dates, i.e. date-term credit, using the discounted cash flow method. Through this model, the researchers found that the optimal order quantity might not occur at a breakpoint in the discount schedule under all-units discounts, while under incremental quantity discounts, the optimal order quantity might fall at a price break in the schedule. Other EOQ extensions apply the model to retail cycle stock inventories (Bassin, 1990), account for conditions of a provisional sale for a buyer (Chen and Min, 1995), account for imperfect quality items (Salameh and Jaber, 2000; Maddah and Jaber, 2008; Khan et al., 2010; Hsu and Hsu, 2013), and handle partial backordering increase (Zhang et al., 2011; Toews et al., 2011; Chung and Cárdenas-Barrón, 2012; Taleizadeh et al., 2013). In addition, several studies extended the EOQ model based on demand as a function of price, demand/supply as a function of time and deterioration rates, lead time, space, promotion, advertising, quality, and holding and carrying costs. Examples of these models include: Ray et al. (2005), Maddah and Noueihed (2017), Shah and Vaghela (2017), Pekgun, Griffin and

Keskinocak (2017), Singh, Khurana and Tayal (2016), Giri and Bardhan (2015), Dordevic et al. (2017), Farhangi and Mehdizadeh (2016), Rajan and Uthayakumar (2017), Hertini et al. (2018), Manna, Dey and Mondal (2017), Hazari et al. (2015), Kumar and Chanda (2017), and Liu, Zhang and Tang (2015). Finally, the different extensions of the EOQ model have enabled it to be applied to various fields, such as transportation (Munson and Hu, 2010; Krichen et al., 2011), supply chain (Hajiaghaei-Keshteli and Fard, 2018; Rezaeiahari and Sharifyazdi, 2016; Xu, Yin and Dong, 2016; Yadav, Pareek and Mittal, 2018), manufacturing (Taleizadeh, Cárdenas-Barrón and Mohammadi, 2014; Pasandideh et al., 2015; Nobil et al., 2019; Nobil and Taleizadeh, 2016; Ramezanian and Saidi-Mehrabad, 2012), and sustainability (Jain et al., 2018; Benkherouf, Skouri and Konstantaras, 2016; Kozlovskaya, Pakhomova and Richter, 2019; Mawandiya, Jha and Thakkar, 2018; Singh, Sharma and Kumar, 2016; Demirel, Demirel and Gokcen, 2016; Kozlovskaya, Pakhomova and Richter, 2015; Liao and Deng, 2018; Hovalaque and Bironneau, 2015).

Despite the important contributions made in the literature towards extending the EOQ model, there are some research gaps that need to be addressed through further research. This is particularly noticeable in the case of inventory management application in the steel manufacturing industry. This industry has special characteristics in terms of the large volume of inventory and the particular storage requirements to avoid deterioration. However, a limited number of studies have developed inventory management models to account for such special characteristics, as the majority of the manufacturing models available in the literature have been developed for a general manufacturing scenario, and not for a specific industry. Consequently, there is a lack of specialised models targeted at addressing the operations of the steel manufacturing industry. Moreover, the few models in the literature that were specifically developed for the steel manufacturing industry assume demand to be deterministic, which does neither accurately depict the nature of demand in this industry nor the variety of products present in such industry. In this context, an inventory management model assuming stochastic demand can help to enhance the operations of the steel manufacturing companies. In addition, the problem of shortage of storage space should also be taken into account. In order to address such an issue, space-dependent demand models that are sensitive to both space and price and have fewer underlying assumptions, can be used. Finally, in general, models only consider the cost minimisation as the objective function. Nevertheless, in order to accurately depict the real-life scenario, other objectives, such as sustainable aspects, in the sense of taking care of the impacts of the current operations without compromising the ability to satisfy future needs, should be also taken into consideration. In fact, to the best of the author's knowledge, no studies have fully explored sustainable aspects in terms of the quantity of or reasons behind waste generation for the steel manufacturing industry. Nevertheless, the increasing regulations imposed on the companies to conduct their business in a sustainable manner make them urged to account for environmental aspects, especially in terms of waste reduction and management.

Based on the above discussion, the main problem in this research study is to manage inventory in the presence of large-volume products and raw materials which require a large storage area. In particular, this becomes harder within the context of the steel manufacturing industry. Such factories need to store a large volume of raw materials, requiring large storage space that is specifically equipped to prevent the deterioration of the final product as a result of various environmental factors, such as humidity. These special requirements prohibit the long-term storage of such products, and increase the storage costs significantly. In addition, the demand, supply and backordering (missed orders due to shortage in the supply), which influence the quantity of raw materials required, are often stochastic in nature in the steel manufacturing industry. Hence, this research's problem is based on both the product's physical characteristics and its special requirements during the inventory holding period. Consequently, there are three sides of the problem regarding the nature of the product that should be addressed:

1. High-volume material that needs a large storage space. Therefore, optimisation management is required to optimise the inventory decision on how and when to order raw material from the suppliers, based on the production process and the market conditions. The aim of this optimisation process is to reduce the storage time needed for raw materials and final products to the minimum possible time, in order to reduce any waste resulting from long storage periods.
2. The high level of energy required to avoid harmful environmental effects on the product's physical characteristics. To address this problem, first, the nature of steel is studied to determine the amount of energy that is required to keep it safe from the effects of humidity and preserve its quality; then, by optimising the storage time for raw materials and final products, the amount of energy needed will be reduced, having a positive environmental effects.
3. Increasing the company's profit. This can be achieved by reducing the storage costs and the amount of waste produced.

Then, this research study assesses how to model the inventory order decisions for the material stock in the steel manufacturing industry under the above circumstances and conditions, by optimising the quantity to be ordered in each period over a 52-week time horizon, and how do we solve this model.

1.3 Research Aims, Objectives and Questions

The overall aim of this research study is to develop a sustainable inventory management model based on the well-known EOQ concept to study and optimise the inventory and order placement decisions over 52-week time horizon periods for high-volume material with limited storage space, such as steel, under stochastic demand, supply and backorders. The proposed model is expected to minimise the high storage and handling costs associated with raw materials and final products of a steel manufacturing company, and to prevent the deterioration of this inventory as a result of different environmental factors, thus maximising the company's profits. In order to do so, the proposed model is developed based on a control system algorithm capable of providing timely recommendations for the storage quantities of both products and raw material. In this way, the decisions regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon are facilitated. In particular, in this research study, two different control system approaches, namely, an open-loop and a closed-loop based on Artificial Neural Networks (ANNs), are considered. Finally, due to the complexity of the addressed problem and its specific characteristics, a PSO technique is used to solve it.

In order to achieve the above aims, a number of objectives have been set for this research study. These objectives are:

1. Model the stochastic nature of the different inventory parameters, such as demand, supply and backorders for high-volume products with limited storage space, when taking the sustainability approach into consideration.
2. Model the manner and nature of the deterioration of the raw materials and final products of the steel manufacturing factory, and optimise the storage time of the inventory in order to reduce the energy cost and, in turn, the storage costs.
3. Analyse the cash flow cycle of the steel manufacturing company and incorporate its different parameters and determinants into the inventory

management model, in order to ensure the efficiency of the production process and maximise the company's profit.

In order to achieve these objectives, this research study is focused on answering the following research questions:

1. How can we develop a model for inventory management within a limited storage space for high-volume material based on the stochastic effect for the inventory variable (demand and supply)?
2. What is the most suitable approach to solve this model to meet the stochastic nature of demand, supply and backorders in inventory control and management?
3. Is there any evidence to suggest that the developed model is robust and can handle different real-life scenarios?

1.4 Proposed Approach

To achieve the above aim and objectives and answer the research questions, the research methodology shown in Figure 1-1 is followed.

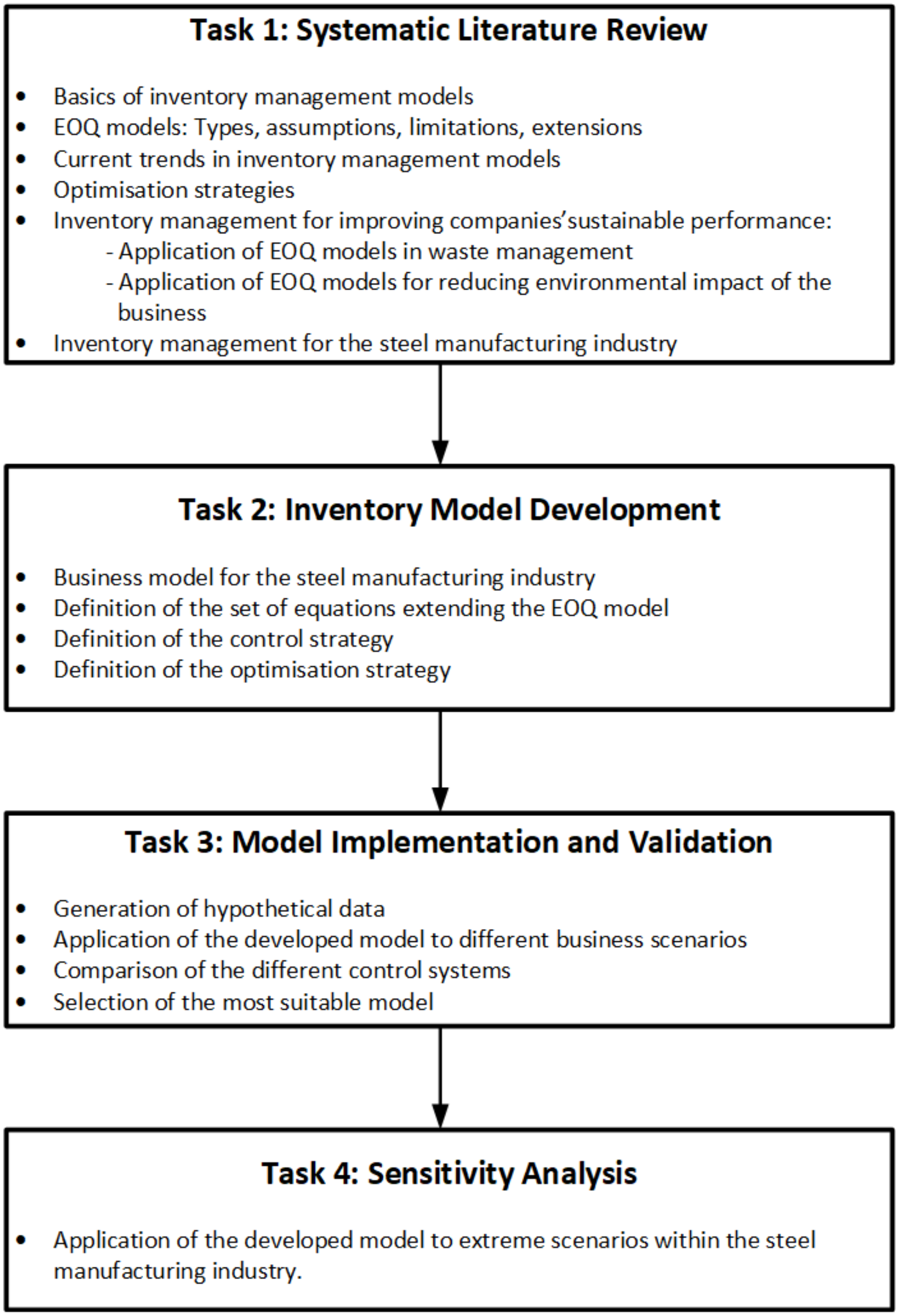

Figure 1-1. Proposed Research Methodology.

As seen from Figure 1-1, this research study starts by conducting a comprehensive Systematic Literature Review (SLR) of the topics related to the research undertaken in this study and the developed model. The SLR explores the current trends in the literature regarding inventory management models, analysing the EOQ model, its different types, assumptions, extensions and limitations. In particular, since the steel manufacturing industry is the subject topic of this research study, special focus is done in studying EOQ model applications within this field. In addition, in order to bridge the gap regarding the lack of sustainable EOQ models, the SLR also analyses the EOQ model's influence on different sustainability aspects, such as waste management, and environmental impact. Through this SLR, the main research gaps are identified and the need for the model developed in this study is highlighted.

The next step in the proposed research methodology is to build a mathematical model that reflects the stochastic nature of the inventory parameters, *viz.*, demand, supply and backorders, for high-volume material with limited storage space when taking the sustainability approach into consideration. As discussed above, the proposed model in this research study is based on the EOQ concept and methodology resorting to two different control systems, namely the open-loop one and the ANN based closed-loop one, and solved using PSO. Once the proposed model has already been developed, and solved by the proposed PSO technique, it is applied to a real-life steel manufacturing industry scenario. In order to do so, hypothetical data (Gasior and Recchia, 2019) is generated based on different average indicators of the steel industry available in the literature (Pardipto and Lussy, 2019; Tseng and Yu, 2019; Tavakoli and Taleizadeh, 2017; Rabieh et al, 2016) as well as on historical trends and publicly available business reports, such as the ones in (OECD, 2017; World Steel, 2018). The performance of each of the proposed control systems (open-loop and ANN) is evaluated in the simulated scenario, in terms of six main parameters, *viz.*, maturity and distribution rates, generated profits per storage unit used, profit generated by each Monte Carlo run, investment strategy, money management, and learning progress. Due to the lack of benchmark results available in the literature, the developed model is validated by comparing these two performances, and the most suitable one in terms of robustness and accuracy is selected. Finally, a sensitivity analysis, in which the selected approach is applied to different real-life scenarios to which a steel-manufacturing company might be subjected, is conducted to examine the robustness of the developed model and its ability to handle these extreme scenarios.

1.5 Contribution to Knowledge

This research study is aimed at supporting the steel manufacturing companies in their inventory management decisions, specifically when they have limited storage space. The model developed in this research study is expected to have an impact on:

1) Reducing the costs of holding inventory for the steel manufacturing companies
2) Reducing the probability of deterioration of the raw materials and final products of a steel manufacturing company as a result of environmental factors
3) Maximising the profits of such companies
4) Improving the sustainability of the steel manufacturing industry as a whole, and the supply chain of this industry in particular.

In addition to these expected benefits, the conducted research study will contribute to the body of knowledge in the inventory and supply chain management areas; these contributions include:

1. *Developing an inventory management model that models the stochastic nature of different inventory parameters at the same time.*

   Unlike previous models, which modelled only the stochastic nature of demand in managing inventory, the model proposed in this research study contributes to the body of knowledge by taking into account the stochastic nature of demand, supply and backordering all at the same time.
2. *Accounting for the environmental impacts of holding inventory for a steel manufacturing company while optimising its inventory management strategy.*

   None of the models developed for the steel manufacturing industry considered the environmental impacts of holding inventory in their objective functions; however, with this factor becoming increasingly important in modern times, the steel manufacturing companies need to consider it during all phases of their operations. Therefore, the proposed model contributes to the body of knowledge by being the first model to include the impact on the environment in the objective function of the inventory management of a steel manufacturing company.
3. *Extending the EOQ model to have the capability to incorporate different business and economic scenarios, and provide robust and accurate results.*

   Several business scenarios were not modelled in the literature, as they were considered too complex. Hence, the proposed model contributes significantly to the body of knowledge by incorporating some of these scenarios, such as sudden disruption to the supply.

4. *Developing an accurate and robust inventory management model for large volume materials based on an ANN control system and solved using the PSO technique.*
   The proposed model expands on the models previously developed in the literature by depicting demand's sensitive nature to both space and price, and basing the model on fewer assumptions by considering many of the complex factors that impact the inventory management of a steel manufacturing company. Moreover, the inventory model developed in this research study is more practical than the traditional EOQ model. On one hand, it is based on a control system algorithm capable of providing timely recommendations for the storage quantities of both products and raw material. In this way, the decisions regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon are facilitated. In addition, using ANNs as the control strategy provides a high learning and generalisation capability, the possibility of handling non-linear variables and missing data, and highly adaptability to changing environments. On the other hand, a new meta-heuristic PSO algorithm is used to solve the model which, under the consider circumstances, results an unconstrained non-integer nonlinear programming model. This algorithm has the capability to provide more accurate results within a shorter computational time when compared to any other meta-heuristic algorithms.

1.6 Thesis Organisation

This thesis is divided over seven chapters. Chapter 1 has introduced the research topic under study. In particular, the main research gaps in the field as well as its main challenges have been discussed. Based on them, the research problem has been defined. In this same line, the main research aims and objectives, as well as the research questions and methodology have also been defined. Chapter 2 provides a comprehensive SLR focusing on the topics that are most relevant to this research study. Through this SLR, the application of the inventory models not only in the supply chain, transportation and manufacturing fields, but also in the fields of sustainability and waste management are examined. In particular, since the steel manufacturing industry is the subject topic of this research study, special focus is done on the application of inventory management models within its context. In

Chapter 3, the detailed research methodology followed in this research study is presented. This chapter outlines the process and steps taken in order to develop and validate the proposed model, including the collection of the relevant data. Chapter 4 focuses on the model development. In the first place, the inventory model is described from the economic and business cycle points of view, and the relevant economic variables are defined. Based on this analysis, the mathematical model is developed, defining the set of equations that extend the EOQ concepts towards reflecting the steel manufacturing industry's dynamics. Then, the control systems used to provide the model with timely data about the business environment are described. Finally, the optimisation algorithms used to adjust the model's parameters are introduced. Chapter 5 presents all the details regarding the Matlab implementation of the developed model for the case of the steel manufacturing factory. In this chapter, the different steps followed for implementing and validating the developed model are outlined. In particular, the results of the implementation of the open-loop and closed-loop models are compared and contrasted in order to determine the best suited one for the addressed application in terms of robustness, accuracy and effectiveness. After determining the most suitable model, this model is applied to three different scenarios that can occur in the steel manufacturing industry, *viz.*, a fixed demand scenario, a fixed supply scenario, and a fully stochastic scenario. The results obtained in each scenario are discussed and compare in order to draw valuable conclusions about the performance of the developed model. In Chapter 6, the robustness of the model is further explored by conducting a sensitivity analysis, where the developed model is implemented in extreme business cases that might face the steel manufacturing company. In particular, a sudden decrease or interruption in the supply of raw materials, a sudden decrease or interruption in demand for final products, and a sudden increase in the costs of raw materials or storage, are considered. Finally, Chapter 7 provides a summary of the main findings of the conducted research and draws conclusions. In this chapter, the contribution of this research study to the body of knowledge as well as its limitations are outlined in detail. In addition, a list of recommended future research areas and topics is provided.

1.7 Chapter Summary

In this chapter, the foundations of this research study have been explained by providing a comprehensive background for this study and the problem statement concerning the topic under investigation. In particular, after discussing the research gaps and defining the research problem, the research aims and objectives to address this problem have been outlined, along with the research questions that will be answered through this research study to achieve these objectives. These research goals and objectives will be achieved over the following chapters of this research study.

2 A Systematic Literature Review about Inventory Management in the Steel Manufacturing Industry

2.1 Introduction

In this chapter, a comprehensive literature review about the current trends in the inventory management field is conducted, making special focus on its application within the context of the steel manufacturing industry which is the subject topic of this research study. The main aim of such literature review is highlighting the previous lessons to be learned and the research gaps in the literature towards developing an inventory management model for the steel manufacturing industry that can make a valuable contribution to the state-of-the-art.

The chapter begins with a detailed background for inventory management and control provided in Sections 2.2. In particular, the need for inventory management and control are highlighted and the basic concepts related to them, including definitions, types, available approaches, associated costs, and challenges are introduced. Section 2.3 introduces the EOQ model as an inventory management tool, its assumptions, limitations and extensions. In Section 2.4, the current trends in the literature regarding EOQ models are explored. In particular, special focus is done on the steel manufacturing applications which are the subject topic of this research study. In addition, the sustainability aspects of inventory management, in terms of the environmental impact of ordering and holding inventory are also discussed. Finally, in Section 2.5 the identified research gaps are highlighted and the summary and conclusions of the chapter are provided.

2.2 Basics of Inventory

This section highlights the basic information required to learn about the inventory, its management, method of classification, costs encountered, and inventory control techniques.

2.2.1 Inventory Management

In the view of Gourdin (2001), the logistics area of inventory has gained enormous management attention in the last few decades, since executives realised the excessive costs of holding additional stocks in warehouses. Therefore, a number of attempts have been made, and management approaches were developed to minimise the excess inventory without compromising the customer service domain. Gourdin (2001) also realised the importance of holding stocks in some situations, such as meeting the needs of global customers, and hence, the goal of the company's management is to hold only what is necessary to achieve this goal. In this regard, Chase et al. (2007) defined inventory as "the stock of any item or resource used in an organisation" (Chase et al., 2007, p.282). According to this perspective, the system of inventory needs a proper set of controls and policies to measure the inventory levels on a regular basis, and adjust it according to the needs of the business. Moreover, the procedures for replenishment and the size of the inventory are also considered important in this research (Chase et al., 2007). In addition, a more comprehensive definition of inventory was provided by Pycraft et al. (2010), as "the stored accumulation of material resources in a transformation system. So, a manufacturing company will hold stocks of materials, a tax office will hold stocks of information, and a theme park will hold stocks of customers." (Pycraft et al., 2010, p.424).

2.2.1.1 Types of Inventory

The role of inventory is considered to be significant for the survival and growth of the organisation, as improper management of inventory levels may result in excess stock, increasing inventory costs, or a shortage of inventory may result in the non-satisfaction of customers. In order to realise the importance of inventory in the company financial statements, Coyle, Bardi and Langley (2003) stressed the value of inventory as an asset on the balance sheet of firms, and used as a means of decline in the investment of the company in the fixed assets of the plant machinery, land and other assets. Moreover, inventory is also important for the activities of production, maintenance and operations. In many situations, increasing inventory resulted in restricting money from being invested in some other productive means. In the case of high inventory stocks, the management of assets becomes crucial, since the company is left with less resources to use on other assets. Therefore, close monitoring is required.

Sallemi's (1997) conclusion was that the critical approach of management is required about less stock or inventory for production. However, inventory shortage can result in the loss of production and occurrence of other costs. Therefore, continuous management and control of inventory has become a sensitive management function, and physical material balance also requires technology upgrades and improvement of handling procedures.

Stock and Lambert (2001) proposed six major forms of inventory, and described their importance and use in the manufacturing sector:

(i) **Fluctuation Inventory:** Fluctuation inventory can be utilised for unpredicted production schedules in situations where predictions of the required quantity of finished products cannot be ascertained. However, businesses with dynamic demand, with the product life cycle in the growth phase, normally require this inventory to manage sudden demands.

(ii) **Anticipation (Speculation) Inventory:** This inventory type is formed for anticipated demand, or in seasonal businesses that expect large sales in particular seasons. The inventory for winter clothing or the Christmas decoration items inventory are some examples of speculation inventory.

(iii) **Cycle (Lot-size) Inventory:** The demand for the inventory is decided in lots. In this type of inventory, the individual units are not considered, as the full lot size is the matter of importance. Nevertheless, the inventory size is normally very high, but the lot size is predetermined.

(iv) **Transportation (In-Transit) Inventory:** In-transit inventories are based on the demand type and nature, and involve the transportation of products from one location to another. The inventories involved in Work-in-Progress (WIP) are considered in this class of inventories, and meant for the type of process of design and layout of the plant.

(v) **Decoupling (Buffer Stock) Inventory:** Buffer stock inventories are used to reduce the burden of stock at various stages of the processes in production. The decoupling period of the sales is responsible for the maintenance of these inventories. These inventories provide major links in the production system and are independent of the other inventories at other stages of the processes.

(vi) **Dead Stock:** The unwanted inventory that is not used for any immediate or long-term purpose. Therefore, more costs are incurred in the storage and maintenance of this inventory. The management sometimes stores the inventory in anticipation of future increases in demand, or because the cost of disposing of the inventory is greater than the cost of storing it. However, customer service is one of the primary reasons to store dead inventory, so that an occasional buyer can receive goods on demand in the future.

2.2.1.2 Types of Inventory Costs

There are different types of costs associated with inventory management. Besides the cost of purchasing the storage items themselves, there are other types of costs that are incurred by firms when attempting to manage their inventory levels. These costs arise from holding and maintaining the inventory, managing it, and ordering it. Hence, other than the inventory price, there are three other major types of inventory costs, namely holding costs, ordering costs, and shortage costs (Vrat, 2014; Axsater, 2015; Ivanov, Tsipoulanidis and Schönberger, 2017).

(i) **Holding Cost:** the cost of carrying excess levels of inventory to cover future demand and production needs. This type of cost includes all the costs incurred as a result of storing the inventory items in the warehouse, inventory handling, and the cost of capital tied up in the inventory. It also includes any applicable taxes, insurance and costs of damage or obsolescence. These costs are variable costs that depend on the quantity of inventory held, and the time span of holding it. These costs differ according to the type of industry and location.

(ii) **Ordering Cost:** These are fixed costs that are associated with ordering the inventory. These costs arise from the administrative efforts required, transportation, and material handling during the ordering process. These are generally fixed costs and do not depend on the order size or quantity.

(iii) **Shortage Cost:** These costs are incurred when an item is required but not present in the inventory. In this case, backlogging will occur, which entails additional administrative and material handling costs. Moreover, in order to convince the customer not to seek another supplier, price discounts might be offered. Furthermore, such costs might impact the cost of other operations within the supply chain or the production process.

(iv) **Price:** This is the actual sum of money paid to acquire the inventory item. If the inventory is bought from another company, then this will be the cost per unit of that product. If it is produced in-house, then it will be the total of all the direct and indirect costs associated with the production of a single unit.

2.2.2 Inventory Control

According to Wild (2017), the term “inventory control” is widely used to organise the procedure of inventory management so that the customer can gain products when needed.

The activities of purchasing, manufacturing, storing, and distributing are mainly based on the marketing and sales function of the company. Therefore, inventory control manages the supply of finished goods, spare parts, raw materials, obsolete parts, and other necessary supplies. Jaber et al. (2009) argued that the functions of logistics, customer services, and production are largely dependent on the efficiency of inventory control. Demand cannot be fulfilled and customer satisfaction cannot be achieved through the mismanagement of inventory, if manufacturing does not coincide with purchasing and sales needs, leaving the operation short of stock.

Operation management research is largely focused on inventory control and its valid approaches in different industries. The researchers conclude the importance of inventory management, considering the evolution of supply chain management and the strategic advantages of inventory control mechanisms. The success of companies from Japan, the United States and European countries is largely due to efficient and upgraded systems of inventory control. The study of Silver et al. (1998) suggested that firms raised the bar of efficiency through coordination-based supply chain activities. However, Mula et al. (2006) highlighted the sharing of information and installation of software support, such as Enterprise Resource Planning (ERP), to coordinate information of variable anticipated demand. In Clodfelter (2010), the benefits of an efficient inventory control system are illustrated. According to Clodfelter (2010), these benefits include: satisfactory service provision to customers throughout the financial year; reduction in investment on work quantity through proper planning and allocation of inventories; discount gained on trade purchases; the insurance of the purchase and the storage of material that matches required product specifications; and finally, managing effective production schedules and new order procedures. Moreover, Clodfelter (2010) asserted that efficient control of inventory assured the proper receipt procedures, safe transactions, and proper storage arrangements for future purposes. However, the achievement of equilibrium is required for inventory purchases and sales of finished goods. Saxena (2003) argued that the costs incurred due to shortage or excess of inventory need to be minimised, so that inventory control remains effective in managing proper inventory turnover. Moreover, cost reduction in inventory control has a major emphasis on reduction of the product cost to facilitate customer satisfaction, increase in sales and maximising of profit.

2.2.2.1 Challenges of Inventory Control

The primary target of inventory control is to ensure client's satisfaction, in the sense that an inefficient stock management can cause conveyances inaccessibility, deficiency orders, lost deals and bottlenecks, which may lead to despondent customers. The second target is to advance the productivity underway or acquiring by limiting the expense of giving sufficient dimension of client benefit; placing excessive significance on client administration can prompt overstocking, which implies that the organisation has excessively tied up its interest in inventories (Biswas et al., 2017). These two targets regularly demonstrate logical inconsistency. Accomplishing an abnormal level of client benefit by maintaining specific inventories results in higher inventory costs and lower productivity. A few times a manager selects an ideal dimension of the client's administration and exercises to control inventory in a way that achieves the dimension of the client's benefit at the lowest cost conceivable. Similarly, this dimension is important in inventory levels by avoiding both overstocking and under stocking capacity. This is an imperative function of the management of a manufacturing firm, ensuring that specific materials are available when the operations need them (Mula et al., 2006). This additionally helps in possibilities regarding augmentation of economy and minimisation of waste, accordingly decreasing losses in the framework. In this way, inventory control is referred to as a framework, which guarantees the supply of the required quality and amount of inventory at an ideal time.

Working capital, which can be lessened by legitimate inventory control, can be enhanced with its effective utilisation into the different procedures of the firm. As per Mathur (1994), inventory control is one of the variables which can be considered "make or break" for any association. The entire significance of inventory control can be outlined by the typical saying that "inventories are the burial ground of a business" (Mathur, 1994). Designing an appropriate and compelling inventory control framework has significance in the adjustment of tasks. It is the basic purpose of numerous divisions with clashing interests and contemplations of long and short-range objectives. The entire expectation of inventory control frameworks is to maintain a balance between excessive and insufficient levels. Budgetary threats are included for an excessive amount of inventory, and insufficient inventory may develop issues for smooth and effective generation and aggressive powers. However, Ouyang et al. (2005) identified that the genuine issue lies in deciding the most ideal inventory measure, and not on decreasing size. This issue is aggravated when the wanted inventory items are more than one in number, and thus expand the inventory control requirements for the organisation.

The adaptability of the firm is influenced by the inventory control productivity it holds. Inappropriate procedures and strategies may prompt uneven and undesirable inventory estimates. A few things may be overstocked, while a few things may face stock-out conditions. In the long run, the benefit of the firm is influenced in light of the fact that inefficient inventory control in the firm will expand the venture levels for maintaining unreasonable inventory or expanded requesting costs, if there should be an occurrence of lower inventory levels with operational bargains. The impact of inventory control and the dimension of venture required are opposite sides of a similar coin. Assortments of measure, for example, ABC examination alongside obsession of standards for inventory holding and reorder costs, helps in legitimate inventory control (Marthur, 1994). Furthermore, Zinn and Charnes (2005) stated that the principal goal of inventory control is to give a high stream of good quality, significant and imperative data/material that will empower retailers/suppliers to provide non-stop and opportune support to end purchasers. However, sudden and disruptive issues, such as stock outs, make inventory management pointless, yet Meng (2006) suggested that such various causes may result in the "Bullwhip Effect".

The next issue in inventory control is material administration. Federgruen and Zipkin (1984) assumed that elements of materials incorporate procurement, warehousing, planning, inventory control of crude materials, and transportation. There are four essential exercises related to material administration: envisioning the necessity of materials, containing and sourcing of materials, condition of materials in the organisation, and finally, the status of materials which need persistent observation. Bowersox et al. (2002) highlighted the next vital inventory control issue, which is the procurement of materials. To help the operational exercises, the producer, distributer, retailer and so forth purchase materials from outside suppliers. From the inventory executives' point of view, procurement fundamentally places focus on connections among purchasers and merchants at key dimensions with higher association of both. Procurement action is the new viewpoint for overseeing inventories. Another imperative parameter is determining the required demand. Demand conjectures depend on current booked requests, deals history, showcasing activities, and data gathered from customers from time to time. Preferably, the required demand is an inward segment of the firm; however, outer accomplices, including suppliers and customers, are additionally engaged in the procedure of better determining such demand. Criticism from customers likewise holds a vital place for showcasing exercises of the organisations. Bowersox et al. (2002) illustrated that the required demand and demand estimating are firmly connected to each other. According to Atheize (2001), the real area of worry for logistics is the genuine development or the system that moves the item. Methods of transportation utilised in moving the item are accounted for by the strategic administrator. Atheize (2001) contended that an immediate connection exists between transportation and the number of stockrooms required for inventory. Jaillet et al. (1997) distinguished another explanation behind the gathering of inventories or crude materials the transportation economies. Lower per-unit transportation rates will be realised if the firm is prepared to purchase the full load limit of the shipment.

2.3 EOQ Model

In this section, the well-known EOQ model, as an inventory management tool, is presented. In particular, the development of the EOQ model, its assumptions, limitations and extensions are introduced.

2.3.1 Initial Development of the EOQ Model

The number of units to order is an important parameter to know at the time of every supply decision in every company. In view of this significance, the model of EOQ has gained importance over the past century. The initial publication on this topic is found in 1913 by Ford W. Harris. The model was based on assumptions of the optimal size of the order to minimise the number of orders, and achieve maximum cost saving. The restriction of holding cost and trade-off with total ordering size are important measures to consider in any inventory control. According to Lucey (1992) and Schroeder (2000), EOQ is defined as the optimal order quantity that balances the minimum inventory holding cost with the cost of reordering. However, the strength of the EOQ model has been accepted in century-long research, and established a basis for the inventory control procedures at later stages of development. Nearly thousands of researches were directly or indirectly based on the EOQ model, and used its assumptions to form decision-making models in different situations.

The aim of the actual Harris (1913) EOQ model is to provide managers with guidelines to order optimal quantities from suppliers. The determination of production quantity is the basis of Harris' (1913) research titled ''How Many Parts to Make at Once''. However, the actual applicability of the model is widely accepted in the batch processing manufacturing models, which require the presence of all materials at the time of processing the production all at once. Therefore, an extension of the EOQ model was developed, the model of Economic

Production Quantity (EPQ), which governs the optimum batch size. Taft (1918) suggested that it is suitable for producing one type of product, one-at-a-time, in which the first formed units are meant for customers' requirements, and the rest are produced to store for the later needs of customers.

2.3.2 History and Evolution of the EOQ Model

As mentioned earlier, relaxing one or more of the EOQ model's assumptions can improve the model's performance and help companies to reduce their inventory costs. Thus, several research studies were devoted to examining which of these assumptions to violate, how to modify the model accordingly, and the results of such violation on the companies' inventory costs. Starting from the late 1950s, such research studies began to gain momentum. The earliest study in the field was the one conducted by Vazsonyi (1957). In this study, it was observed that utilising the EOQ as the economic lot size had led to some scheduling errors. Hence, a nonlinear mathematical programming technique that takes into account the time spent and the productivity of labour on each machine was proposed to mitigate these scheduling errors. This technique used a well-ordered procedure that calculated the order quantities under the previously-mentioned conditions. In the next decade, Crowther (1964) derived a formula to calculate cost reductions from the seller and buyer perspectives by using the EOQ model for the required quantity to be bought in order to receive a discount. At the same time, several other studies aimed at modifying the EOQ after comparing its performance against other inventory control methods. For instance, Schussel (1968) developed a model called Economic Lot Release Size (ELRS) to determine the ideal lot sizes. This model starts at the lowest level of inventory and moves up to the final product; at such a point, the lot size and the total cost are calculated. This process is then repeated with the use of a developed algorithm, until two consecutive values of the above parameters fall within a pre-specified range. Other similar studies include Kaimann (1968), Philips and Dawson (1968), and Hoffmann (1969), who pointed out the problems in the EOQ's model formulation, and used a dynamic programming algorithm, as well as Bayesian statistics, to increase the accuracy of the calculation of order quantities and reorder points.

From the beginning of the 1970s, the EOQ research evolved into extending the model capabilities to calculate several other parameters. Lippman (1971) extended the original EOQ model to include several setup costs, such as transportation costs. Moreover, Moore (1971) extended the model to improve the forecasting of replacement items; Trippi and

Lewin (1974) included the present value of the discounted costs over an infinite time horizon to minimise the holding and ordering costs. Langley (1976) derived three possible values from the EOQ model, namely optimistic, pessimistic, and most likely values, by utilising the maximax, maximin, and minimax regret strategies and the Laplace criterion, while Ram Mohan (1978) included the working capital in the EOQ model formulation. Finally, Liberatore (1979) developed a stochastic lead-time generalisation of the EOQ model with demand backlogging under the non-interchangeable parts assumption. The developed model defines the expected total cost as a function of the constant demand rate, the number of time units of demand satisfied by each order, the time differential between the placing of an order and satisfying it, and the lead-time probability density function.

Another type of research, which flourished during this decade, concerns the relationship between Materials Requirement Planning (MRP) and the EOQ model. Chamberlain (1977) argued that MRP is the best model for inventory control management, and disregarded the use of the EOQ. Yelle (1978a; 1978b) examined how the EOQ lot sizing rule compares to that of the MRP in the context of a multi-level lot sizing challenge, and suggested the use of uneven and expanding lot sizing sequences to minimise the inventory costs. Furthermore, Kropp et al. (1979) addressed the MRP's sensitivity issue by utilising the EOQ model, since it is not influenced by any imprecisions in the demand and cost estimates. This field of research continued to gain momentum during the 1980s under different contexts. Choi et al. (1984) compared the EOQ's performance with that of the different MRP systems and observed that the EOQ was the lesser performer among the MRP systems. On the contrary, Rubin et al. (1983) compared the EOQ approach to the Total Setup Lot Sizing (T-S) model developed by Kuzdrall and Britney (1982) in the case of quantity discounts, and found that a modified EOQ had a better performance than the T-S method; similarly, Boucher (1984) found that a modified EOQ was better in the context of group technology systems. Other comparisons were made by Drake et al. (1986) and Patterson and LaForge (1985), with a fixed charge heuristic and incremental part-period algorithm, respectively. Lastly, Williams et al. (1985) compared the performance of the EOQ under conditions of serially correlated demand sequences, while Ritchie and Tsado (1986) compared this performance under the linear increasing demand.

Several extensions to the EOQ model were developed. These extensions included: accounting for a temporary one-time price discount (Tersine and Price, 1981), adding nonlinear holding costs (Weiss, 1982), considering the discounting rates (Gurnani, 1983;

Clarke, 1987), adding multiple setup costs (Aucamp, 1984), and including constant inflation and simple interest (Kanet and Miles, 1985). Furthermore, Bigham (1986) modelled ordering costs as an increasing step function. Lastly, Replogle (1988) developed a modified EOQ model that accounts for the impact of reducing setup costs through learning. Other models examined how variation in the sizes of production loads impacted inventory costs that were derived from the EOQ, while taking into account the impact of overtime use whenever the regular capacity is exceeded (Axsater, 1980; 1981; Goyal and Evans, 1981). At the same time, other research studies extended the use of the EOQ model to other areas, such as the impact of these models on deteriorating items, which was accomplished by considering the changes in products' shortages and deterioration rates under both deterministic and probabilistic demand conditions (Elsayed and Teresi, 1983). Das (1984) suggested that changing price rates, supply conditions and expansion issues found in the developing countries recommended modifications to the traditional EOQ.

During the 1990s, developing extensions to the EOQ model remained the most researched area; some examples of these extensions follow. Lev and Weiss (1990) relaxed the infinite horizon and the static costs to develop a model that incorporates both finite and infinite horizons with changes in cost. Cheng (1990) integrated the product pricing and order sizing to maximise profits given the storage space and inventory investment constraints. Carlson et al. (1996) extended the EOQ model to include quantity discounts when all costs are incurred on different dates, i.e. date-term credit, using the discounted cash flow method. Through this model, the researchers found that the optimal order quantity might not occur at a breakpoint in the discount schedule under all-units discounts, while under incremental quantity discounts, the optimal order quantity might fall at a price break in the schedule. Other extensions offered by research studies include: applying the model to retail cycle stock inventories (Bassin, 1990), and accounting for conditions of a provisional sale for a buyer (Chen and Min, 1995).

Finally, in the new millennium other EOQ research has been published, such as extending the model for imperfect quality items (Salameh and Jaber, 2000; Maddah and Jaber, 2008; Khan et al., 2010; Hsu and Hsu, 2013). Moreover, interest in models that handle partial backordering increased, as reflected by the cases of Zhang et al. (2011), Toews et al. (2011), Chung and Cárdenas-Barrón (2012), and Taleizadeh et al. (2013).

2.3.3 Assumptions of EOQ Models

The major assumptions of the EOQ model determine the constraints for demand, time, availability and unit costs. The notation of the equations followed in this model is generated with the help of these constraints, as explained below:

- **Constant demand (D)**: The constant, linear and deterministic demand is assumed for items in the EOQ over a certain time period. Sometimes period demand variability, such as seasonal changes in demand, affect the applicability of the model. The argument of Silver et al. (1998) holds that a small variation in demand over a constant interval of time is considered fit for applicability, as demand is near linear. However, the major changes over less time need a model that holds for variations, for instance, the use of the Wagner–Whitin algorithm in a model.
- **The unit cost (p):** This unit cost should stay uniform over a certain period of time and consider transactions at a fixed price. Long-term contracts with suppliers to gain raw materials or goods at a certain price can help in fulfilling this assumption. However, changes in exchange rates and the economic conditions of the country significantly affect this price constraint.
- **Lead times (L):** Constant and known lead time is another important constraint for receiving orders. For close and large suppliers, this assumption is valid, as large quantities can be delivered on time, but for far away suppliers or seasonal products, the supply cannot remain constant with time. Therefore, the variable lead times result in lower applicability of the EOQ model.
- **Order size (S):** The cost of the firm’s ordering is considered to be self-regulating and self-regulating with the size of the order quantity.
- **Cost of order holding (r):** The company's yearly holding cost rate is settled and independent of the measure of the quantity ordered. Subsequently, the expense of holding a unit in inventory for a whole period (indicated by H) can be computed as per Equation 2-1:

  $$H = r * p \tag{2-1}$$

  where r is the cost of order holding, and p is the unit cost.

- **No financial or capacity limitations apply for the firm or its supplier.** This is particularly applicable for make-to-stock products where accessibility is prompt in a supplier's dispersion focus. It is additionally applicable for cheap things for which the firm has adequate money stores to pay for orders (Teng, 2009).

The firm looks to determine the optimal order quantity (Q*) in light of these assumptions, which minimises its aggregate yearly inventory costs. The total amount of money that the firm should pay to the supplier (p*D) for the periodic supply of the inventory is not related to the order quantity, since there are no quantity limits in the original model. In a similar way, stock-out expenses are not critical in light of the fact that the firm is expected to fulfil the majority of the interest when it happens. Yearly ordering expenses and the yearly holding costs are the two remaining components of the aggregate yearly applicable expense. We can set up the following capacity representing the aggregate yearly pertinent expense as an element of any order quantity, Q, as shown in Equation 2-2:

$$TARC\ (Q) = (\frac{S * D}{Q}) + (\frac{H * Q}{2}) \qquad 2\text{-}2$$

The main term above represents the yearly ordering expense by multiplying the expense per order, S, by the quantity of orders per year, D/Q. The second term is the yearly holding cost, which is the product of the expense of holding a unit in inventory for a year, H, and the average inventory level, Q/2. The holding cost is applied based on the normal inventory level, on the grounds that the quantity of units in inventory is always changing: a few units spend a significant amount of time in inventory, while others spend just a little time.

The second subsidiary is positive for all estimations of Q, i.e. the aggregate yearly important cost work above is raised; along these lines, the expense minimising estimation of Q can be found by setting the first derivative of Equation 2-2 equal to zero and solving for Q*. While Harris (1913) utilised this math-based approach to determine the optimal quantity, succeeding specialists have utilised different techniques to obtain the equivalent optimal arrangement (Minner, 2007). Every one of these arrangement approaches yields the following optimal order quantity (Equation 2-3), which is regularly mentioned as the EOQ.

$$Q^* = \sqrt{\frac{2DS}{H}} \qquad 2\text{-}3$$

Thus, the EOQ model is considered to be a crucial part of the historical backdrop of operations reviewed, since it is one of the principal published applications of a scientific model in business basic leadership. The original EOQ model is still significant on the grounds that it is still generally utilised in practice. Despite everything, due to its strength and simplicity, it functions admirably in practice, although some organisations utilise this model incorrectly in circumstances where it is not the best practical arrangement (Drake and Ptak, 1988).

2.3.4 EOQ Model Implementation

The limiting nature of the early assumptions of the EOQ model make the model applicable to few items in real business practice. The criticism from Woolsey (1988) shows that the parameters set in this EOQ model design are very difficult to use, and shows the inappropriateness of the model to all products. For instance, the inaccurate modelling of the holding cost, which does not consider obsoletion or deterioration, makes it essential to estimate this cost from previous performance data of inventory control. The study of Woolsey suggested the use of Uniform Order Quantity (UOQ) for the determination of best order size that matches the whole supply chain. Moreover, in other situations in which the performance of the EOQ model is argued by the researchers to be effective, this performance was only possible after ignoring or violating certain conditions to fulfil the technical requirements.

The application of the EOQ model is still found in abundance, as compared to the newly formed complex models. This is based on the reason that the costs incurred in the application of EOQ in real business situations are lower compared to the amplified costs incurred while applying complex models. According to Fulbright (1979), the EOQ model is actually robust with regard to errors during estimation of the cost parameters, as opposed to the criticism raised by Woolsey (1988). The cause of this discrepancy is the relative difference in the importance and preferences of any item kept in the company inventory. Many items are unimportant in view of the competition faced by the company, and do not require the application of complex models to estimate the cost of inventory, despite the significant cost savings generated by the latest complex models of inventory control. The vast majority of material required on a regular basis in the production process is insignificant. However, for important and preferred products, more complex models should be used in place of the EOQ model.

The associated risk is discussed in the study of Wilson (1977). Regarding restrictions on production capacity of batch size, obsolescence risk, and lack of coordination in production scheduling, the importance of the EOQ model is significant compared to the complex models, but needs adjustments to manage constraints in real business situations. Thus, Fuller (2003) expressed that the application of the EOQ model shows that EOQ is the Order

Quantity for minimisation of carrying costs and ordering costs. However, Morrison and Jossep (1994) discussed the application of EOQ as the means of minimising the acquisition cost and the whole annual cost. Nevertheless, the argument of Sallem (2003) is based on the fact that the obsolescence of inventory storage and spoilage costs makes it inefficient to store inventory for a long time. Therefore, time is another important factor to consider when appraising the EOQ model. Similarly, Cannon and Crandall (2004) declared that the recent complex models share the same goals as the EOQ model: to balance the ordering and inventory holding costs. The research of Wilson (1977) illustrated that recent models, as well as EOQ, assist in providing a better explanation of inventory costs trade-off.

Regarding the universal applicability of the EOQ model besides the shortfall of rigid conditions, Cannon and Crandall (2004) provided scenarios in which EOQ can be widely or sporadically used: the model is found ultimately useful in establishing inventory cost estimates for make-to-stock items carrying uniform demand with stable holding and ordering costs for a long period of time. However, the customization of the EOQ model was suggested in the case of variable quantity discounts involvement and variable shipments of a single order made by suppliers; thus, the production and shipment constraints affect the inventory's holding and ordering costs.

In conclusion, the application of EOQ in the industry for over a century is evident, but modifications and variations in the constraints are implemented by the researchers. The customization of the model with respect to the conditions of the business is also found in the literature. Moreover, the researchers also expressed concerns over the performance of the adapted model of EOQ over the original developed model of Harris (1913).

2.3.5 Types of Inventory Models

Across the history of EOQ model research, several studies extended the model by relaxing one or more of its original assumptions. In this section, studies that extended the model based on demand as a function of price, demand/supply as a function of time and deterioration rates, lead time, storage space, rebate offered, product quality, and service quality are examined in detail. After examining these different models, a background on how the EOQ model can be extended to minimise costs and maximise profits is presented, which will help in revealing the various methodologies that can be used to develop the subject model of this research study.

2.3.5.1 Inventory Models with Various Price-Dependent Demand Rate

One of the first areas in which several extensions of the EOQ model were developed is the area of demand for a product changing with the price of that product. In these models, the relationship between the price and consumer demand, and the optimal pricing strategy for the firms were explored. Ladany and Sternlieb (1974) conducted one of the first studies that extended the EOQ model to include demand rates that vary according to the selling price of the product. In their model, they took into consideration the interaction between the EOQ and different pricing policies to determine the order quantity that maximises the net profit for the company. Before developing the model, several assumptions were made, including the assumptions that the demand has a deterministic rate that depends on the selling price; that the demand depends on a demand curve that has uniform elasticity; that the supply is ordered in a single batch; that the selling price is based on a fixed mark-up; and that the unit cost decreases either linearly or hyperbolically. Through this model, the computed net profit was greater than that computed with the original EOQ model.

Ray, Gerchak and Jewkes (2005) conducted a more recent study that examined the relationship between the demand for a product and its selling price and incorporated this relationship in the EOQ model. In this study, the researchers considered a case of a firm selling a single product based on mark-up pricing and developed an extension to the EOQ model. Moreover, linear and log-linear demand functions were considered in this study. Through this model, it was proved that, from a profit point of view, for highly price-sensitive customers with non-linear demand, managers should not reduce the price too much and try to be aggressive. In addition, if managers did not determine the optimal batch size, the impact of such an action on the profits is not significant in a model where demand varies according to the selling price, except for cases when the setup cost is high, or the demand is not linear.

The following research studies extended the EOQ model by investigating demand as a function of the selling price, while assuming the demand function to be deterministic: Fibich et al. (2003) and Chou and Parlar (2006), who used a linear deterministic demand function; Jeuland and Shugan (1988) and Agrawal and Ferguson (2007), who used a power deterministic demand function; Hanssens and Parsons (1993) and Song et al. (2008), who expressed the deterministic demand function in an exponential manner; Chen et al. (2006), who used a logarithmic deterministic demand function; and Chen and Simchi-Levi (2012), who used a logit-based deterministic demand function.

All the above models assumed the demand function to be a deterministic one; however, a number of research studies extended the EOQ model further, beyond this assumption, to include demand functions that have random probability distribution functions (Huang, Leng and Parlar, 2013). Federgruen and Heching (1999) extended the model to include demand distribution that depends on the item's price. The assumptions for this model included: the price is a function of the state of the system, a replacement order can be placed at the beginning of a period, and the inventory stock is fully backlogged. Furthermore, Petruzzi and Dada (1999) incorporated the relationship between demand and the selling price in an EOQ model for a newsvendor business with robust results. In this model, the researchers examined various forms of uncertainty, namely additive, multiplicative and hybrid. Through their case study, it was found that if the uncertainty is in an additive form, then the optimal price will not be higher than the one obtained from the deterministic model; nevertheless, if the uncertainty is in a multiplicative form, the optimal price will not be lower than the one obtained from the deterministic model. Moreover, the developed model showed that a single-period model can be perfectly applied to a multiple-period problem, which increases the usefulness and applicability of the developed model.

Chen and Simchi-Levi's (2004) model is another that treated the demand function as stochastic. In this study, demand is assumed to be a random variable, with distributions depending on the product's selling price and, similar to previous studies, pricing and ordering decisions are taken at the beginning of the period, all shortages are backlogged, and the ordering cost encompasses both fixed and variable costs. Through the developed model, the researchers demonstrated that, when the demand model is additive, the optimal policy will be the one in which the inventory is managed based on the policy stating that an order is only placed for more inventory when the inventory level at the beginning of the period is below a specific reorder point. Moreover, the price in this case is best determined based on the inventory position at the beginning of the period. On the other hand, when the demand model is multiplicative plus additive, the optimal policy will be one in which the order level of the desired level of inventory minus the current level is made when the inventory level at the beginning of the period is less than a specific reorder level. Finally, other stochastic demand models present in the literature include those developed by Kocabıyıkoglu and Popescu (2011), Phillips (2005), and Agrawal and Ferguson (2007).

Other demand models are named "willingness to pay demand function"; these models were based on the idea that consumers have a heterogeneous willingness to buy a product from a firm at a specific selling price that is less than the maximum price they are willing to pay for that product (Huang et al., 2013). One of these models was developed by Kalish (1985). The developed model incorporated the uncertainty arising from feedback on experience of the product by reducing its value accordingly.

Furthermore, there are Poisson flow models; these models incorporate customers' buying process and any changes in their price preferences in terms of being willing to pay for the product. Zhao and Zheng (2000) developed a model based on the theory that customers with a changing reservation price distribution over time arrive according to a non-homogeneous Poisson process, and that the probability function of this distribution changes over time. When applying this developed model on a numerical case study and comparing its performance against conventional models, a revenue improvement of 2.4–7.3% was achieved. The developed model, represented by the solid black curve, yields higher return percentage than the original models, for every respective number of items. Other studies that modelled the demand function as a Poisson flow function include Bitran and Mondschein (1997), and Xu and Hopp (2009).

All the above models assumed that there is a single firm selling that given product, whereas in real life the presence of competitors selling similar products is the norm. Hence, several models were developed where demand is dependent on the selling price in a competitive multi-firm environment. Some of these models that account for competition are the ones developed by Anderson et al. (1992), Singh and Vives (1984), and Vives (1999), in which demand follows a linear function, and both the impact of the product's price and the impact of the price of the competitors' product are accounted for, although the magnitude of the former is higher.

As seen from the above review of the various price-dependent models, the procedure and assumptions for the model development differ according to the nature of the demand function, as well as the competitive environment. In addition, from the wide array of demand functions used, several advantages and disadvantages of each model can be deduced, which help when selecting the appropriate function for the model to be developed in this research study.

2.3.5.2 Inventory Models with Changing Time-Dependent Demand Rate

In addition to demand rates varying with the selling price of the product, rates varying with time have also been explored in previous literature, and extended models have been developed accordingly. The main driver behind such models is the case of goods with finite shelf lives, resulting in the goods' loss or deterioration, which is defined by Shah and Shukla (2009) as the decay, spoilage, or loss of utility of the product. Some examples of such products include fish, medicine, vegetables and airline tickets, whose lifespan starts to shorten as soon as they are produced. This large array of products with finite lifespan increased the importance of the effect of deterioration and perishability in many inventory systems; consequently, many research studies started extending the conventional EOQ model to include the impact of time on the demand for the product. The earliest models in this domain were developed by Resh, Friedman and Barbosa (1976) and Donaldson (1977), who considered an inventory model with linear demand over time. In the former study, the researchers extended the EOQ model to include deterministic demand that starts at the origin and linearly increases over time. Before formulating their model, the researchers made three assumptions on which the model was based. First, the inventory replacement orders are made promptly based on the number of inventory items required; second, the researchers used a determinate and well-defined planning horizon for the model; and third, replenishment, carrying and shortage costs are all included in the formulated model. Hence, a model is derived, which determines the optimal schedule of replacement inventory to minimise the costs when the inventory level becomes zero. Through the study, the researchers proved that for a given number of required replacements "m", there is a unique vector of "m" time intervals that minimise the total cost. They then developed an algorithm that determines the unique optimal value of "m", and the unique optimal scheduling of replacements for "m" using the derived mathematical formula. Moreover, the scope of the formulated model was further expanded by the researchers with the inclusion of a product that has an increasing rate of demand, while, simultaneously, its market is diminishing. Donaldson (1977) used the replacement cycle and the cycle time rather than the replacement quantity to derive the demand using dynamic programming methods.

Another important study that considered a linear demand model varying with time is the study by Bose, Goswami and Chaudhuri (1995). However, unlike the previous models, this one allowed for shortages and backlogging, while also including the effects of inflation and time-value of money. The assumptions on which this model was based include: a constant deterioration rate over time, an infinite replenishment rate, and a finite time-horizon with a number of reorder points. In addition, this comprehensive model also considered three different types of costs, namely production cost, carrying cost and shortage cost. For the production cost, it is assumed that the total cost increases as a result of the internal inflation rate, while the unit purchase price rises due to the external inflation rate. Furthermore, the carrying cost type consists of opportunity and out-of-pocket costs that are not related to the operations, such as insurance, taxes and storage. The model is then developed and can be solved by any iterative method. When applying the model to two numerical cases, a case that allowed shortages and one that did not, it was found that both the reorder number and the system cost increase significantly in the no-shortage case; however, the scheduling period is longer when shortage is allowed. Finally, the researchers conducted a sensitivity analysis regarding the degree by which the reorder number and the optimum cost are affected by several independent variables. This is shown in Table 2-1 below:

Table 2-1. Sensitivity analysis for the reorder number and the optimum cost (Bose et al., 1995).

Model Outcome	Highly Sensitive	Moderately Sensitive	Not Sensitive
Reorder Number	Finite Time Horizon	Replacement Cost	- Demand Rate - Purchase Cost - Fraction of inventory that deteriorates over time - External and Internal Holding Costs - External and Internal Shortage Costs - Internal and External Inflation Rates
Optimum Cost	- Purchase Cost - Finite Time Horizon - External Inflation Rate	Demand Rate	- Replacement Cost - Fraction of inventory that deteriorates over time - External and Internal Holding Costs - External and Internal Shortage Costs - Internal Inflation Rate

Giri, Goswami and Chaudhuri (1996) developed another model, in which the demand rate, deterioration rate, holding and ordering costs are assumed to be continuous functions of time. The main assumptions behind this model are: the inventory system is for a single item; shortages are allowed and are completely backlogged; a finite planning horizon, no repair or maintenance of deteriorating items are allowed; the replenishment periods are constant; and the lead time is zero. The model is used to derive the replacement rule which minimises the total costs. Similar to the previous study, it was found that the system cost and reorder number increase considerably in the no-shortage case. Finally, through a sensitivity analysis, it was shown that the developed model is highly sensitive to any modifications in the time-horizon, while any inaccuracies in the demand rate and shortage cost have only a slight impact on the model, and changes in the cost per unit and the fraction of inventory that deteriorates over time do not affect it at all. Dave and Patel (1981) developed another model that assumed that demand changes linearly with deteriorating time. In their model, the planning horizon is finite; the replenishment rate is infinite with no lead time; shortages are not allowed; the unit cost, holding cost and replacement cost are all constant; and a constant fraction of the inventory on hand deteriorates over time. Through a numerical example, the researchers demonstrated that this model leads to a reduction in the total annual cost and number of deteriorating units per year. Moreover, through the sensitivity analysis, several conclusions can be made. First, the optimal value of the number of replenishments increases with the increase in the time horizon and the fraction of the inventory that deteriorates over time. This number does not change in response to a change in any other parameter. Second, the optimal value of the scheduling period increases with a decrease in the fraction of the inventory that deteriorates over time. Third, the cost increases with a decrease in the time horizon and the fraction of the inventory that deteriorates over time.

Another set of models aimed at extending the EOQ model when demand varies with the deteriorating time exponentially. One of these models is the Hariga and Benkherouf model (1994). The formulated model in this study was based on a number of assumptions: replacements take place instantly and at an infinite rate; the deterioration rate is constant; there is only one product's item kept in stock over one year; no shortages in the number of items are permitted; and the costs include a constant ordering cost, and holding and deterioration costs per unit. Through this model, six heuristic procedures were developed and compared based on the percentage of cost deviation from optimality, measured as the percentage increase in the total cost above the optimal cost value, and the computational time needed. From this study, several conclusions were made, as follows: the average cost performance of equal replenishment cycle procedure is worsened as the time horizon increases, with no clear trend for the other heuristics; the extended Silver-Meal procedure performs poorly, in terms of cost, in declining markets; and the average percentage of cost deviation for the extended least-cost approach and the extended least-unit cost procedure improves as the fraction of deteriorating goods increases. However, no clear impact of this parameter can be observed on the cost performance of the other heuristics, and for large ordering cost, the average cost performance worsens for the equal replenishment interval procedure and the extended Silver-Meal procedure, while the performance of the extended least-unit cost procedure is improved as the ordering cost increases.

Based on the above, it can be concluded that according to the cost performance measure, the extended least-cost approach performed best, followed by the extended least-unit cost procedure. On the other hand, the worst performing heuristic is the constant demand approximation, followed by the equal replenishment interval procedure, then the extended Silver-Meal procedure and the constant demand approximation procedure. Finally, Mirzazadeh (2010) took this model extension a step further, and modelled the demand as a function of the items whose deterioration is based on different practical situations. In general, the model is developed based on stochastic internal and external inflation rates; however, in practice, these rates depend on a number of economic, political, social and cultural variables, such as labour cost, raw materials cost, exchange rates, taxes, liquidity and unemployment rates, among others. The derivation of the model is based on the assumptions that the internal and external inflation rates are random variables with well-defined distributions, shortages are allowed and fully backlogged, the demand rate is known and constant, the replacement is instantaneous with no lead time, the time-horizon is a determinate one, and the fraction of the inventory on hand that perishes over time is constant. The solution of the model computes the number of needed replenishments and the percentage of time in which the inventory cycle can be filled from the existing inventory (k). Finally, the sensitivity analysis revealed the following observations:

1- When the internal inflation rate increases, the number of replacements decreases and “k” increases.
2- When the external inflation rate increases, the number of replenishments and “k” increase.
3- When the mean values of the normal distribution function of the inflation rates increase, the optimal expected present value of the cost increases.
4- The changes in the standard deviation of the inflation rates do not affect either the number of replenishments or “k”.
5- The number of replenishments is highly sensitive to changes in the demand rate per unit time, the ordering cost, and the fixed time horizon.

6- The number of replenishments is slightly sensitive to the internal shortage cost, and insensitive to the internal and external holding costs and the external shortage cost.
7- The optimal value of “k” is highly sensitive to changes in internal and external holding costs and the external shortage cost.
8- The overall system cost is highly sensitive to changes in the demand rate per unit time, the purchase cost, and the fixed time horizon, while it is insensitive to the other parameters of the model.

In conclusion, the extensions of the EOQ model based on time-dependent demand differed according to the nature of the demand function adopted by the different studies. However, all the above models were only concerned with the scenario in which competition is non-existent, which reflects a limitation of such models. Nevertheless, several insights were gained on the influence of certain assumptions on the model formulation, which help in the process of developing the model, the subject of this research study.

2.3.5.3 Inventory Models with Various Lead-Time-Dependent Demand Rate

Besides the change of demand with the deterioration time, another time parameter that impacts demand is the lead time, which is the time interval between the placement and the receipt of the order by the customer (Albana, Frein and Hammami 2017), because timely customer service is an important competitive advantage for firms. Hence, besides the selling price, firms should also be focusing on optimising its delivery time, because, on the one hand, customers consider this parameter when they make their purchasing decisions; and on the other hand, sometimes customers are willing to pay a premium for fast and reliable delivery. This emphasises the relationship between consumer demand and lead time, which highlights the importance of modelling this relationship.

When extending the EOQ model to include the change in demand based on lead time, these developed models can be sorted into two main categories: single firm models and competition models (Huang et al., 2013). With regard to the former, different types of models were used, including linear, Cobb–Douglas, Multi-nominal Logit, and willingness to pay models (Huang et al., 2013). The pioneer study developed by Palaka, Erlebacher, and Kropp (1998) is an example of a linear model. In this model, the researchers modelled the firm’s operations as a simple M/M/1 queue, which follows Kendall’s notation of the arrival process/the service time distribution/the number of servers. Hence, in an M/M/1 queue, there is a single server, the arrival distribution of the customers follows Poisson distribution, and the distribution of the service time follows an exponential distribution, and maximises the revenues minus the total variable production costs, holding cost and late penalty costs by utilising the quoted lead time, capacity utilisation and price. Through this model, it was shown that capacity utilisation should be lowered when customers are more sensitive to lead times; the firm incurs higher congestion-related costs, and the late penalty is higher. Moreover, through a conducted sensitivity analysis, the following conclusions were made:

1- As lead-time sensitivity decreases, the price drop becomes higher. This can be explained by the fact that when the firm's service level exceeds the minimum requirement, it tends to reduce the quoted lead times instead of reducing prices.
2- Any increase in the unit holding cost will lower both the optimal arrival rate and the optimal cited lead time.
3- When the industry service level is small and non-binding, the increase in the industry service level has no impact on the optimal price. On the other hand, when the service level constraint becomes binding, the optimal price decreases at an increasing rate as the industry standard service level increases.
4- Errors in parameter estimation that result in higher than optimal demand levels lead to an increase in costs and vice versa. At the same time, the effect on the firm’s revenues depends on the elasticity of both the price and the lead time.
5- When the unit holding cost is underestimated there is a larger drop in optimal profit than when it is overestimated.
6- When the service level constraint is non-binding, underestimating the penalty cost can cause profits to decrease at an increasing rate.
7- Finally, the sensitivity to errors in estimating the industry standard service level is dependent on the service level constraint.

Another extension of the model was conducted by Albana et al. (2017), who extended the model to include a client rejection policy when the number of customers (K) present in the system is sufficient. Hence, the researchers modelled the firm’s operation as an M/M/1/K queuing system, which is a finite queuing system where K is the number of customers that the system can accommodate; although, for simplicity, K was assumed to be equal to 1. Moreover, the assumptions behind this model included a constant capacity, the customers’ arrival process being a Poisson process with customers served on a first-come first-served basis, and the customers’ processing times being exponentially distributed.

Furthermore, the objective of this model is to maximise the firm’s net revenues, which entails maximising its expected revenues while minimising its total congestion and lateness penalty costs. In addition, in order to assess if such a queuing system is more profitable than the simple $M/M/1$ system, a comparison was conducted between these two systems under two scenarios: an $M/M/1/1$ without penalty and holding costs, and an $M/M/1/1$ with penalty and holding costs. Through this comparison, the following observations were deduced (Albana et al., 2017):

1- Having a rejection policy can be more profitable at low market potential and high lead-time sensitivity, even when there are no penalty and holding costs.
2- When the penalty and holding costs are included, there are more cases in which a rejection policy becomes more profitable, because the lead time gets longer when we accept all clients, which results in high congestion costs.

Another type of lead-time developed models uses a multi-nominal logit function to depict the relationship between this parameter and the demand rate. Ho and Zheng's (2004) model is an example of such functions. In this model, the researchers assumed that the firm's customers are sensitive to lead time; hence, the objective was to maximise the demand rate while fulfilling the customers' lead-time expectations.

Regarding the willingness to pay demand functions, Zhao, Stecke and Prasad (2012) extended the EOQ model to investigate the impact of variable lead time on the firms' profitability. The researchers achieved this objective by comparing two different strategies that are commonly used by firms. These strategies are Uniform Quotation Mode (UQM), where only a single price with a corresponding lead time is offered to customers, and Differentiated Quotation Mode (DQM), where different combinations of prices and lead times are offered to customers. In this study, two models were developed for lead-time- and price-sensitive customers. In these models, a Poisson process was used to model the customers' arrival processes, while the production system was modelled as an M/M/1 queue system. Furthermore, with each model, three marketing scenarios were considered, namely lead-time-sensitive focus, price-sensitive focus, and no focus; the model included both the production and capacity costs.

In a competition setting, i.e. multi-firm models, Pekgun, Griffin and Keskinocak (2017) developed a model where two firms compete on both price and lead time, with the objective of examining the impact of decentralisation on the firms' profitability. There were two categories of assumptions behind the developed model: first, the general model assumptions, including fixed capacity, M/M/1 queuing system, and linear demand function;

and second, multi-firm assumptions, including all parameters being known to both firms, a unit increase or decrease in a firm's price or lead time having a stronger impact on its own demand than that of the competitor, and if one of the firms can increase its demand when it sells at cost with the shortest lead time, then the other firm can achieve similar results. Based on these assumptions, two models were developed, one for centralised decisions and one for decentralised decisions, and their generated profits were compared against each other. From this comparison, several observations were made. The first of these observations is that, for identical firms, if both firms follow a decentralised strategy, the lead time becomes longer, the price lower, and the demand higher than if both firms follow a centralised strategy. On the other hand, if one of the firms adopts a centralised strategy while the other adopts a decentralised one, then the centralised firm quotes higher prices and lower lead times, resulting in lower demand. Furthermore, the relationship between the intensity of competition in price and lead time plays an important role in selecting the strategy that generates more profit.

Unlike the EOQ extensions that were based on time-dependent demand, some lead-time-dependent models considered a multi-firm competitive environment. This provided insights into how the various models are developed and the assumptions behind them, which can be utilised when developing the subject model in this study.

2.3.5.4 Inventory Models for Limited Space Storage Area

In addition to price- and time-dependent models, several extensions for the EOQ model were developed to account for the limited storage and/or display space for large-scale items, as this parameter might influence demand (Huang et al., 2013). Huang, Lai and Shyu (2007) incorporated both a limited storage scenario and partial permissible delay in payments, and extended the EOQ model to deduce the optimal retailer's lot-sizing policy while minimising the cost. In their model, it is assumed that the retailer will rent a second warehouse when he runs out of storage space in order to store excess items. Further assumptions for the model include: constant demand, infinite time horizon, no shortages are allowed, and partial payment is made when the order is placed, then the remaining balance is paid at the end of the credit period. After developing the model and applying it to a numerical example, the following conclusions were made. First, for a fixed fraction of the delay payments permitted by the supplier per order (α) and a fixed unit stock holding cost of the rented warehouse per year (k), increasing the storage capacity (W) will result in a significant increase in the quantity ordered by the retailer. Second, a similar trend is observed when increasing the value of (α)

while keeping (k) and (W) fixed, while a reverse trend is observed when increasing the value of (k) while keeping (α) and (W) fixed.

A similar model was developed by Yen, Chung and Chen (2012), in which two levels of trade credit were considered, along with own and rented warehouses scenarios, to find the optimal cycle time that minimises the firm's cost. Two levels of trade credit were incorporated in this model: (1) the retailer settles the payments for all units sold and uses the profits for other purposes, and (2) the retailer pays the supplier the amount owed whenever he/she collects money from sales. Moreover, the results of the sensitivity analysis conducted on the developed model are shown in Table 2-2.

Table 2-2. Sensitivity analysis for the developed model (Yen et al., 2012).

Parameter	**Change**	**Impact on Order Quantity (T) and Total Annual Cost of the Retailer (TRC)**
Interest rate earned	Increase	Decrease
Customer trade credit period	Increase	Increase
Interest rate charged	Increase or Decrease	No Impact
Purchasing cost	Increase	No Impact
Holding cost of own or rented warehouse	Increase	Decrease (T); Increase (TRC)
Storage capacity	Increase	Decrease (T); Increase (TRC)
Demand rate	Increase	Decrease (T); Increase (TRC)
Retailer's trade credit period	Increase	No Impact
Ordering cost	Increase	Increase
Selling price	Increase	Decrease

More recently, Sana (2015), considering the need to rent a warehouse when the quantity of items exceeds the retailer's own warehouse storage, took the problem a step further and extended the EOQ model to one in which demand is a random variable. In this study, the average cost functions for three different scenarios order size exceeds the capacity of own warehouse, order size does not exceed the capacity of own warehouse, and absence of own warehouse are derived for both continuous and discrete demand functions. In order to derive these functions, the researcher assumed that replacement size is infinite when not considering the lead time, and shortages due to uncertain demand are permitted and, in that case, lost sales are considered. In this same line, Singha, Buddhakulsomsiri and Parthanadee (2017) attempted to identify the reorder point and the optimum order quantity when there is a shortage in the storage capacity for a single-item inventory, while minimising the total inventory management cost, which consists of ordering, shortage, holding and storage costs. Under this scenario, the retailer must rent a new warehouse to store excess items and is charged on a per-unit basis that is higher than the in-house storage rate. In this model, both inventory policies of continuous and periodic reviews are considered, as well as two types of shortages: lost and backlogged. The developed model assumed stochastic demand and is solved through an iterative method, and optimal solutions are reached via exhaustive search. Finally, a comparison between the two inventory policies revealed that at optimal solutions, both policies replace the order quantities, the reorder point is higher for the periodic review policy, and the periodic review has higher total costs and longer cycle length, despite the fact that the continuous review policy exhibited higher holding and ordering costs. At the same time, a comparison between backlogged and lost shortages revealed that the former has higher replacement order quantities, while the latter has higher reorder point.

Ghosh, Sarkar and Chaudhari (2015) extended the space-dependent EOQ model to include the case of multiple-item inventory. In this model, the researchers assumed that the demand rate is dependent on the item's stock, no shortages are allowed, replacements are instantaneous with no lead time, and all of the holding costs, ordering costs and shortage costs remain constant over time. Through a sensitivity analysis for the developed model, it was found that:

1- When demand increases, the order quantity and the cycle time increase; hence, the total cost increases.
2- When the setup cost increases, the order quantity and the cycle time increase; hence, the total cost increases.
3- When the holding cost increases, the order quantity and the cycle time decrease; however, the total variable cost increases.
4- When the required storage quantity increases, both the order quantity and the cycle time decrease.

Mondal, Garai and Roy (2018) extended the space-dependent demand models to include a space constraint in an intuitionistic fuzzy environment through the application of intuitionistic fuzzy programming. The objective of this single-item model is to minimise the firm's Total Average Cost (TAC) while accounting for both the holding and production costs. Hence, the assumptions behind this model include constant demand rate, instant replacements at an infinite rate, no lead time, and no shortages allowed. Based on these assumptions, an intuitionistic fuzzy model was developed and compared against fuzzy and crisp models. Through this comparison, it was observed that the intuitionistic fuzzy model leads to a lower TAC at higher levels of demand and production quantity. Next, a sensitivity analysis for the three models was conducted to examine the impact of the available storage space on the TAC computed from the three models. Based on this analysis, two observations were made. First, as storage space decreases, the TAC decreases in all three models; second, the optimal TAC is always the lowest in the intuitionistic fuzzy environment.

Finally, some other studies also developed space-dependent models under various scenarios and conditions. For instance, Singh, Khurana and Tayal (2016) developed a model in which the demand is dependent on the shelf space of the item with allowable credit and partial backlogging. Giri and Bardhan (2015) developed a single-vendor single-buyer model for a single product when the demand is dependent on the limited display space available for the retailer. Dordevic et al. (2017) extended the space-constrained EOQ model further to include multiple products, using a meta-heuristic approach under a combinatorial optimisation problem. Farhangi and Mehdizadeh (2016) used a mixed integer and nonlinear programming to formulate a multiple products model. Moreover, Ouyang et al. (2005) incorporated a permissible delay payment period and unit production cost in the space-dependent demand model, in which storage space is limited and the retailer must rent another warehouse. In addition, Mohanty, Kumar and Goswami (2016) extended the two-warehouse scenario to include non-instantaneous deteriorating products in a stochastic framework; and Sekar, Uthayakunar and Mythuradevi (2017) and Tiwari et al. (2017) accounted for inflation when modelling a two-warehouse scenario. Finally, Tiwari et al. (2018) developed a space-dependent inventory model, but instead of renting another warehouse, the retailer stores the extra item in an unlimited capacity backroom.

Here the various space-dependent models and how different scenarios can evolve as a result of limited storage or shelf capacity have been analysed. The objectives of this type of models are similar to the ones devised in this research study:

- Storage space is limited
- The item is perishable
- The objective is to maximise profits.

Nonetheless, although cost minimisation and profit maximisation were targeted by these models, none included optimising the storage space, as they assumed the rental of a second warehouse in which the excess items will be stored.

2.3.5.5 Further Extensions of the EOQ Models based on the Demand-Dependent Parameter

In Sections 2.3.5.1, 2.3.5.2, 2.3.5.3 and 2.3.5.4, EOQ extensions based on the most-examined parameters that impact the consumers' demand have been discussed. However, there are more parameters, whose impact has been studied in literature, but to a lesser extent. The first of these parameters is the presence of a rebate or promotion. The presence of this parameter can direct demand towards a particular product, as consumers will be encouraged to buy this product to receive a reward (Huang et al., 2013). Hence, Pattnaik and Gahan (2018) extended the EOQ model to determine the optimal replacement quantities when promotional efforts are present and reflected in terms of increased demand and promotional cost. Gahan and Pattnaik (2017) developed a fuzzy EOQ model to determine the impact of a promotion policy on optimising the retailer's profit. Furthermore, Pattnaik (2015) incorporated promotional efforts in the developed price-dependent model in which demand declines with price, thus concluding that the presence of a promotion can boost the retailer's profit, especially for deteriorating items. In addition, Soni and Suthar (2018) extended the model with promotion-dependent demand by considering demand to be stochastic. Other stochastic demand models in which demand is dependent on promotional efforts include those developed by Maihami and Karimi (2014), and Roy, Sana and Chaudhuri (2015). Avinadav et al. (2017) divided the demand function into three independent multiplicative components of selling price, products' age and promotion investment. Similarly, Rajan and Uthayakumar (2017) extended the EOQ model by assessing the impacts of the promotional efforts on demand to determine the optimal replacement schedule and order quantity to maximise profits. Finally, Hertini et al. (2018) used Potryagin's Maximal Principle, which is an optimal control principle to find the optimum solutions when variables change over time, to extend the EOQ model to include salesman's initiative, i.e. promotional efforts. De and Sana (2015) developed a promotion-dependent intuitionistic fuzzy EOQ model.

Different types of promotions are also considered in some models. For instance, Yang, Liao and Shi (2015) optimised the order quantity for both a rebate programme and an EDLP, and found that the rebate programme is more effective in encouraging demand when consumers are price-sensitive, while the EDLP is not necessarily better for low price-sensitive consumers. Tsao (2015) found that, to maximise profits, it is better for retailers to use a buyback policy, rather than off-invoice or scan-back policies. Similarly, Chen, Chen and Bidanda (2016) found that a buyback and minimum supply quantity contract yield more profit than an advance payment contract in a decentralised decision model.

Another stream of model extensions related to promotion concerns those that consider a return policy. An example of such models is the one developed by Noori-daryan and Taleizadeh (2018). In their model, two scenarios in a supply chain, consisting of a supplier, manufacturer and wholesaler, are considered for a single item. In the first scenario, there are two return contracts between the outside supplier and the supplier, and the manufacturer and the wholesaler; in the second scenario, the first return contract is the only one applicable. The objective of this model is to maximise the profit for the entire supply chain by optimising the order quantity of the supplier and the selling prices of the manufacturer and wholesaler. Through these models, Noori-daryan and Taleizadeh found that the first scenario yields higher profit than the second one.

In a related type of EOQ models, the promotion efforts specifically consist of advertising efforts, and demand is dependent on these efforts. Bhunia et al. (2015) developed a deterministic inventory model in which demand is dependent on the frequency of advertisements in both electronic and print media. In this model, they also assumed a single deteriorating item in a two-warehouse storage scenario, and the problem was modelled as a mixed integer nonlinear constrained optimisation problem. Moreover, Chanda and Kumar (2016) developed a fuzzy EOQ model for a company that sells technology products, in which demand depends on the advertising cost, and found that the optimal strategy is to hold inventory for a short period and at low cost when it is possible to invest in advertising. Manna, Dey and Mondal (2017) developed an EPQ model in which demand is dependent on advertisement, and increases with time but at a decreasing rate. The objective of this model is to maximise the total profits to deduce the optimal production rate. Furthermore, Hazari et al. (2015) modelled demand as an increasing function with the expansion of the advertisement policy. Based on the bi-fuzzy nature of the selling price, holding cost and advertisement cost, the authors used a bi-fuzzy technique to convert the problem into equivalent crisp problem. Finally, in order to solve the problem with constraints, they resort to the Generalised Reduced Gradient (GRG) method, which is a generalisation of the reduced gradient method allowing nonlinear constraints and arbitrary bounds on the variables, and to the Pontryagin's Maximum Principle (PMP) (Boltyanski et al., 1998), which allows finding the best possible control for taking a dynamical system from one state to another, especially in the presence of constraints for the state or input controls.

Several other advertising-dependent models were developed while taking into consideration a different set of variables. For example, Kumar and Chanda (2017) modelled demand as a hazard rate function; Geetha and Udayakumar (2015) assumed partial backlogging based on the waiting time for the next replacement inventory; Shah and Vaghela (2017) included inflation in their model; Shah, Chaudhari and Jani (2018) assumed demand to be quadratic for non-instantaneous deteriorating items; Rathore (2019) included the process reliability factor; and Gupta, Biswas and Kumar (2018) included the market power structure and quality in their model.

The final category of types of EOQ models to be explored in this review is EOQ models based on quality. There are two types of models that fall under this category, product quality and service quality demand-dependent EOQ models. With regard to the product quality model, the researchers included this parameter in their models due to the fact that improving quality requires R&D, which increases the cost; hence, a trade-off exists between benefit and cost when attempting to determine the optimal quality level which impacts consumers' demand (Huang et al., 2013). An example of such models is the one developed by Maiti and Giri (2015). In their model, demand for the product is linearly dependent on its quality in a directly proportionate manner. In their study, the researchers assumed a closed-loop supply chain, in which a manufacturer sells a product to a retailer, and a third party collects these products from the consumers and sends them back to the manufacturer to recycle them. The model was applied to five different scenarios; it was found that a retailer-led decentralised strategy results in a win-win situation for all the supply chain players. Modak, Panda and Sana (2015) considered a one manufacturer, one supplier scenario, in which demand depends on the product's quality, and tried to maximise the retailer's profit for both the centralised and decentralised strategies. For a similar supply chain structure, Seifbarghy, Nouhi and Mahmoudi (2015) developed a model in which demand is linearly dependent on the quality of the final product to maximise the retailer's profit. Through this model, they found that the entire supply chain's profit is higher under the centralised strategy. Moreover, Liu, Ahang and Tang (2015) devised an inventory model for perishable goods in which the demand depends on the product's quality, which deteriorates continuously. Feng (2019) developed another model for perishable goods. This model is based on the scenario in which the quality and physical quantity of the product deteriorate at the same time, and the demand rate increases with the quality level. To maximise profit in this scenario, this research study utilised a dynamic optimisation model, and the problem was solved using Pontryagin's maximum principle. Similarly, Rabbani, Zia and Rafiei (2016) considered the scenario of the quality and physical quantity of the product deteriorating at the same time, and developed a quality-dependent demand model to maximise the total profit of the system by calculating the optimal replacement cycle, discount rate and initial price.

All the above models are applicable for the manufacturing companies; however, for service companies, a demand rate that depends on service quality in terms of speed and convenience must be modelled (Huang et al., 2013). Very few studies have developed these models, Hou, Koster and Yu's (2018) model being one of the few examples. The researchers developed a model for an online retailer in which demand depends on delivery service quality. The aim of this model is to reach the optimum investment in service quality that optimises the retailer's profit. Furthermore, Xia, Xiao and Zhang (2016) developed a model to study the impact of investment in in-store assistance on the retailer's demand, while Xiao and Qi (2012) modelled service quality in terms of delivery time and reliability in satisfying the announced delivery time. In their model it was assumed that the customers arrive randomly, and that the time it takes the manufacturer to produce the product is random too.

In conclusion, there are various types of demand-dependent EOQ models in which demand changes as a result of a change in one or more parameter. Each of these types of models has its own application in real-life scenarios as modern-day business continues to evolve. From all these types, several assumptions and/or conditions can be mapped in the subject case of this research study.

As seen from the above discussion, several extensions of the EOQ model assumed stochastic demand, and proved that the EOQ model still holds under this assumption. Whether this demand is price-, time- or space-dependent, the EOQ model can be extended to accommodate such a demand type. This fact was highlighted by Maddah and Noueihed (2017), as they assumed that demand occurs at random under a renewal process that is independent and identically distributed with no lead time. In order to examine if the EOQ model holds under these conditions, the researchers followed all the EOQ model's assumptions except for the demand one mentioned above. Consequently, through their developed model, they found that the optimal order quantity that results in the minimum cost incurred can be calculated using an EOQ formula.

2.4 Systematic Literature Review about current trends in EOQ models for the steel manufacturing industry

This section presents a SLR conducted to explore the different applications of the EOQ in the steel manufacturing industry. In the first place, the current trends in the literature of EOQ models are reviewed. Then, the focus is made on the particular subject topic of this research study, i.e., EOQ models applied within the context of the steel manufacturing industry and their capability of improving sustainable aspects of the company in terms of waste and environmental impact management. The SLR is performed following the methodology adopted by Shekarian et al. (2017):

1- Relevant studies collection (Section 2.4.1)
2- Descriptive analysis of the collected dataset (Section 2.4.2)
3- Category selection (Section 2.4.3)
4- Content analysis (Section 2.4.4)

2.4.1 Data Collection and Literature Search

In this first step, the most relevant studies in the state-of-the-art are revisited and collected in order to be able to make the most appropriate contribution to the topic under study. In particular, for the preliminary stage, the approach presented in Huang et al. (2013), in which the various categories of the different types of EOQ models, based on the parameter on which the demand depends were defined, is used. For the subsequent stage, different fields, such as transportation, supply chain, manufacturing and sustainability within the steel manufacturing industry context are taken into account. Using these approaches as the foundation for the outline of this SLR, several new branches of research are added to align the SLR towards the purpose of this study. Consequently, five research questions are formulated for the preliminary and the specific stages as shown in Table 2-3.

Table 2-3. Research questions.

SLR: EOQ Models	SLR: EOQ Applications

1- What types of EOQ models have been developed based on the demand-dependent parameter? 2- Within these types, what are the demand functions adopted by different researchers? 3- What are the different fields in which the EOQ model has been applied? 4- Within the examined studies, what is the optimisation technique used by the researchers? 5- What are the limitations of the current state of the art?	1- What are the environmental impacts of ordering and holding inventory? 2- What are the types of inventory management models used in the steel manufacturing industry? 3- Within these applications, what are the demand functions adopted by different researchers? 4- Within the examined studies, what is the optimisation technique used by the researchers? 5- What are the limitations of the current state of the art?

In order to answer these questions, a search process is designed which defines how the search is conducted, the inclusion/exclusion criteria, and data analysis procedures. The following electronic databases are used as the primary sources for examined literature:

- Scopus
- Taylor and Francis
- Emerald
- Elsevier
- Wiley
- Science Direct

To locate the relevant studies, the search terms shown in Table 2-4 are utilised:

Table 2-4. Research terms.

EOQ Models	**EOQ Applications**
"EOQ Model", "EOQ Model Extensions", "Price dependent Demand Model", "Time dependent Demand Model", "Space dependent Demand Model", "Promotion dependent Demand Model", "Advertising	"EOQ Model in sustainability", "Inventory management in the steel industry", "Environmental impacts of inventory", "Waste reduction in inventory", "waste management in inventory", "EOQ Model

dependent Demand Model", "Quality dependent Demand Model", "Deterministic EOQ Model", "Stochastic EOQ Model", "EOQ Applications", "Inventory Cost" and "Inventory Management Policy".	with emissions", "EOQ Model with recycling", and "EOQ Model with remanufacturing".

Table 2-5 shows the inclusion/exclusion criteria used in each case:

Table 2-5: Inclusion/exclusión criteria.

EOQ Models	EOQ Applications
1- The paper develops a new EOQ model 2- The developed EOQ model is relevant to this study's research questions 3- The paper explains the model's development thoroughly, lists all its assumptions, and highlights the differences from previous models 4- The paper provides an application of the newly developed model	1- The paper clearly applies the EOQ model to the steel manufacturing industry or analyse the sustainability aspects of applying the EOQ model. 2- The paper demonstrates that the application of the model led to a breakthrough in that given field 3- For waste reduction and management, the inclusion criteria included: • One of the developed model's objectives is to manage or reduce inventory • The results of the developed model's implementation led to waste reduction or better waste management.

2.4.2 Descriptive Analysis

Only the studies that actually contributed to answer the research questions listed in Table 2-3 are considered for the SLR. In order to identify such studies and ensure their quality, the abstracts of all the studies collected based on the process described above are examined thoroughly. This step trimmed the number of studies to 88 and 61 for the EOQ models and their applications, respectively.

2.4.3 Category Selection

In this step, the collected data is categorised in different classes. Table 2-6 shows the main defined classes for the studies collected for the preliminary and specific stages of the SLR.

Table 2-6: Categories for the collected data.

EOQ Models	**EOQ Applications**
1- Models with a price-dependent demand rate (P): Studies where an EOQ model is developed in which the demand rate depends on the item's selling price. 2- Models with a time-dependent demand rate (T): Studies that developed an EOQ model in which an item has a finite shelf life, i.e. it experiences deterioration over time, and the demand rate is thus affected as time passes and the product deteriorates. 3- Models with a lead-time-dependent demand rate (LT): Studies that developed an EOQ model in which customers are sensitive to the waiting time; hence, the lead time of the item has a major impact on the demand rate for that item. 4- Models with a space-dependent demand rate (S): Studies that developed an EOQ model in which	1- EOQ applications in the steel manufacturing industry: Studies that applied the EOQ model in the steel manufacturing field. With regard to the EOQ applications in sustainability, the following three classes are used: 2- EOQ application in sustainability: Studies that applied the EOQ model in a sustainable setting or under sustainability regulations. 3- Environmental impacts of inventory: Studies that discussed, highlighted, or proved the different kinds and types of environmental impacts of ordering or holding inventory. 4- Waste reduction and management from inventory: Studies that discussed and developed models that were aimed to reduce or manage waste through better inventory management.

the retailer has limited storage space, and the demand rate for the item depends on how large this space is. 5- Models with a promotion-dependent demand rate (PR): Studies that developed an EOQ model in which the retailer uses particular promotional efforts to induce demand for a certain product. 6- Models with an advertising-dependent demand rate (A): Studies that developed an EOQ model in which the retailer specifically uses advertising efforts to induce demand for a certain product. 7- Models with a product-quality-dependent demand rate (PQ): Studies that developed an EOQ model in which consumer demand depends on the quality of the product. 8- Models with a service-quality-dependent demand rate (SQ): Studies that developed an EOQ model in which consumer demand depends on the quality of the service received.	

After the categorisation shown in Table 2-6, the studies are further evaluated towards finding common characteristics according to the nature of the demand function and the type of optimisation technique used. This process combines both deductive and inductive approaches to reach the most comprehensive and accurate categorisation of the studies, so that a content analysis can be carried out.

2.4.4 Content Analysis

In this section the collected studies addressing the current trends in EOQ models and their applications in the steel manufacturing industry are analysed.

2.4.4.1 EOQ models

The EOQ model studies are classified into classes according to the parameter on which demand depends, as shown in Table 2-7. An extension of Table 2-7 can be found in Table A 1 (Appendix A), where all the references to the corresponding studies have been included.

Table 2-7. EOQ model studies by parameter

Type	Number of Studies
Price	25
Time	9
Lead Time	6
Space	15
Promotion	14
Advertising	10
Product Quality	6
Service Quality	3

As seen from Table 2-7, the price-dependent demand models form the lion's share of studies that developed an extension for the EOQ model, with a total of 25 studies, equivalent to 28% of the total number of EOQ model studies. This class is followed by the space-dependent and promotion-dependent models, with 15 and 14 studies, respectively. On the other hand, the parameter that has received the least attention in the literature is service quality, with only three studies. This is somehow expected, as inventory models, in general, deal with products rather than services.

According to the reviewed studies, the overwhelming majority of the EOQ models proposed in the literature assumed that the demand function is deterministic. In particular, 60 studies, which is equivalent to 68% of the total EOQ model studies assumed deterministic demand.

On the other hand, only 28 studies, which is equivalent to 31% of the total studies, consider stochastic demand. The references for each of these studies can be found in Table A 2 (Appendix A). The allocation of these models among the eight identified classes of the EOQ model is shown in Figure 2-1.

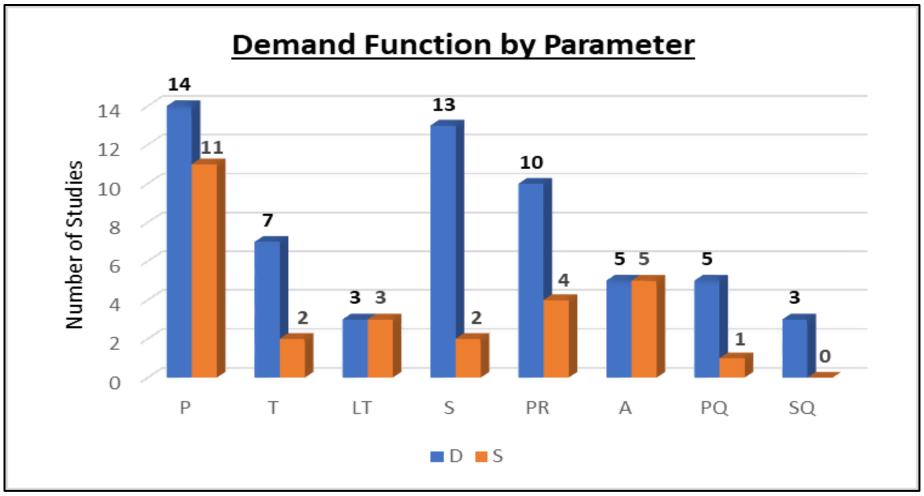

Figure 2-1. Number of demand function studies by parameter.

As seen from the above figure, the lead-time- and advertising-dependent models have an equal number of deterministic and stochastic models, while the space-dependent models show the largest gap between the two types, with the deterministic models being dominant.

Another important characteristic of the developed models examined in this literature review is the type of optimisation technique used. Solving complex inventory management models constitutes a separate (and often far more complex) task than developing the models themselves. In this context, the use of different optimisation techniques has become increasingly popular, since it allows addressing multiple concepts for multiple dimensions of decision making by producing objectively verifiable and quantifiable outputs (Stadtler and Kilger, 2002); it allows quantifying important aspects of the inventory process (Ivanov and Sokolov, 2005); it is low cost. Moreover, these techniques have demonstrated to be equally robust in capturing most of the major decision-making challenges faced at the managerial level, such as raw materials purchasing and storage, production planning, and other challenges in the inventory management processes (Shapiro, 2001).

Traditionally, Mixed-Integer Linear Programming (MILP) methods have been used to solve EOQ models. Nevertheless, relaxing the traditional EOQ model assumptions has led to even more complex non-linear EOQ-related models, making the MILP method to be no longer suitable for solving them (Rabieh et al., 2016). In this new context, nonlinear programming, multi-objective programming, fuzzy mathematical programming, stochastic programming, heuristics algorithms, metaheuristics models and hybrid models, have gained popularity. These approaches have been used at the strategic, tactical and operational levels of inventory management to improve the organisational, industrial and commercial sustainability of the process, and bridge the gap between theory and management in practice (Wang, 2010). For instance, Cohen and Lee (1989) used mixed integer non-linear programming technique to solve their extended EOQ model, while Paksoy et al. (2010) utilised a fuzzy nonlinear multi-objective mathematical model to solve a model for a supply chain network. Another example of a solving technique is the fuzzy inequalities linear membership function used by Hu and Fang (1999), and a new fuzzy linear programming-based methodology was used by Vasant et al. (2005). In (Pasandideh et al., 2010) the EOQ problem is formulated as a Non-Linear Integer-Programming (NLIP) model and genetic algorithms are used to solve it. In (Zhao et al., 2006), the EOQ model is solved by using a new PSO algorithm that combines gradient acceleration and penalty functions. In (Nazari-Heris et al., 2018), a number of heuristic derivative-free global optimisation methods has been listed as follows:

1. Ant Colony Optimisation (Dorigo and Blum, 2005).
2. Simulated annealing, which is a generic probabilistic meta-heuristic (Kirkpatrick et al., 1983).
3. Taboo search, which is an extension of a local search that is capable of escaping from local minima (Glover, 1986).
4. Evolutionary algorithms, for example, genetic algorithms and evolution strategies (Holland, 1992).
5. Differential evolution, which is a method that optimises a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality (Storn and Price, 1997).
6. Swarm-based optimisation algorithms, for example, PSO, social cognitive optimisation, and multi-swarm optimisation (Kennedy and Eberhart, 1995).
7. Memetic algorithms, which combine the global and local search strategies (Moscato, 1989).
8. Reactive search optimisation, which integrates the sub-symbolic machine learning techniques into the search heuristics (Battiti and Brunato, 2010).

9. Graduated optimisation, which is a technique that attempts to solve a difficult optimisation problem by initially solving a greatly simplified one, and progressively transforming that problem, while optimising until it is equivalent to the difficult optimisation problem (Thacker and Cootes, 1996; Blake and Zisserman, 1987; Mobahi and Fisher, 2015).

Figure 2-2 shows the number of studies for each specific optimisation technique adopted in the literature. As seen from Figure 2-2, a large variety of optimisation techniques are used in the literature, as each one is seen as the best fit for the developed model based on the model assumptions. Nevertheless, the mixed integer technique is the most widely used technique in the examined studies, with 14 studies opting for this technique. On the other hand, only the study by Tiwari et al. (2017) used the PSO technique to solve the developed model, while two studies used commercial software to solve their developed models.

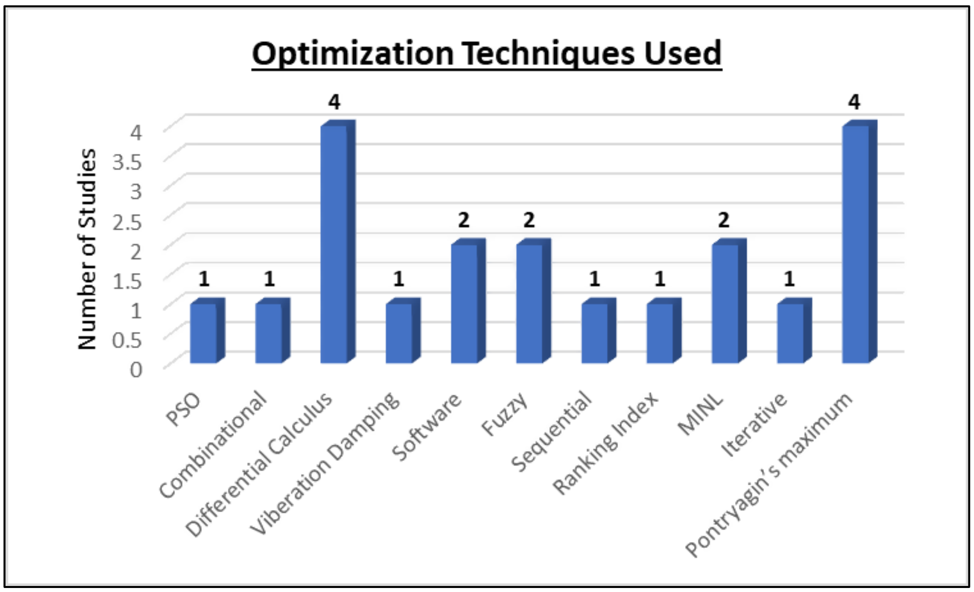

Figure 2-2. Optimisation techniques used in the examined studies.

Finally, the analysis of the reviewed studies revealed an increasing trend in the field of inventory management towards using machine learning techniques to train EOQ models. In general, machine learning techniques allow using up-to-date data input to adjust calculations and predictions, in such a way that the model becomes better suited to the business the more it is used. In this way, machine learning allows optimising the performance of tracking technology in inventory management and offering more accurate data to assist in planning for the future. In particular, ANNs have become very popular for inventory management applications since they have a high learning and generalisation capability, they can handle non-linear variables and missing data, and they can adapt to changing environments. In fact, different approaches based on ANN, addressing different aspects of inventory management, such as determining the optimal ordering and recovery policy (Koh et al., 2002), optimising replenishment policy (Wee and Chung, 2009) and determining the optimal lot size in inventory control (Chiu, 2003), can be found in the literature.

In this research study, the proposed model extends the traditional EOQ model to account for the specific characteristics of the steel manufacturing industry based on a control system algorithm capable of providing timely recommendations for the storage quantities of both of products and raw material. This facilitates the decisions of the factory's management regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon. In particular, in this research study, two different control system approaches, namely, an open-loop and a closed-loop based on ANNs, are considered. In addition, the PSO technique is used to solve the developed model. To the best of the authors knowledge, although having demonstrated to be well suited to develop applications within a short period of time and to assist in gaining better results in a faster and cheaper way when compared with other methods, even involving fewer adjustments to the optimisation parameters (Yin, 2003; Onwubolu and Babu, 2013), PSO techniques have not been fully explored for solving EOQ models. In this research study, the PSO technique is chosen to solve the developed since 1) it is a relatively simple algorithm that is easy to implement on Matlab (Nasri et al., 2007), and 2) the obtained results demonstrate that this method converges to the sub-optimum, and that this sub-optimum does not change a lot after we add more particles into the model. In the following sections, the basic principles of the ANN and PSO methods are introduced, respectively.

2.4.4.1.1 ANN

Neural networks are a set of algorithms, modelled loosely after the human brain, that are designed to recognise patterns. They interpret sensory data through a kind of machine perception, labelling or clustering raw input. The patterns they recognise are numerical, contained in vectors, into which all real-world data, be it images, sound, text or time series, must be translated.

An ANN is based on a collection of connected units or nodes called artificial neurons. Each connection, like the synapses in a biological brain, can transmit a signal to other neurons. An artificial neuron that receives a signal then processes it and can signal neurons connected to it. The signal at a connection is a real number, and the output of each neuron is computed by some non-linear function of the sum of its inputs. The connections are called edges. Neurons and edges typically have a weight that adjusts as learning proceeds. In addition, neurons may have a threshold such that a signal is sent only if the aggregate signal crosses that threshold. Typically, neurons are aggregated into layers. Different layers may perform different transformations on their inputs. Signals travel from the first layer (the input layer), to the last layer (the output layer).

A typical structure of an ANN with one hidden layer is shown in Figure 2-3. As seen from Figure 2-3, each circle represents an artificial neuron (depicted in Figure 2-4) that collects input signals and produces output signals which act as inputs for each neuron of the following layer. In general, the weights of the ANN are fine-tuned during a back-propagation algorithm; however, in the case of the model developed in this study, due to the lack of an "actual" parameter of business controls, the error cannot be estimated. As a result, the PSO is used.

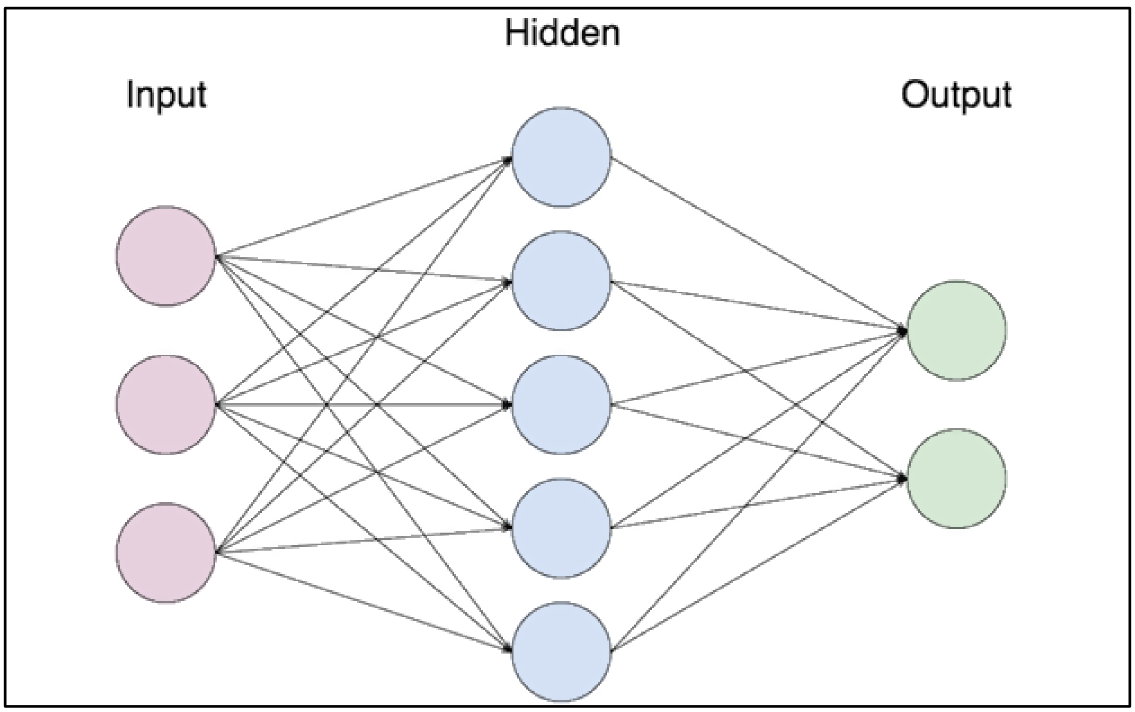

Figure 2-3. Scheme of an ANN with one hidden layer (Kriesel, 2007).

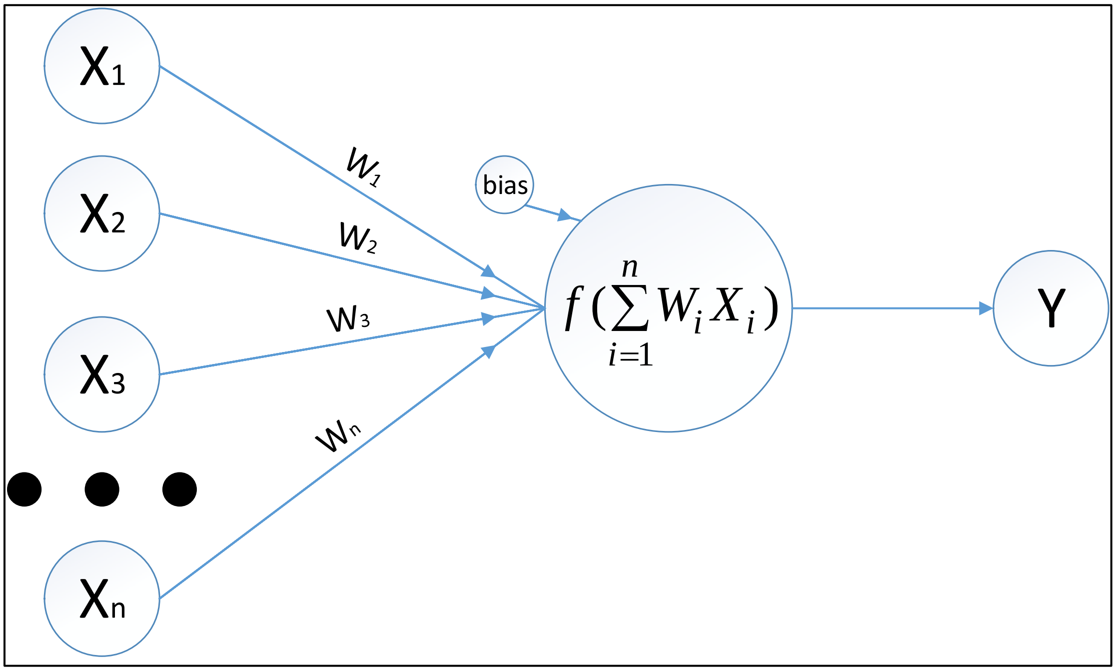

Figure 2-4. Scheme of an artificial neuron (Kriesel, 2007).

As seen from Figure 2-4, the neuron collects the input signals X_1, X_2, X_3…X_n and multiplies each signal with the corresponding weight W_i (which are parameters of the neuron). The results of these multiplications are summed together to reach a weighted sum, which is transformed according to an activation function $f(\cdot)$ in order to add nonlinearity to the model and reduce the high-value outputs. Each layer hidden layer, as well as the output layer have a corresponding activation function. There exists several activation functions in the literature of ANNs. Selecting the best suited activation function for each layer is one of the most important tasks when designing an ANN.

Suppose the activation function for the hidden and output layer are given by $f_{hidden}(x)$ and $f_{output}(x)$. The calculation of the controls for the generic ANN with one hidden layer is given by Equation 2-4:

$$U = f_{output}(W_2(f_{hidden}(W_1.I + B_1)) + B_2 \qquad \text{2-4}$$

where W_1 is the weights matrix from the input layer to the hidden layer, B_1 is a bias vector for the hidden layer, W_2 is the weights matrix from the hidden layer to the output layer, B_2 is the bias vector for output layer, I is the vector of inputs, and U is the vector of controls.

2.4.4.1.2 PSO

As discussed above, the use of the PSO techniques to solve the inventory management problems has become very popular in recent years. The main advantages of this technique are its ability to find the maximum profit function even if this function is non-differentiable by all the control parameters and/or discontinuous in fewer adjustments to the optimisation parameters (Yin, 2004). The basic form of the PSO algorithm works by having a population (called a swarm) of candidate solutions (called particles). The position of each particle is some solution of the problem. So, if, for example, we need to minimise a function of 5 arguments, then particle space will be 5-dimensional. To seek the optimal solution, each particle moves in the direction of its previous best position (pbest) and the global best position (gbest) in the swarm, hence it can be expressed by the following equations (Liu, Abbas and Tan, 2019, p.15):

$$pbest(i,t) = arg \min_{k=1,\dots,t} \left[f\left(P_i\ (k)\right)\right], i \in \{1, 2, \dots, N_P\}, \quad 2\text{-}5$$

$$gbest(t) = arg \min_{i=1,\dots,N_P} pbest(i,t), \quad 2\text{-}6$$

where i denotes the particle index, N_P the total number of particles, t the current iteration number, f the fitness function, and P is the position of the particle in n-dimensional search space.

Moreover, all the particles move around in the search space according to a few simple formulas, and the velocity V and position P of the particles at time t are updated by the following equations:

$$V_i\ (t+1) = \omega V_i\ (t) + c_1 r_1\ (pbest(i,t) - P_i\ (t)) + c_2 r_2\ (gbest(t) - P_i\ (t)) \quad 2\text{-}7$$

$$P_i\ (t+1) = P_i\ (t) + V_i\ (t+1), \quad 2\text{-}8$$

where ω is the inertia weight used to balance the global exploration and local exploitation, r_1 and r_2 are uniformly distributed random variables within the range of [0, 1], and c_1 and c_2

are positive constant parameters called “acceleration coefficients” (Liu, Abbas and Tan, 2019, p.15).

Therefore, the particles move according to their own best-known position in the search space and the entire swarm's best-known position. Thus, the movements of the swarm will be continuously guided when new improved positions are discovered, until a satisfactory solution will eventually be discovered, albeit this is not guaranteed. Figure 2-5 provides the detailed scheme of the PSO algorithm.

As seen from Figure 2-5, at each iteration, a set of different vectors of weight are considered. Then, the best control system is selected for this set of vectors and, consequently, the vectors of weight of the other systems will be changed to converge with the best system selected. Hence, through this network, an output of the best control system over all sets of vectors is obtained.

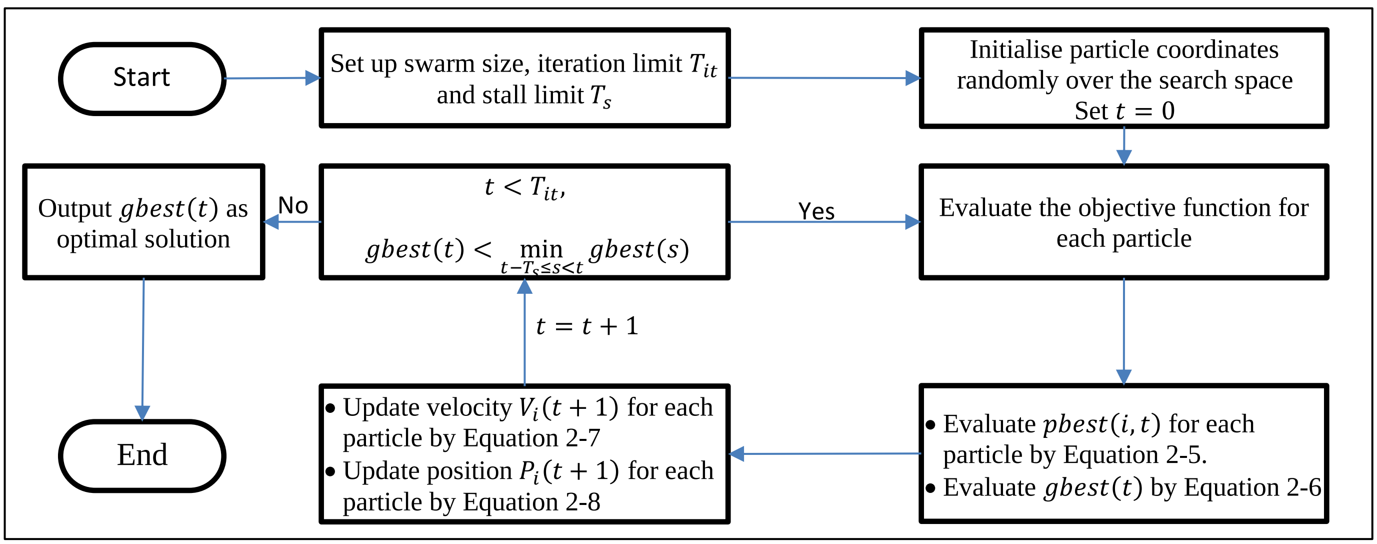

Figure 2-5 Detailed scheme of PSO algorithm.

2.4.4.2 EOQ Applications

The total number of studies classified under this category is 54 studies. These studies are classified according to the field in which the models were applied, as shown in Table 2-8. An extension of Table 2-8, including the corresponding references can be found in Table A 4 (Appendix A).

Table 2-8. EOQ application studies by field.

Type	Number of Studies
Sustainability	45
Steel	9

As seen from Table 2-8, the application of the EOQ model is popular in the sustainability field, as the 45 studies demonstrate. On the other hand, only a limited number of EOQ applications in the steel industry has been found, which suggests that there is a lack of studies in this field, and that it requires further research. In order to better understand the different aspects associated with sustainability that the found studies address, they are further sub-classified. There are 45 studies under the sustainability field, in addition to seven studies that outlined the environmental impacts of ordering and holding inventory without actually developing new models. This brings the total to 52 studies, which are further classified under the sub-categories shown in Table 2-9. An extension of Table 2-9, including the corresponding references can be found in Table A 5 (Appendix A).

Table 2-9. Sub-classification of sustainability studies.

Type	Number of Studies
Emissions	14
Remanufacturing / Recycling	19
Waste Reduction and Management	12
Environmental Impacts	7

As seen from the above table, remanufacturing and recycling form the majority of studies in the sustainability field with 19 studies, followed by the reduction of emissions. On the other hand, waste reduction and management have only 14 studies, accounting for 27% of the examined studies, which suggests that there is room for more research in this area to examine all the associated aspects of waste generation as a result of ordering and holding inventory, and how to minimise this waste generation. Finally, as in the case of the EOQ models in general, the majority of studies addressing EOQ applications in the steel manufacturing or analysing their sustainability implications assumed deterministic demand. In particular, 64% of them assume deterministic demand, whereas only the 36% consider demand as stochastic. In addition, in the most popular optimisation technique used in these cases is also the mixed integer technique, whereas only two studies used fuzzy technique, and three used the PSO technique.

In the following sections the studies found in the SLR addressing the relationship between the EOQ models and the sustainable performance of the companies as well as the application of the EOQ models to the steel manufacturing industry are discussed in detailed.

2.4.4.2.1 Sustainable Inventory Management

Over the past two decades were several efforts have been directed towards combating global warming and other environmental issues, the topic of sustainability and the impacts of firms' operations on the environment gained much attention from researchers and scholars. Consequently, firms started to incorporate sustainability practices in their operations, which added new cost and benefit components that were not previously accounted for. Therefore, it became essential to extend the EOQ model to incorporate such new parameters, in order to be able to accurately model the firms' operations and maximise their profits and/or reduce their costs. In order to have a better understanding of the need to extend the EOQ model to cover this area, the sustainability impacts of inventory management are discussed in this section.

The supply chain stage has four main impacts on the environment, *viz.*, the consumption of precious resources, the generation of waste, the consumption of energy needed to store and handle inventory, and the greenhouse gas emissions resulting from the consumption of resources and energy (Liao and Deng, 2018; Fichtinger et al., 2015; Hariga, As'ad and Shamayleh, 2017). Although the consumption of resources is necessary to acquire raw materials for manufacturing, by poorly managing inventory, firms sometimes order excess amounts of raw materials, which they never use, wasting precious resources and applying pressure to the environmental ecosystem (Liao and Deng, 2018). In addition, the excess usage of natural resources has other negative environmental impacts that are associated with the process of extracting these resources, such as air pollution, soil contamination, water pollution and greenhouse gas emissions (Blass, Chebach and Ashkenazy, 2017). Regarding waste generation, one of the main sources of industrial waste is the ordering of unnecessary quantities of inventory that are not used in production and have to be discarded (Fercoq, Lamouri and Carbone, 2016). For the third and fourth impacts, as the quantity of inventory ordered and stored increases, more energy is needed in terms of electricity, heating, cooling, etc. in order to preserve it (Fichtinger et al., 2015; Uzturk and Büyüközkan, 2016), and more greenhouse gases are emitted due to the need both to transport and preserve the inventory (Hariga et al., 2017). Moreover, a significant amount of CO_2 emissions from logistics activities, approximately 13% of the overall supply chain emissions, are caused by the storage and material handling processes in the warehouses (Ries, Grosse and Fichtinger, 2016).

Incorporating sustainability measures in the inventory ordering problem started to gain popularity at the start of the millennium (Soleymanfar, Taleizadeh and Zia, 2015; Hariga et al., 2017). Since the proper management of inventory and raw materials can lead to lower consumption of resources and energy (Liao and Deng, 2018), and/or emissions reduction (Hovelaque and Bironneau, 2015), the inventory management policy of a company should be directly linked to its environmental performance (Konur, Campbell and Monfared, 2016). As a result, sustainable raw materials management and green inventory management have become necessities. According to Blass et al. (2017), sustainable raw materials management assists companies to manage their natural resources and use them in production from economic, social and environmental perspectives. Marklund and Berling (2017) defined green inventory management as finding ways to efficiently manage inventory in terms of costs and emissions. Therefore, the ordering of the optimal quantity of inventory that will be used in production will help to preserve natural resources and reduce emissions and waste, which are proportional to the number of items held in stock.

As discussed above, waste generation is one of the major impacts of imperfectly managed inventory for manufacturing firms. Hence, a number of research studies attempted to develop models to assist companies in managing their inventory in a way to reduce the amount of waste generated from ordering and holding inventory (Malladi and Sowlati, 2018). In general, there are two types of inventory management models that incorporate waste in their objective function, namely, models aimed at managing waste and models aimed at reducing it (Malladi and Sowlati, 2018). Regarding waste-management models, Elbek and Wohlk (2016) developed a model for the collection of waste glass and paper at a number of collection points to schedule the emptying and transporting operations in such a way to minimise cost while preserving service quality and fulfilling capacity constraints. In their study, the researchers developed a heuristic solution method to solve this planning problem on a daily basis. Another waste management study was conducted by Habibi et al. (2017)

for a generic product. In their model, the researchers modelled the operations of the collection-disassembly problem of a remanufacturing company by optimising both the lot sizing and vehicle routing through a two-phase iterative heuristic. Furthermore, it is assumed that the structure of the product is known, the vehicle has a fixed capacity, and there is a penalty cost for any unmet demand. Following the successful results of this model, Habibi et al. (2018) applied this developed model to managing electrical and electronic equipment waste.

Regarding the waste reduction models, most of these models address perishable goods, i.e. manage the quantity of perishable goods to prevent them from perishing and turning into waste (Malladi and Sowlati, 2018). Hiassat et al. (2017) developed a multi-objective model to determine the number and location of required warehouses, the inventory level at each retailer, and the routes travelled by the transportation vehicles in order to preserve the quality of the perishable products and reduce waste. Moreover, in this model, it is assumed that demand is deterministic, all vehicles have the same capacity, the manufacturer or the retailer never run out of stock, holding costs vary slightly across time, and quantities of inventory at the retailers are limited by the capacity at retailers' warehouses/shops and the shelf-life of the products. A genetic algorithm was used to solve this problem, and through a numerical application, it was found that it provides promising solutions at medium and large instances. Another model aimed at reducing waste resulting from excess inventory of perishable products was developed by Soysal et al. (2015), who developed a model to reduce the waste generated by food products. Furthermore, Soysal et al. (2018) considered the wastage cost of perishable products when developing an inventory management model for food products. Similarly, Azadeh et al. (2017) developed an inventory model for a single perishable product that deteriorates at an exponential rate while being stored at a warehouse. The objective of their model is to identify the optimum inventory replenishment policy that would minimise waste resulting from spoilage in the warehouse. In addition, Janssen et al. (2018) developed a perishable goods micro-periodic inventory replenishment model with stochastic demand, deterministic lead time, mixed FIFO and LIFO issuing policies, and imperfect items. The results of implementing the model in the food industry showed a reduction in the waste generated by 66%, with a decrease in cost when compared to the traditional inventory policy. Another model that aimed to address the problem of packaging waste of products was developed by Iassinovskaia, Limbourg and Riane (2017). In their model, the use of Returnable Transport Items (RTIs) to reduce packaging waste is considered in a scenario of one producer who distributes products to a number of customers in RTIs and collects the empty ones for reuse. The model was aimed at minimising the total cost while satisfying the inventory level constraints for each customer in each period under deterministic demand. The results of this research showed that using RTIs can result in minimising costs, and thus reducing the amount of waste generated. Moreover, Kazemi et al. (2017) used order-up-to-level policy to develop an inventory management model to minimise blood waste under deterministic demand. Finally, Timajchi et al. (2018) developed an inventory model for hazardous waste material, with the aim of reducing exposure to these materials through a reduction in accident incidents. This model is a bi-objective model, as it was also aimed at minimising the total cost of logistics, which includes ordering, transportation, delivery, pickup, shortage and inventory holding costs.

Teunter (2001) developed one of the earliest extensions of the EOQ model to incorporate sustainability parameters. In this model, dumping costs, modelled through different holding cost rates for manufactured and recovered goods, were incorporated to determine optimal batch quantities for both types of goods. Moreover, Dobos and Richter (2000; 2003; 2004; 2006) devoted their research efforts to applying EOQ to the manufacturing of recycled products. With the increase in the popularity of the application of EOQ in sustainability, several new research topics, related to sustainability, started to emerge. First, Teunter (2004) used the EOQ to determine the optimal lot size for the recovery of returned goods. Gou et al. (2008) extended the model further to discover the optimal delivery batch size in open-loop reverse supply chains that consist of one centralised returns centre and various collection points. Second, a wide array of models was developed to manage inventory for firms that manufacture recycled products. Alinovi et al. (2012), Zhang and Jonrinaldi (2017), Jain et al. (2018), Benkherouf, Skouri and Konstantaras (2016), Kozlovskaya, Pakhomova and Richter (2019), Mawandiya, Jha and Thakkar (2018), Singh, Sharma and Kumar (2016), and Turki et al. (2017) aimed at helping decision makers operating in the recycling industry by developing an EOQ model for a system that manufactures original and recycled goods. For instance, Benkherouf et al. (2016) extended the EOQ model to include remanufacturing and refurbishing activities for recycling firms. In this model, the researchers assumed a scenario in which used products are returned to the firm by customers, and are classified as either remanufacturable or refurbishable, and for each scenario, the total cost to the firm is minimised by determining the optimal inventory level of used items. Moreover, in this model, demand is assumed to be stochastic, the remanufacturing rate is known and constant, the rate of returned items is proportional to demand, and no shortage is allowed. Third, the optimal level of inventory for remanufacturing firms is another area that gained considerable attention in recent research studies. Demirel, Demirel and Gokcen (2016) incorporated the regulations of recovery, dismantling and recycling of end-of-life vehicles imposed on the automotive industry to optimise the logistics of such companies. Kozlovskaya, Pakhomova and Richter (2015) incorporated the switching cost into their model, which is incurred when a firm switches from repair to manufacturing and vice-versa; and Shekarian et al. (2016) developed a reverse inventory model for a remanufacturing process. Furthermore, Liao and Deng (2018) developed an environmentally sustainable EOQ model for remanufacturing firms to determine the remanufacturing ratio that minimises the inventory cost, maximises the sustainability benefits of remanufacturing, and coordinates forward and reverse logistics. In this model, all the environmental parameters, costs as a result of holding and acquiring inventory, and profits resulting from remanufacturing are converted to the economic equivalent to provide easy comparison of the model's results. Through numerical examples, the researchers proved that when the holding cost of the finished goods is low, the optimal inventory strategy is to remanufacture as much as possible, and vice versa.

A fourth area of sustainability that started to attract attention with regard to inventory management models, as a result of the current global warming trend, is incorporating greenhouse gas emissions in the inventory optimisation problem. In general, according to Fichtinger et al. (2015), the research studies in this area can be divided into three streams in accordance with how emissions are integrated into the inventory models. These streams are:

1- Emissions are converted into a monetary cost in the form of a carbon tax, carbon trading within a carbon cap-and-trade system, or internal (virtual) steering cost, which can be included in the objective function.
2- Reducing emissions is considered as a second objective in a multi-criteria optimisation approach.
3- Emissions are integrated as a constraint within the inventory optimisation model.

Several approaches have been proposed in the literature addressing the carbon emission issue. Hovalaque and Bironneau (2015) developed a model that took into account carbon emissions associated with the firm's operations. Their model was developed assuming a scenario where a single retailer buys a single product with deterministic demand and including both the holding and ordering costs of inventory. Their model's aimed at determining the optimal inventory quantities that maximise the firm's profit and minimise its carbon emissions. Shu et al. (2017) extended the EOQ model to include carbon emissions resulting from manufacturing/remanufacturing activities and the transportation of products. In particular, they proposed to determine the manufacturing and remanufacturing quantities with and without carbon constraints, assuming the scenario depicted in Figure 2-6. According to this scenario, once the original product is manufactured, it is sold to customers. Customers use the product, and then they return it to the firm. The returned products are then inspected towards deciding whether they are suitable for remanufacturing or they should be sent to waste treatment. When comparing the ordering quantities of manufacturing and remanufacturing obtained when applying carbon constraints to the described scenario, it was found that they are the same. In addition, the presence of carbon constraints has also demonstrated to reduce the total manufacturing and remanufacturing costs as well as the carbon emissions.

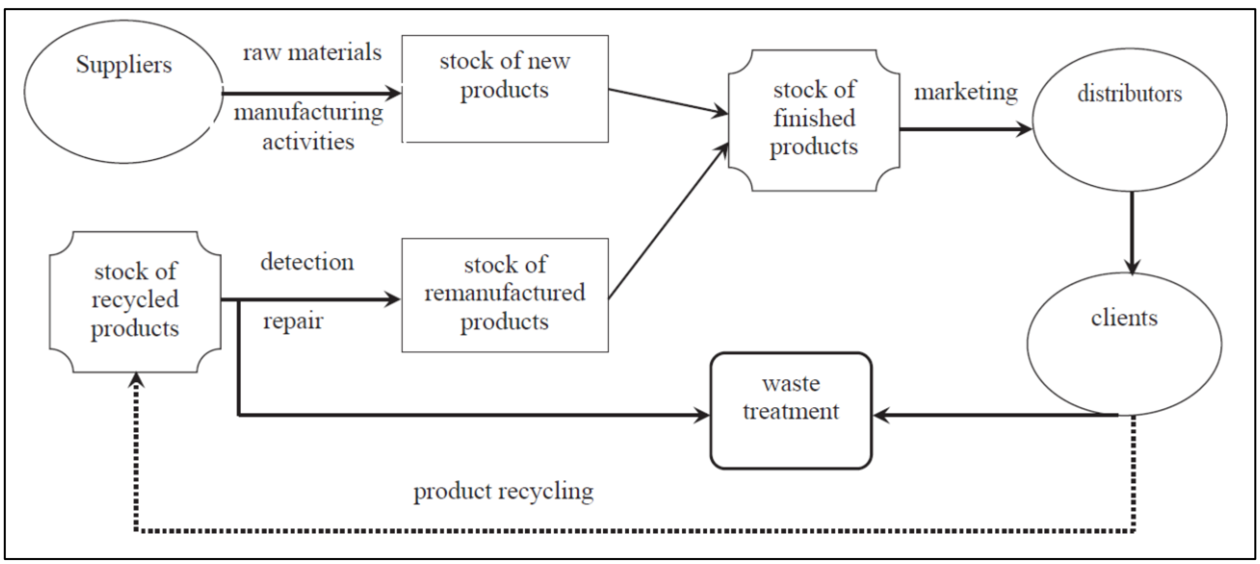

Figure 2-6. Manufacturing/remanufacturing scenario assumed in (Shu et al., 2017).

More complex models which incorporated carbon emissions were developed as follows. Soleymanfar et al. (2015) considered emissions generated during the entire inventory management's life cycle while allowing partial backordering. Hua et al. (2016) considered perishable goods with freshness-dependent demand. Bozorgi (2016) included a multi-product inventory scenario in which each product requires specific storage conditions in their inventory model. Konur et al. (2016) considered a stochastic inventory model with multiple suppliers. Cheng et al. (2017) developed a model to minimise inventory and routing costs while considering the environmental impacts of holding inventory. Lee, Yoo and Cheong (2017) took into account a stochastic lead time for the inventory and multiple transportation modes. Wangsa (2017) included emissions resulting from both industrial and transport activities. Alinaghian and Zamani (2019) developed a bi-objective model to reduce emissions while minimising the inventory costs. Tiwari, Daryanto and Wee (2018) considered deteriorating goods with imperfect quality. Finally, Bazan, Jaber and El Saadany (2015) extended the model further to include both the energy consumed and greenhouse gas emissions resulting from manufacturing and remanufacturing operations. The objective of their model is to determine the quantity of products to be manufactured per cycle, the number of manufacturing and remanufacturing batches per cycle, and how many times an item may be remanufactured, minimising total cost for the firm. The results of their model showed that to minimise the total financial and environmental costs, the firm needs to collect more used products to remanufacture, while reducing the number of times each product is remanufactured.

The social aspect is another area of sustainability that was the focus of a number of research studies and extended models. For example, Nozick and Turnquist (2001) attempted to examine the optimal level of inventory based on an objective function that tries to maximise customer responsiveness while minimising costs. Moreover, Rahimi, Baboli and Rekik (2017) developed a multi-objective inventory model that aimed at maximising profit, while minimising greenhouse gas emissions, and minimising the service level measured through the rate of delays, the rate of backorder, and the rate of backorder frequency.

2.4.4.2.2 Inventory Management in the Steel Manufacturing Industry

A specific area of particular interest in this research study is the application of inventory management models in the steel manufacturing industry. As this industry is one of the most capital-intensive industries, inventory management has a significant impact on its financial performance (Shardeo, 2015). It is a challenging task for these companies to match the ordering quantities of raw materials with the stochastic nature of demand (Singh and Mondal, 2016). In this context, mathematical modelling becomes essential towards understanding the dynamics of the business environment of the steel manufacturing industry and predicting the future outcomes within the system. In fact, using mathematical modelling to address the problem of inventory management for a steel manufacturing companies can help them to organise and optimise the flow of resources, including funds, information, materials and goods. Moreover, the use of mathematical modelling can also help the company in facing the complexities associated with managing and controlling the globalised supply chain of today's manufacturing world, in terms of ordering, pricing and transporting the inventory.

There is a number of inventory management models developed in the literature to assist steel manufacturing companies in their inventory ordering and holding policies. Xiong and Petri (2005) developed one of the earliest models for the inventory management of steel manufacturing companies. In their study, the researchers developed a fuzzy model based on fuzzy logic theory combined with the basic EOQ model, and demand was assumed to be stochastic over a 52-week planning horizon while incorporating lead time. Other inventory management models for steel manufacturing companies were developed by Liu, Tang and Song (2006), Tang, Liu and Liu (2008), and Zhang et al. (2011). More recently, Zhang et al. (2015) developed a model to optimise order planning and inventory matching for finished and unfinished products for a steel manufacturing company. In their model, multiple objectives were considered, which include minimising inventory matching costs, production capacity balance costs, delivery penalty costs and order cancellation costs. The researchers used three different algorithms to reach the optimal solution, namely PSO, local search, and improved PSO. Through the application of these algorithms, as seen in Figure 2-7, Zhang et al. found that the improved PSO is the best performer, as it converges with the optimal solution more effectively with the increase in the number of iterations. In this figure, it is clear that local search performs well in the first 1000 iterations, but the rate of improvement in the solutions slows down after 1000 iterations, and remains stagnant after 4000 iterations. Initially, the PSO improves the solutions very quickly, then the improvement slows down after 5000 iterations, unlike the improved PSO, which continues its improvement until the point of 12000 iterations.

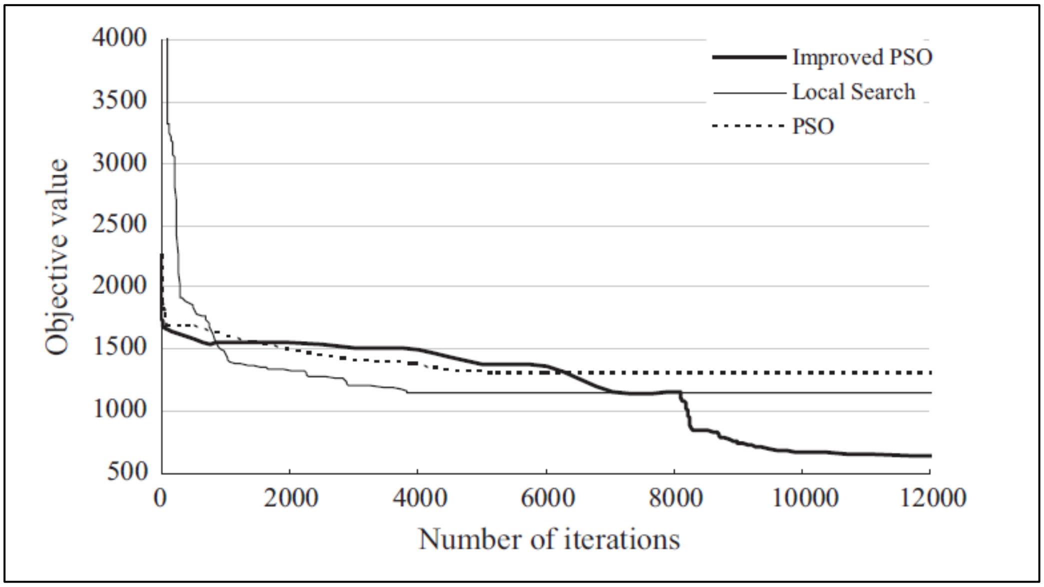

Figure 2-7. Convergence curve for the three algorithms (Zhang et al., 2015).

Rabieh et al. (2016) developed another recent model for managing inventory at a steel manufacturing company. The researchers developed two mixed-integer non-linear programming models to minimise the procurement costs of a steel manufacturing factory in Iran. Their model assumed the following: demand is deterministic over the entire planning horizon, storage space is infinite, purchasing cost is constant, and constant safety-stock level. Through comparing the model results with the company's actual data, it was found that the developed models led to a reduction of 10.9% and 7.1% in the total procurement costs of the company over two consecutive years.

Lately, Bula, Medina and Sierra (2018) designed an inventory management model to take into account the service level within the scrap casting process in steel manufacturing. Through implementing the model in a real-life case study, they found that the model maintained the level of service at the required 99%, while, at the same time, it decreased the costs associated with inventory management by 25.09%.

As seen from the above review, despite the importance of inventory management for the steel manufacturing companies, a limited number of models have been developed to assist these companies in this process, as the majority of the models were more concerned with production planning. In addition, none of the above models considered the scenario of limited storage space for both raw materials and final products. Hence, our current study addresses this gap by developing a robust model that takes into consideration the unique nature of the steel manufacturing industry and the large size of its inventory and final products.

2.4.5 Research Gaps

From the literature review conducted here, it has been found that there are eight categories of extensions of the EOQ model, which are based on the parameter on which the demand rate depends. These categories are:

- price-dependent
- time-dependent
- lead-time-dependent
- space-dependent
- promotion-dependent
- advertising-dependent
- product-quality-dependent
- service-quality-dependent

Within each of these categories, different models are developed taking into account two types of demand functions:

- deterministic
- stochastic

Despite the important contribution of the models developed in the literature, several research gaps that need to be addressed through further research have been identified. The main identified research gaps are listed as follows:

- The deterministic category of models is the one that received most attention (Fibich et al., 2003; Chou and Parlar, 2006; Jeuland and Shugan, 1988; Agrawal and Ferguson, 2007; Hanssens and Parsons, 1993; Song et al., 2008; Chen et al., 2006; Chen and Simchi-Levi, 2012), being a lack of stochastic models.
- The majority of the proposed models in the literature are based on log-linear functions (Ray, Gerchak and Jewkes, 2005; Chen et al., 2006), which are not sufficiently flexible in terms of deriving clear results for the optimal solution.
- There are several problems of shortage in storage space. In this regard, more accurate models can be developed by exploring this problem using space-dependent

demand models that are sensitive to both space and price, and have fewer underlying assumptions behind them (Đorđević, et al., 2017).

- In general, the objective functions in the space-dependent models are based on minimising cost. However, in real-life situations, other objectives should be taken into consideration. For instance, more sustainable operations, and the trade-off between sustainability and cost should be also considered (Malladi and Sowlati, 2018).
- There is a need for using better optimisation techniques when developing the models. In fact, only one study reported to use the particle swarm technique. Using this optimisation methods can help to obtain better results in a faster and cheaper way when compared with other methods. In addition, it also requires fewer adjustments to the optimisation parameters (Yin, 2003; Onwubolu and Babu, 2013).

In addition, particular research gaps related to inventory management in the steel industry which has special characteristics in terms of the large volume of inventory and the special storage requirements to avoid its deterioration, have been identified. The main research gaps identified in this context are as follows:

- Only a limited number of studies have developed inventory management models to account for the special characteristics of the steel manufacturing industry (Shardeo, 2015; Singh and Mondal, 2016; Xiong and Petri, 2005; Liu, Tang and Song, 2006; Tang, Liu and Liu, 2008; Zhang et al., 2011; Zhang et al., 2015; Rabieh et al., 2016; Bula, Medina and Sierra, 2018), as most of the manufacturing models were developed for a general manufacturing scenario, and not for a specific industry.
- There is an urgent need to develop sustainable inventory management models to be used within the steel industry, especially those in which demand is space-dependent to account for the large volume of steel Malladi and Sowlati (2018).
- Almost all the reviewed models were specifically developed for a steel manufacturing company who assumed demand to be deterministic, which does not accurately capture the nature of demand in this industry or the variety of products present.
- Only the model developed in Xiong et al. (2005) assume a non/deterministic demand.
- No reviewed studies explored the quantity of and reasons behind waste generation in the inventory for the steel manufacturing industry.
- None of the steel industry models accounted for any aspect of sustainability in their objective functions.

Based on the above research gaps, there is an urgent need to develop inventory models that can more accurately depict the real-life scenarios. This is particularly true for the steel manufacturing industry, which has special characteristics that should be taken into account, such as space-dependent demand and large volume of steel. In addition, the current focus on sustainable aspects, and the increasing regulations imposed on the companies to conduct their business in a sustainable manner, increase the need for an inventory management model for the steel industry that accounts for sustainability aspects, especially waste reduction and management. The study conducted in this thesis is significant in the field of inventory models developed for steel manufacturing applications, since it addresses the research gaps identified in the SLR conducted in this section by depicting the real-life scenario of such industry more accurately. In particular, the demand is treated as stochastic in order to develop an EOQ space-dependent demand model capable of optimising an objective function that is not just aimed at minimising costs, but it is rather aimed at maximising profit while minimising adverse environmental impacts. Finally, an ANN based closed loop control system provides the model the possibility of being periodically updated according to the current business scenario and the PSO technique allows to optimise the model's variables.

2.5 Chapter Summary

In this chapter, a comprehensive literature review about inventory management have been conducted. On one hand, the main concepts of inventory management and control, as well as the different approaches proposed in the literature have been discussed, making special emphasis in analysing the well-known EOQ model, its types, assumptions, extensions and limitations. In addition, the current trends in machine learning and optimisation algorithms applied to inventory models have also been discussed. On the other hand, special focus has been done on steel manufacturing applications, which is the subject topic of this research study. Finally, the importance of the concept of sustainability has been highlighted, studying the environmental impacts of ordering and holding inventory, which laid the foundation for the need for inventory management models that account for different aspects of sustainability. In particular, three different sustainability topics have been deeply explored, *viz.*, emissions, remanufacturing and waste reduction.

According to the SLR conducted in this chapter, there are several research gaps that need to be addressed through further research. Generally speaking, it has been found that many of the EOQ models proposed in the literature do not accurately depict the real-life scenarios involved in the manufacturing industry. For instance, most of the available models assume deterministic demand, and their objective functions are only based on minimising cost. However, in real-life situations, the demand is stochastic, and other objectives, such as more sustainable operations, should be considered. In addition, in the specific case of the steel manufacturing industry, which is the topic of this research study, the special characteristics in terms of the large volume of inventory and the special storage requirements to avoid its deterioration, should also be taken into account. However, a limited number of studies have developed inventory management models accounting for such characteristics, as most of the manufacturing models were developed for a general manufacturing scenario, and not for a specific industry. In this context, there is an urgent need for developing specialised models targeted towards the operations of the steel manufacturing industry, especially those in which demand is space-dependent to account for the large volume of steel; as well as for taking into account sustainable aspects, such as the environmental impacts of ordering and holding inventory.

The research study presented here is significant in the field of inventory management models for the steel manufacturing industry, as it will help to address the research gaps identified in the literature reviews conducted in this chapter. In particular, a model with stochastic demand and the aim of minimising storage cost and time using PSO, which, in turn, will assist in minimising the negative environmental impacts of ordering and holding inventory in the steel industry, is developed.

3 Research Methodology

3.1 Introduction

In the next chapters, different aspects and areas related to the developed model and the overall purpose of this research study is explored. Before explaining the model's development, this chapter explains the detailed methodology adopted for this research study. As discussed in Chapter 1, the inventory management application is, by nature, a complex task. This is particularly true for the case of the steel manufacturing industry application addressed in this research study. In this context, it is necessary to carefully design the conducted research towards successfully answering the research questions formulated in Section 1.4, and successfully achieving the research objectives. The research design allows the researcher to rely on a well-defined plan for implementing the research strategy, in terms of research sites, and data collection procedures (MacMillan and Schumacher, 2001). Generally speaking, an experimental research approach using a descriptive method is adopted to conduct this research study through the development of a mathematical model, based on the well-known EOQ model, which is capable of capturing the entire business parameters of the research problem under study. Using the experimental approach allows the researcher manipulating one or more variables, while controlling and measuring any change in other variables. In particular, the experimental approach uses standardised procedures that ensure high internal validity when an experimental group is compared to the control group on the dependent or outcome variable (Ross and Morrison, 2004). In this way, it ensures that the differences between groups are attributed to changes in the model's environment rather than external factors. This is crucial for the research study conducted here since the problem of the inventory management in the steel manufacturing industry involves the storage of large-volume products and raw materials, attempting to minimise the high storage and handling the associated costs while maximising the profits and preventing the deterioration of the inventory which depends on different environmental factors.

A systemic approach has been adopted for conducting the research in this thesis. Figure 3-1 shows an outline of the research flow. In the first stage, the problem at stake is thoroughly defined. In this research study, the problem faced is the limited storage of large-volume raw materials and final products that deteriorate over time as a result of environmental factors. Hence, an inventory management system is required which can minimise the storage, handle the costs of these materials and maximise profit for the company. In order to fully understand the current needs of the steel manufacturing industry, a SLR is conducted towards exploring the current trends in the field. Based on the findings of the SLR, the identified research gaps and the specific characteristics of the research problem, the research design is established. The main aim of this study is to handle the storage problem arising from the high volume of raw materials and final products being purchased and produced in the special context of the steel manufacturing industry where limited storage space is available and inventory is subjected to deterioration. In order to address such a problem, a quantitative approach which is characterised by systematically studying a phenomenon, using highly structured data collection methods towards gathering quantifiable data (numbers) that can be analysed resorting to statistical techniques (Saunders et al., 2000). Using a quantitative-based research allows the researcher to obtain statistically significant and highly generalisable results, which are essential characteristics for developing a model as it is the case of the conducted research. In this line, on one hand, the necessary data to achieve the research objectives is collected. In this study, hypothetical data (Gasior and Recchia, 2019) is generated based on different average indicators of the steel industry available in the literature (Pardipto and Lussy, 2019; Tseng and Yu, 2019; Tavakoli and Taleizadeh, 2017; Rabieh et al, 2016) as well as on historical trends and publicly available business reports, such as the ones in (OECD, 2017; World Steel, 2018). On the other hand, a model-based quantitative research is conducted towards developing a mathematical model capable of optimising the purchasing and production activities of the company based on the available storage space. More specifically, the developed model should reduce the storage costs and minimise the waste associated with the manufacturing process resulting from the stochastic nature of customers' demand. In this study, a model extending the EOQ concepts to take into account the specific steel manufacturing industry's characteristics is developed based on a control system algorithm capable of providing timely recommendations for the storage quantities of both products and raw material. In this way, the decisions regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon will be facilitated. In particular, two different control system approaches, one based on an open-loop and other one based on an ANN are used. The main reason for using two different control systems strategies lays in the possibility of determining the most robust one, and reach an acceptable trade-off between model's accuracy and complexity. Finally, PSO techniques are used to optimise the model's parameters.

Once the proposed model has already been developed, computer-based approaches capable of implementing it are designed. In this study, as mentioned in Chapter 1, the developed mathematical model is coded in Matlab. In particular, in order to implement it, the mathematical model is divided into 25 main different equations. In addition, a user interface is developed using the Graphic User Interface (GUI) tool of Matlab to provide managers a simple and useful tool to help them in their decision-making process. Finally, the developed model is validated. In order to do so, the performance obtained with each of the proposed control approaches, namely the open and closed-loop ones, are compared. In this way, the best suited one in terms of technical capabilities, such as robustness, accuracy and efficiency, is selected. Once the best approach has already been selected, a sensitivity analysis is performed. In particular, the developed model is applied to different scenarios characterised by the existence of extreme business or economic conditions, and its performance is evaluated. It is important to highlight that, at this stage, the model's applicability and practicality could also be evaluated by the companies' managers who will be using it in real-life scenarios. Finally, after validating the accuracy and practicality of the model, it could be fully implemented at a real-life steel manufacturing company.

The following sections provides an insight into each of the different research phases depicted in Figure 3-1. In Section 3.2, the collection of the hypothetical data used to validate the developed model is introduced. In Section 3.3, the design of the proposed mathematical model is described in detailed. In Section 3.4, the validation of the developed model is introduced. Finally, the chapter summary is provided in Section 3.5.

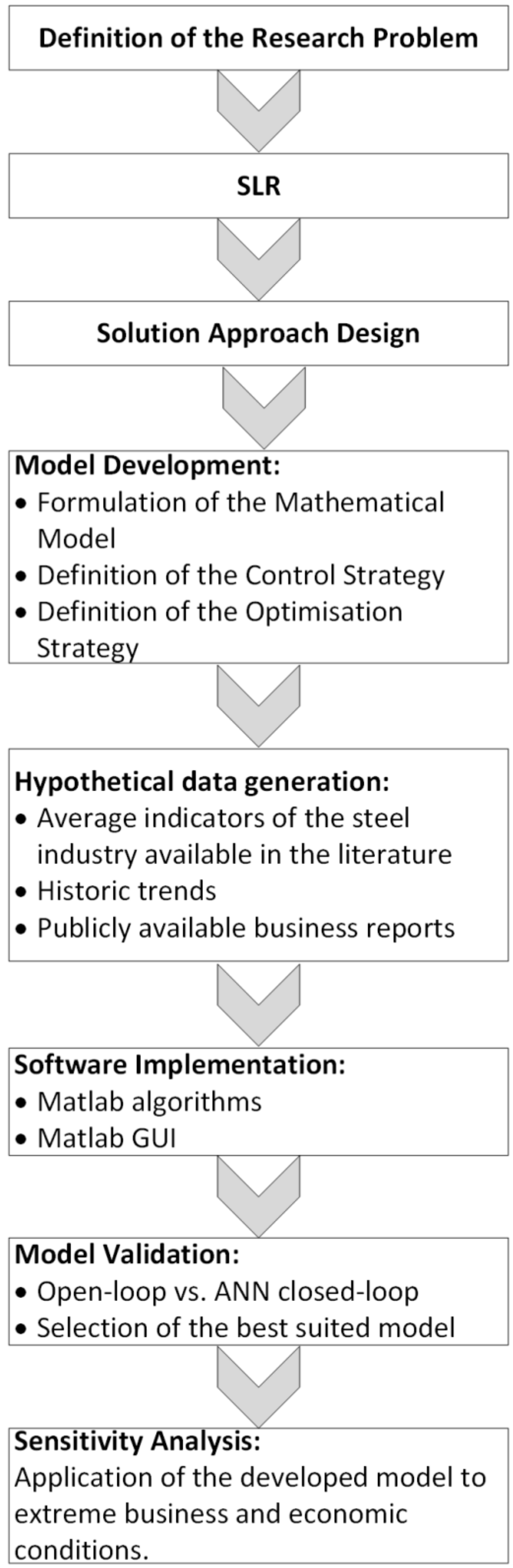

Figure 3-1. Research outline.

3.2 Data Creation and Simulation

In this research study, data about the steel manufacturing industry is required in order to test the developed model after formulating it. One of the most challenging tasks in the steel manufacturing application addressed in this research study is to deal with the lack of available real-life business data. In order to bridge this gap, hypothetical data (Gasior and Recchia, 2019) is generated based on different average indicators of the steel industry available in the literature (Pardipto and Lussy, 2019; Tseng and Yu, 2019; Tavakoli and Taleizadeh, 2017; Rabieh et al, 2016) as well as on historical trends and publicly available business reports, such as the ones in (OECD, 2017; World Steel, 2018). On one hand, publicly available trial data from open source data and previous literature concerning steel indicators like steel prices, steel product demand and storage expenses is used to simulate the needed data. This data can be interpreted as a fair representation of the average current trends of the steel manufacturing industry. In order to ensure the reliability of this data, a strict inclusion criterion for relevant sources and literature is put into place. Based on this criterion, only previous research studies that have been recently published (in the last 10 years) are taken into account. In particular, these studies should have collected their data from primary sources, such as finance, procurement, field or store managers of a steel manufacturing company who have in-depth knowledge and control over the supply chain operations of such companies, are taken into account. In addition, in order to be included, this data is required to have been used in a corresponding research study providing reliable and accurate results. On the other hand, data based on historic records is also used to collect hypothetical data. This data source is quite effective in reducing the cost of the study, as it substitutes the need to collect data through field studies or personal interviews. Finally, it is important to highlight that using publicly available data to simulate and test the developed model is crucial not only in terms of reliability but also towards ensuring the replicability of the obtained test results. In addition, it also allows the obtained results to be comparable to other ones in the state-of-the-art obtained for the same test data, as well as to be potentially used as benchmark results.

Figure 3-2 defines the different types of hypothetical data, as well as their interconnections, that are collected from the described sources. As seen from Figure 3-2, three types of data, viz., inventory, production and selling data are considered. Regarding inventory data, information about the quantity of raw materials ordered and stored is required to test the model's effectiveness. Similarly, the quantity of final products produced and the physical characteristics of these products are collected with a view to inputting requirements for the model, in order to optimise the company's operations. In the case of the selling data, this information is used to determine how many units of the final products were sold, how long it took to sell them, the demand for the final products, and their selling prices and expenses.

Finally, it is important to highlight that, although the validity of the hypothetical collected data can be inferred by the rigorous inclusion criterion described above, the quality of the collected data will be further validated through the accuracy of the obtained results. In particular, if the model implementation achieves reliable and logical outputs based on the collected data for production planning and inventory control parameters, it will further reinforce the validity of the collected hypothetical data.

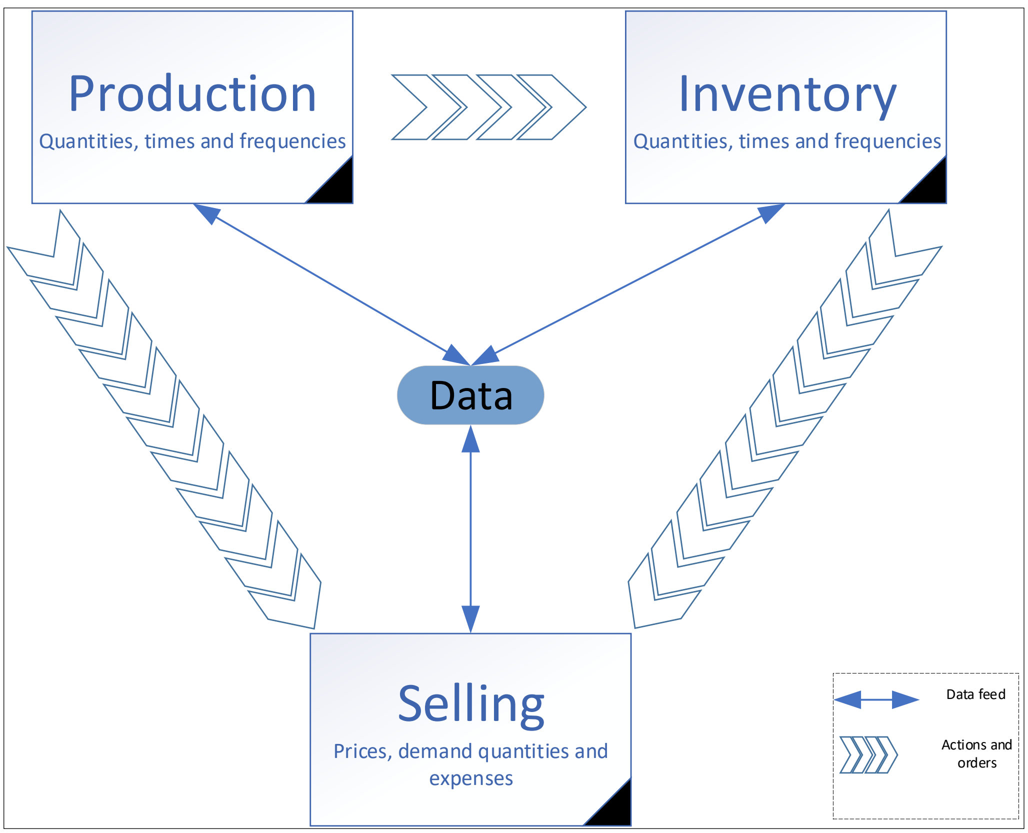

Figure 3-2. Interconnections between different aspects of business data.

3.3 Design of the Inventory Management Mathematical Model

Mathematical models, are used to describe some real-world processes. However, the mathematical model describe "simplified" behaviour of the real process. For instance, when the business scope of the steel-consuming factory is modelled, it would be necessary to take into account the weather forecast which affects storage costs in the sense that if it is hot outside, more air cooling will be needed. Nevertheless, predicting the weather forecast is extremely complex. In this context, storage costs can be modelled as stochastic taking into account different parameters based on historical values for the sake of finding “the golden mean” between the accuracy and the complexity of the model.

In this research study, the inventory management for a steel manufacturing company is mathematically modelled extending the well-known EOQ model. As discussed in Section 2.3, despite its usefulness, the EOQ model does not accurately depict the real-life complexities of today’s business environment. For instance, it assumes that raw material prices and storage costs are deterministic and do not change over time; and its optimisation goal is focus on minimising storage costs rather than on maximising profits. The developed model is then based on the EOQ concept, relaxing some of its assumptions, including the stochastic nature of demand, in order to be more applicable to the steel manufacturing industry. Several steps are involved in the design of the proposed model. In the first step, the key components of the steel manufacturing processes should be identified in order to define the best suited business and economics indicators to include in the model. The identified components will represent the different model parameters. Once these parameters have already been defined, the model structure is designed. The proposed model extends the EOQ concept by including different assumptions capable of capturing the steel manufacturing industry’s dynamics. In this stage, the set of equations modelling the research problem are defined. Once the mathematical model has already been developed, it is necessary to design a control strategy in order to provide the model with the necessary information regarding the business parameters. In particular, the control system should be able to provide measurements and estimations of storage quantities of both products and raw material from the steel manufacturing company. Having these inputs available helps the model to handle timely data about the most important parameters and process them accordingly towards making the best decisions regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon. As introduced in Section 1.4, two different control approaches are used in this research study, namely the open-loop and the ANN based closed-loop ones. Through these control systems, the parameters of the model are adjusted so that the model can better reflect the real-life scenario. In the case of the open-loop system, the parameters are adjust once, and the same parameters are used to the whole planning horizon independently the changes in the business environment. In the case of the ANN based closed-loop approach, feedback is introduced, allowing the model's parameters to be periodically updated taking into account the current business scenario as well as any change that may occur. Figure 3-3 shows a scheme of the closed-loop proposed inventory management model. In this case, whole model works as follows:

- The mathematical model, uses the set of equations describing the steel manufacturing company's dynamics to compute the current model's parameters.
- The current model's parameters are introduced in the ANN.
- The ANN compute the control variables (output of the ANN) by optimising the objective function (company's profit) in terms of the received parameters' values.
- Based on the computed control variables, the PSO algorithm calculate a new set of weights for the ANN.
- Using the new set of weights, the ANN updates the control variables' values.
- The new control variable values are sent to the model, so that it can be updated and the corresponding decisions can be made.

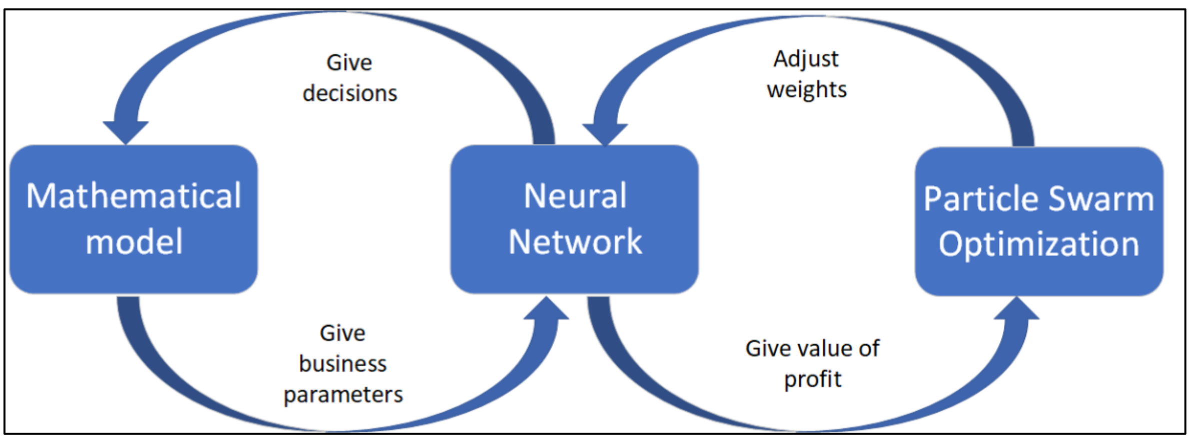

Figure 3-3. Scheme of the proposed inventory management model.

3.4 Validation

One technique to test and verify the solutions of the developed model is to compare the results with already published research results of similar models. Unfortunately, this technique could not be used in this research study due to the unique complexity of the developed model. Instead, in order to validate the model, two different validation techniques are used. First, two solutions of the same model are compared against each other, in terms of optimality. One solution is derived through the use of an open-loop model, while the other is derived from the neural network closed-loop model. Furthermore, the model is verified by applying it to different cases with different sets of stochastic variables that cover the most frequent behaviour of raw materials and final product costs and demand. Some of these cases depict different scenarios of non-stationary stochastic demand, while other cases reflect the stress test scenario, with sudden termination of demand or supply. Such test cases are carried out for sensitivity analysis. Second, a neural network closed-loop control system is applied under several extreme scenarios, which depict extreme economic and business conditions, to track the neural behaviour of the model using different data schemes for the worst case scenarios. Through these two validation techniques, the effect of the production/inventory decisions on supply chain performances can be accurately depicted, and this validation showed that the operational adjustments improve the sustainability of the supply chain and inventory management in the proposed models. This is crucial for the developed models to be widely accepted by practitioners as well as researchers in the field since, as discussed in Section 2.4.4.2.1, taking care of sustainable aspects has become extremely important when performing inventory management.

3.5 Chapter Summary

In this research study, an inventory management model based on the extension of the well-known EOQ model is developed for a steel manufacturing factory. This inventory management model considers the stochastic nature of demand and storage costs, hence capturing the real-life conditions of the steel manufacturing company's business environment. In this section, the research methodology followed to develop the proposed model has been introduced, describing each of the involved steps from the research problem definition to the implementation of the model developed to address it. In particular, the adopted approaches towards creating and simulating the required hypothetical data, the design of the proposed model and the validation protocol have been described in detail.

4 Mathematical Modelling of Inventory Management for High-volume Material within a Limited Space

4.1 Introduction

This chapter describes the development of the proposed mathematical model for the production and inventory planning of a steel manufacturing factory. This model covers all aspects of the steel production process, from the purchase of the necessary raw materials to the setting of the selling price of the final product. The model takes into account both the order and storage queues. The former is essential to determine the time needed to convert raw materials into final products, whereas the latter is essential to model the inventory movement inside the factory through the adoption of the FIFO inventory management system. In particular, the FIFO management system is applied to both raw materials and final products in the sense that the first raw materials received will be the first to enter into production, as well as the first item produced will be the first sold.

The chapter is organised into five main sections. Each of them provides details about a particular aspect of the developed model. First, Section 4.2 provides, in detail, the economic and business nature of the studied system, which are crucial to understand the assumptions behind the developed model. This section starts by reviewing the business cycle of the steel manufacturing factory under study in order to identify the crucial business parameters required for the estimation of the business efficiency. Here, the relationship between the developed model and the EOQ model is highlighted. Section 4.3 describes the whole model development. In particular, Section 4.3.1 defines the model’s parameters based on the business analysis performed in Section 4.2. For any business model, there are two types of parameters, namely stochastic parameters that change throughout the planning horizon, and deterministic parameters that are fixed throughout the planning horizon. In this section, these parameters are defined, their importance is explained, and their application in the scope of the steel manufacturing factory is described. In addition, the different assumptions and constraints used and applied in developing the model are presented. These constraints are included in the software implementation of the model to make the results more plausible from an economic point of view. Section 4.3.2 describes the proposed strategies to control the mathematical model. The main goal of the control system is to provide timely recommendations for the storage quantities of both of products and raw material. This will also facilitate the decisions of the factory’s management regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon. As already introduced in the previous chapters, two different control systems are proposed in this research study. In this section, the difference between them, in terms of their working mechanisms, advantages and limitations, are analysed. Finally, the mathematical model is developed, in terms of defining the set of equations modelling the steel manufacturing factory's dynamics in Section 4.3.3. In Section 4.3.4, the optimisation algorithm used to adjust the model's parameters is presented. Finally, Section 4.4 summarises the entire chapter. In particular, it analyses the business model, its strengths and possible weaknesses. In addition, it discusses about the proposed control systems, and provides recommendations based on the numerical experiments.

4.2 Business Logic Explanation

The main problem in this research is the presence of large-volume products and raw materials which require a large storage area. In addition, when managing the inventory for a steel manufacturing factory, the problem becomes even more complicated, due to the high storage costs and the deterioration of the final product as a result of various environmental factors, such as humidity, prohibiting the long-term storage of such products. Hence, the problem of this research is based on both the product's physical characteristics and its special requirements during the inventory holding period. In this context, there are three sides of the problem concerning the nature of the product, as introduced in Section 1.3:

1. High-volume material that needs a large storage space. Therefore, optimisation management is required to optimise the inventory decisions of how and when to order the raw material from suppliers, based on the production process and the market conditions. The aim of this optimisation process is to reduce the time required to store raw materials and final products to the minimum possible time, in order to reduce any waste that results from long storage periods.
2. The high level of energy required to avoid harmful environmental effects on the product's physical characteristics. As discussed in Section 2.4.4.2.1, the high consumption of energy is one of the main environmental impacts of inventory management, being crucial to reduce it. In order to address this problem, first, the nature of the steel is studied in order to determine the amount of energy that is required to keep it safe from the effects of humidity and preserve its quality; then, by

optimising the storage time for raw materials and final products, the amount of energy required will be reduced, having positive environmental effects.

3. Increasing the company's profit, which can be achieved by reducing the storage costs and the amount of waste produced. In the case of the steel manufacturing factory, the storage costs are stochastic in nature, as several factors can change the necessary requirements to preserve raw materials and final products from deterioration. For example, in hotter weather, more air cooling will be necessary, which means more energy consumed and greater costs.

Furthermore, the main problem facing the steel manufacturing factory under study can be broken down to the following points:

1. The factory needs to use its limited storage area of inventory to meet the stochastic demand within determined supply.
2. The factory needs to use its limited storage area of inventory to cope with frequent stochastic backorders.
3. The factory needs to reduce the storage cost of the high-volume product which requires extra energy to face the environmental conditions.
4. Raw materials and final products cannot be stored for more than one week due to the space limitations and the cost of the extra energy required.
5. The factory needs accurate estimation of the expected demand, supply and backorders for a weekly resolution.
6. The factory needs accurate estimation of stochastic demand to synchronise it with supply chain management in order to achieve a sustainable process between stochastic demand and stochastic supply.

As a result, this research proposes the development of a model that optimises the space and material distribution in the inventory section within the time and area specification. In particular, the research is mainly focused on optimising the required quantity in a limited storage area to meet the required demand and backorders, based on the stochastic nature of the process, in order to give a timely approximation of the optimised quantity on a weekly basis. In order to help in this process, the value of the output variables of the model will be compared with the value of the input parameters of the model, allowing the model to be updated and corrected towards improving its outcomes. The choice of a weekly planning period is based on several business-specific conditions of the steel manufacturing industry. In particular, the following ones can be mentioned:

- The factory has limited storage space, which makes precise planning necessary to achieve a timely approximation; hence, a weekly basis is considered a suitable timeframe to balance supply and demand.
- The operational and production plan for the factory under study is based on a weekly calendar.
- The physical nature of steel and its energy requirements makes the application of timely solutions necessary in order to avoid any defects and deterioration in the products.
- Determining the required optimal quantity of both raw materials and final products on a weekly basis will help in avoiding congestion in the storage area, and/or having empty storage spaces in the presence of high demand and backorders.

Therefore, in this research, a model is developed to help a complex industrial steel manufacturing company to determine its optimal financial investment, steel purchasing strategy, and sales and pricing management for a one-year (52-week) period in order to increase its annual net profit. The one-year timeframe is assumed to be sufficient to observe the seasonal effects that result in price and demand fluctuations, while any larger planning horizon will reduce the accuracy of the future selling price and demand forecasts.

The problem under study deals with three different cases in the inventory section:

1. The stochastic nature of demand.
2. The stochastic nature of supply.
3. The stochastic nature of backorders.

The first step in developing this model is to review the business cycle of the steel manufacturing factory in order to identify the crucial business parameters needed for the estimation of business efficiency. Figure 4-1 outlines the business cycle of the steel manufacturing factory. Basically, the business cycle consists in purchasing and storing raw material composed of steel and then performing value-adding activities to convert it to other steel products through manufacturing/fabrication processes. The facility is a steel structure facility, storing steel products (channels, beams, flat steel, etc.) and performing manufacturing/fabrication activities, such as welding, shearing, cutting, painting and finishing.

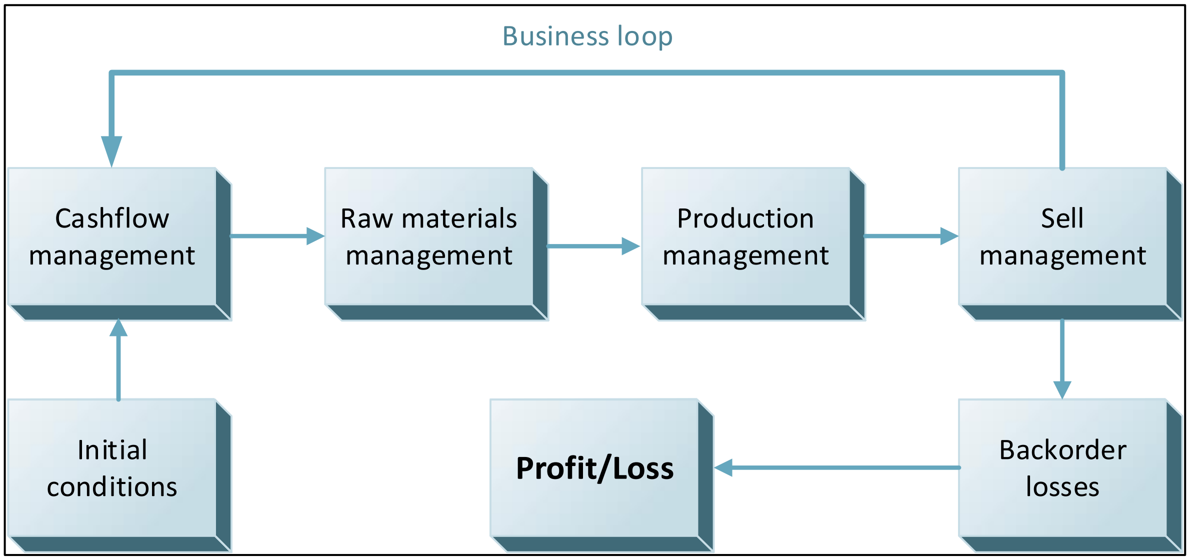

Figure 4-1 Diagram of the business scope of the steel factory.

As seen from the above figure, all weekly business activities need to be grouped into logical components to facilitate the analysis process. Accordingly, the main business cycle of the steel manufacturing factory consists of the following four components:

1. Cash-flow management block: This block consists of the activities performed when the business owner decides on how much profit to keep in the business and how much to distribute to the shareholders as dividends; it also involves any interest payments required. The input for this block includes the initial financial conditions of the system.
2. Raw materials management block: This block consists of the activities performed when there is a need to decide on the amount of raw materials to be purchased.
3. Production management block: This block consists of the activities performed during the estimation of the optimal production quantity.
4. Selling management block: This block consists of the activities performed when the managers attempt to set the price for final products in order to maximise their profit while, at least, covering expenses.

Based on the business cycle introduced above, the company's management must decide on the level of investment, the quantity of raw materials to buy, the quantity of final products to be produced, and the selling price of final products, on a weekly basis.

In addition, the business environment of the steel manufacturing factory is characterised by a set of unique economic parameters and market indicators; these indicators can be divided into two major groups:

1. Deterministic (fixed) indicators: These are the parameters/indicators whose values are fixed over the planning horizon and can be predicted with great precision. Examples of fixed economic parameters include: tax rate, demand elasticity and inflation rate, while examples of fixed market indicators include the deterioration rates of raw materials and final products, the probability of critical defects in the final products, the maximum storage capacity, and staff salaries.
2. Stochastic indicators: These are the parameters/indicators whose future values cannot be predicted accurately due to the randomness of their occurrence. Hence, for these parameters, future trends and expected possible deviation from these trends can only be predicted through the use of historical data. Examples of the stochastic parameters include demand for the final products, the market prices of raw materials and final products, and storage costs.

In general, most crucial business indicators are stochastic in nature, which makes the process of developing a strategy that maximises the company's profit even more difficult. Hence, it is essential to extend the business scope segment of Figure 4-1 as shown in Table 4-1 to investigate all the factors that affect the steel manufacturing business and analyse the company's cash flows and drivers for realising profits or losses.

Table 4-1. Business loop of a single working period (one-week)

Business Loop Component	Sub-steps
Cash flow Management	• Select the percentage of profits to be reinvested in the business and the percentage to be distributed to shareholders. • Adjust the value of the available funds according to the inflation rate. • Add accounts receivables (up credit) and deduct account payables (down credit). • Update the up credit and down credit with any payments made.
Raw Materials Management	• Decide on the quantity of raw materials to be ordered. • Add the value of the down credit needed to order raw materials. • Add the quantity of raw materials to be stored.

Production Management	• Select the quantity of the final products to be delivered and the quantity available for sale after a specific number of periods. • Reduce the available funds by the quantity of products ordered while taking into account that any overtime production will cost more than regular production.
Selling Management	• Select the selling price of the final product. • Calculate the maximum quantity of goods that can be sold in a period. • Adjust for up credit. • Reduce the available funds with the money needed to repair or reproduce the defective products.
Final Steps	• Reduce the available funds by the storage costs of raw materials and final products. • Reduce the actual quantities of raw materials and final products according to their respective deterioration rates over time.
Backorder Adjustment	• Calculate the profits lost due to late deliveries or additional orders of raw material.

Table 4-1 reveals the relationships between the available funds, raw materials and final products in the steel manufacturing industry, as well as other existing relationships in this business; some of these relationships are:

1. Factors: purchasing raw materials, reduction due to inflation and taxes, production costs and storage costs, affecting the level of available funds.
2. The quantity of raw materials increases after purchasing and deteriorates over time as they move to production, and costs extra in the case of backorders.
3. Final products are produced from production lines and wait in storage until they are sold. These products are the only source of profits for the company.

These relationships are used to construct the mathematical model in the next sections and further test it, in the next chapter, on a hypothetical case. Hence, to maximise the company's net profit at the end of the year, the different transactions that contribute to realising profits or incurring losses have to be identified. The net profit for the company includes:

1. Invested money: These are funds invested into securities and not used in the business operations. The goal of this investment is to protect the value of the funds available to the company from the effects of inflation.

2. Cash money: This is the balance of funds available for buying raw materials and/or making production orders.
3. Up credit: This is money owed by customers to the company for the products purchased.

On the other hand, the company can incur losses due to a number of factors:

1. Cash debt: When the company's cash flow is negative, the company starts incurring interest on any outstanding loan balances.
2. Down credit: The money that the company owes to suppliers of raw materials.
3. Backorder extra loss: The money that the company loses because of delayed and/or failed supplies.
4. Cost of remaining raw materials: This is price of raw materials remaining in storage at the end of the planning period.
5. Cost of goods in storage: This is the storage cost of the final products that are not sold at the end of the planning period.
6. Cost of raw materials that are still in the production lines at the end of the planning period.

Finally, in order to accurately model the business cycle of the steel manufacturing factory, the business limitations, as a result of the economic conditions, must be taken into account. These limitations are:

1. The company can make investments, purchase raw materials and produce final products only if it has positive cash-flow. Hence, the maximum quantity of raw materials that can be ordered is calculated, so that after these activities, the company will still have a positive balance of free cash.
2. The company cannot sell the final product at a price that is higher than the maximum allowable price set by governments and/or anti-monopoly regulators.
3. The company is not allowed to directly borrow money from a bank to purchase raw materials.
4. The company uses limit storage space and one market price policy, so the FIFO principle is applied to both raw materials and final products.

It is important to take into account, that inventory optimisation is a supply-chain management method used to avoid having excess levels of inventory, while maintaining the appropriate amount of inventory, where needed, to meet consumers' demand and revenue goals (Iyer, 2012). Traditionally, companies used to manage inventory using the “binge-and-purge” inventory cycles, which involved over-purchasing of inventory to accommodate potentially large demand spikes. This management method produced a lot of waste, as extra items were thrown away or sold at a huge discount (Jackson et al., 2018). On the contrary, inventory optimisation methods attempt to minimise the level of inventory needed, which extends vertically along the supply chain (Iyer, 2012). Hence, in today’s business environment, inventory optimisation is considered a core competence for mid-sized and large corporations, as it provides the potential to save millions of dollars in working capital by reducing the quantity of stored inventory without jeopardising operational efficiency and sales (Willems, 2014). Although there are many optimisation models that aimed at calculating the optimal quantity of stored inventory, such as EOQ and EPQ models, the dynamic lot size model, and the newsvendor model, all of these models have their limitations, complicating their usage in real life. For example, production rarely has fixed and well-defined demand over the planning horizon; hence, in real life, these variables are considered to be stochastic in nature, which would suggest that the traditionally used models are inapplicable, as they all assume that the demand rate is known. To overcome these limitations, the developed model is a joint pricing and inventory management model for high-volume products and raw materials with stock-dependent demand under the conditions of stochastic quantity deterioration of the current inventory, product degradation over time, and the possibility of a positive end-of-cycle inventory level. Moreover, the model incorporates economic assumptions that are as close to real life as possible to reach the optimal solution. In this line, unlike the previously developed stock-dependent models of Dordevic et al. (2017) and Tiwari et al. (2018), which aimed at minimising the storage cost only to maximise the profit, the developed model’s objective function is more complex, as it simultaneously maximises profit from the sold goods and minimises storage, interest and backorder costs. In particular, the objective function of the developed model is to increase the company’s total net worth, which is affected by:

1) The amount of free and invested funds at the beginning of the planning horizon.
2) The amount of up credit and down credit at the beginning of the planning horizon.
3) The quantity of raw materials in storage at the beginning of the planning horizon
4) The quantity of final products in storage that have been ordered and not yet delivered.

Nevertheless, the most important components of the company's total net worth are those related to money, i.e. free and invested funds, up credit and down credit, while, on the other hand, the quantities of raw materials and final products at the end of planning horizon are not the crucial factors in the objective function. In this context, the objective function of the developed model is only indirectly related to optimal storage cost, using a solution from a Pareto optimal set (Brownstein, 1980), which allows maximising income while keeping storage expenses low. Finally, the developed model has the advantage when compared to previous stock-dependent models in the state-or-the-art, such as the ones in Singha et al. (2017) and Farhangi and Mehdizadeh (2016), that it is aimed at finding the optimal order quantity not only of raw materials but also of finished products. Figure 4-2 shows the relationships between storage space, the stochastic variables and the decision variables (controls) included in the proposed model.

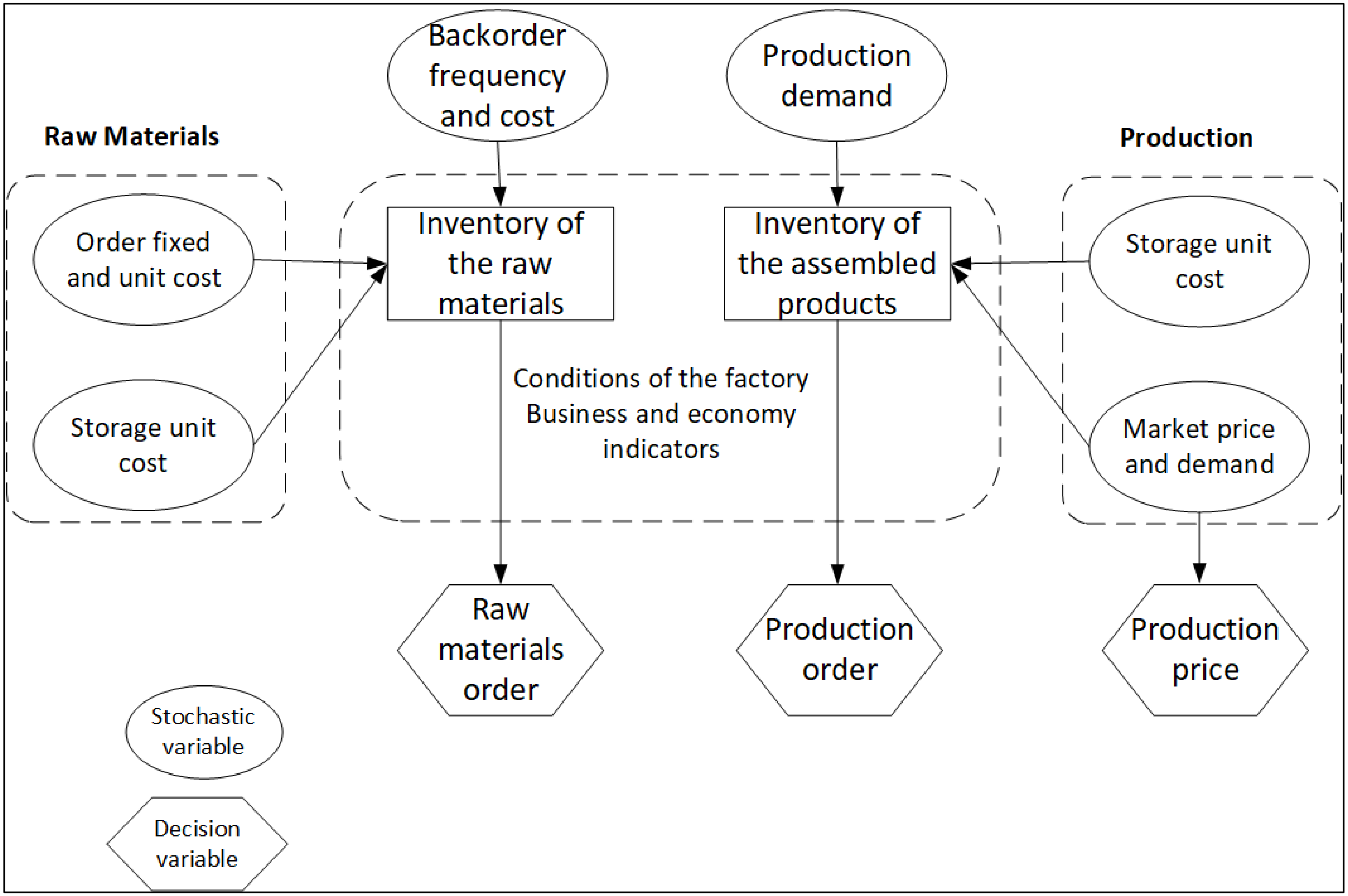

Figure 4-2. Inventory influence diagram.

As seen from the above figure, the level of stored raw materials and the purchase cost of these raw materials, which is stochastic in nature, are the main factors that influence the raw material order policy. At the same time, the quantity of the final assembled products and their unit costs and demand, which are again stochastic in nature, are the main factors that influence the selection of the final product order policy. Finally, regarding the price setting of the final products, the main factor influencing this decision variable is the quantity of final products held in storage, i.e. if the factory has a large quantity of final products in stock, then it will have to reduce its selling price to be able to sell them in time and reduce the corresponding storage costs.

Figure 4-3 shows the production assembly and the storage system when applying the relationships in Figure 4-2 in a steel manufacturing factory. As demonstrated in Figure 4-3, most of the steel manufacturing factories need to continuously adjust their production orders (represented by the dotted arrows) according to the level of the final products in storage. Therefore, when the quantity of final products in storage is high, the general managers of the steel manufacturing factories must either instruct the production manager to reduce the level of production, i.e. produce less products, or instruct the sales manager to increase efforts to sell the final products, or sell them at a discount. Nonetheless, in most cases, and in the absence of an optimisation management process, the general managers of the steel manufacturing factories must adjust their production levels manually, based on their previous experiences, without the help of accurate control systems that take into account the market and economic conditions, which can ultimately hurt the company's profits.

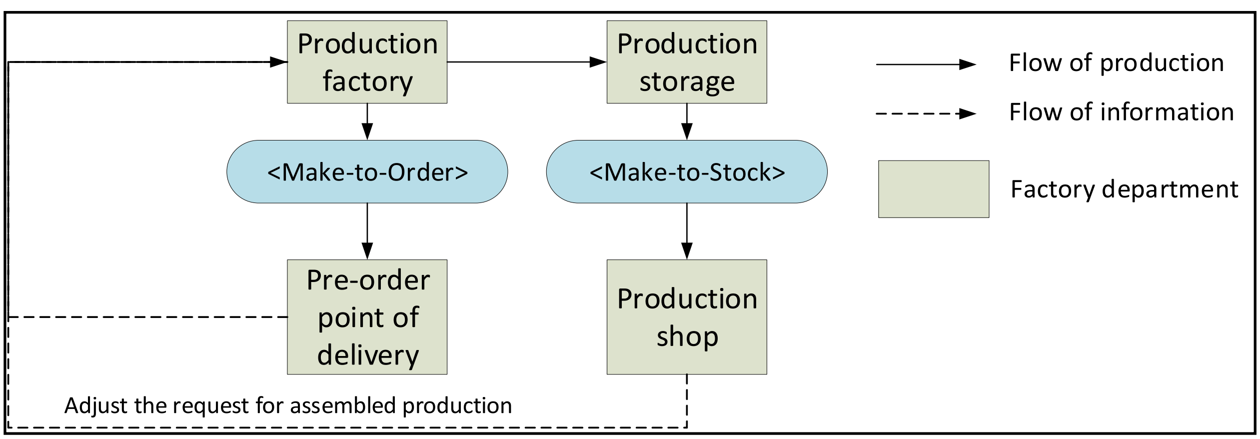

Figure 4-3. Production assembly and storage system.

As a result, to create a control system that operates and provides managerial advice, an efficient analysis of storage efficiency must be conducted by utilising knowledge of the following parameters:

1. The inventory turnover: The period of time between the purchase of one item of raw material and the use of that item in production.
2. Final product turnover: The period of time between producing a unit and selling it.

To answer these questions, the model assumes a storage queue of First-In-Fist-Out (FIFO) principle, in which the first raw material purchased is the first one to be used in production, and the first final product produced is the first one to be sold. This FIFO principle is used by the selling companies as they want to sell or use the oldest product in order to prevent it from deteriorating and causing losses (Khan et al., 2018). Therefore, the storage of raw materials and final products in the developed model also follows the FIFO principle. Thus, the maturity of each item of raw material is tracked and further analysed in the order to test the model and the control system.

Figure 4-4 shows the flow of funds within the steel manufacturing factory's business cycle. As explained earlier, the only source of income for the factory is the profit earned from selling the final products (up credit), which is reduced by the applicable tax rate. On the other hand, there are several sources of expenses for the factory, which include down credit (money for purchased raw materials), storage fixed and unit costs, production costs, and backorder losses. Finally, other economic parameters, such as inflation and interest rates, can also contribute to additional expenses for the factory. These type of factors were rarely considered in past EOQ or EPQ models since they complicate the model's computations significantly. In this research study, the use of ANNs to build the closed-loop control approach allows including more variables to the model without further complicating its calculations. In particular, when using ANNs adding extra parameters to the model will not alter the numbers of the inputs and outputs of the model, remaining the ANN architecture the same. In this way, when using the ANN based control approach, the resulting model will be suitable to model more complex real-life scenarios.

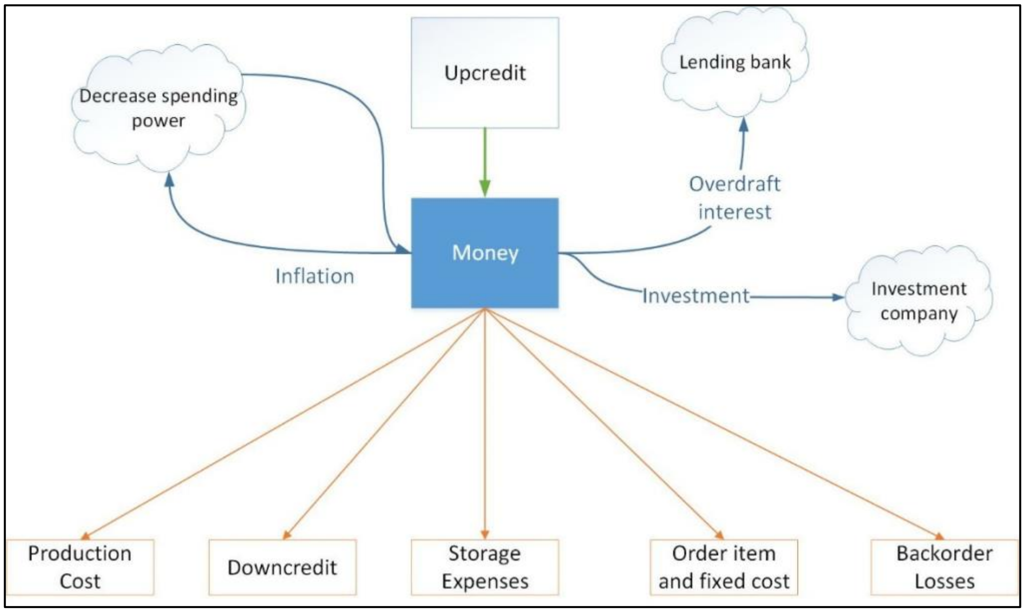

Figure 4-4. Scheme of money flows in a steel-consuming factory.

Finally, Figure 4-5 displays the flow of raw materials from the time they are purchased to the point at which they enter into production. During this lifecycle, raw materials and final products might need to be stored in the warehouse, thus the company will incur the following expenses:

1. Fixed storage costs, such as electricity and air conditioning.
2. Unit storage costs, such as rent for extra square meters of storage, and salaries for warehousing workers.
3. Deterioration costs: when items are not sold, and kept in storage for a long time, they deteriorate, hence the factory incurs costs for items that are not used.

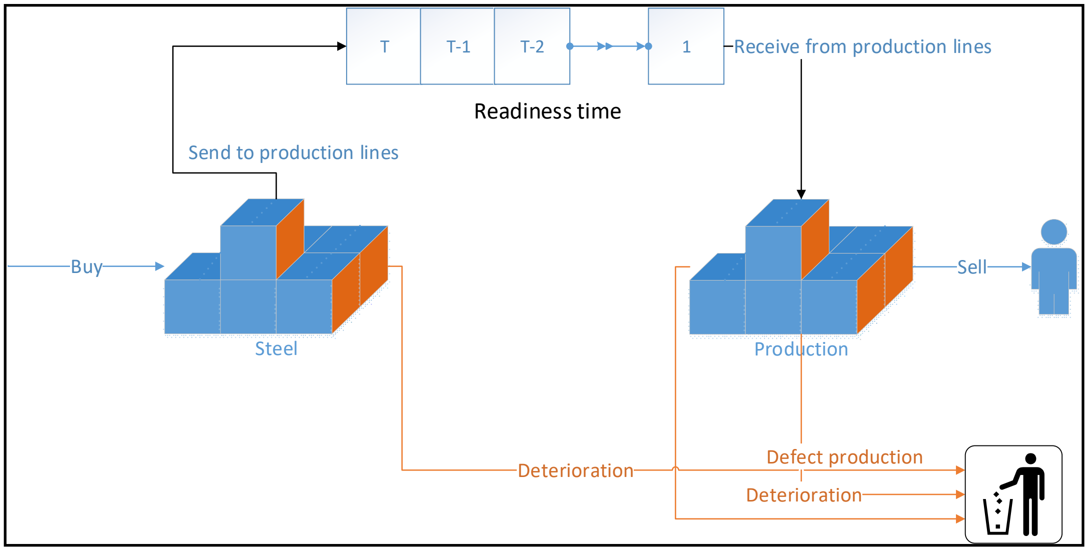

Figure 4-5. Scheme of steel flows in a steel manufacturing factory.

Here, the logical strategy would be to move all the available inventory to the production lines; this would lead to extra quantities of the final product that cannot be sold instantly. This shows that there are no "simple" strategies that lead to high income and low storage costs simultaneously.

4.3 Development of the Steel Manufacturing Inventory Model

In this section, the development of the inventory model for the steel manufacturing industry proposed in this research study is described. Section 4.3.1 provides the mathematical definition of the model's assumptions and constraints. In Section 4.3.2, the control system approaches used to implement the model are described. In Section 4.3.3, the economic model is actually converted into a set of equations modelling the steel manufacturing dynamics. In Section 4.3.4, the optimisation algorithm used to adjust the model's parameters and solve the equations introduced in Section 4.3.3 is developed.

4.3.1 Stochastic and Fixed Variables / Parameters Used in the Model

In this section, the types of variables, the used assumptions and the model's constraints are outlined. Concerning the types of variables, since the model of a steel manufacturing factory described in Section 4.2 covers all the aspects of this business, many parameters related to raw material purchase, investment, production ordering, marketing and storage should be taken into account. Some of these parameters, such as production time, are unique for each type of steel manufacturing business, and can vary from several weeks to several months, while others are related to the economy in general, such as tax rate and inflation rate. Hence, all variables used in the developed model can be categorised under four groups as follows:

1. Fixed variables: Input variables that define the economic and business parameters that do not change over time.

2. Stochastic variables: Input variables that define the business indicators that change over time.

3. State of business: Variables for the following business week which are calculated based on the fixed variables, current value of stochastic variables, and current controls.

4. Controls: Decision variables which are calculated based on the current value of the stochastic variables and current state of business.

4.3.1.1 Assumptions

Figure 4-6 shows the interactions between three critical aspects for the steel manufacturing inventory management, viz., the supply, the demand and the backorders of the steel production process.

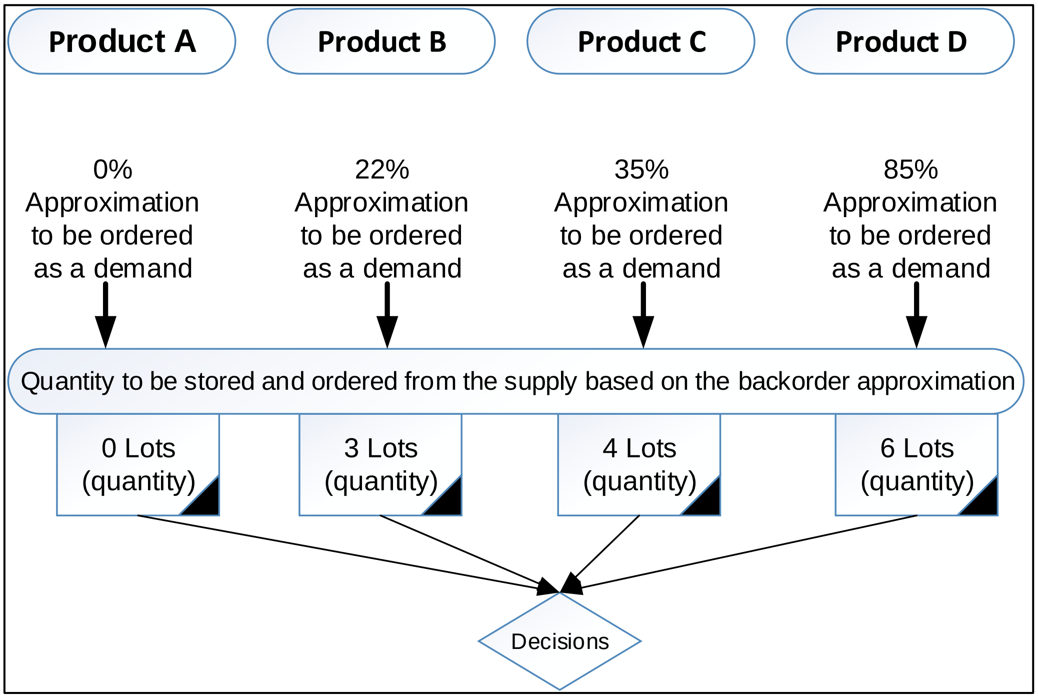

Figure 4-6. Interactions between supply, demand and backorders.

In order to succeed in fulfilling the specific inventory requirements of the steel manufacturing, the variables shown in Figure 4-6 as well as their interactions should be properly handled. In this line, different assumptions, which affect each other and interact with other sub-assumptions, govern the steel manufacturing process modelling. These assumptions are summarised in Table 4-2, illustrated in Figure 4-7, and described in detailed as follows:

- Assumption 1: According to Figure 4-6, first assumption considers a fixed supply determined hypothetically. On the other hand, the demand and backorders for the final products are assumed to be stochastic in order to reflect the case of a commercial environment for the known and continuous supply of material, as well as to show the effect of the stochastic nature on its continuity. In addition, most of the orders in a given period of time will be fulfilled. This approach gives managers more flexibility to place orders based on the demand level.
- Assumption 2: Within the context of the second assumption, demand is categorised as deterministic, while there is no continuity or assurance regarding the supply chain. This implies that there is a stochastic limit to the quantity of raw materials that the factory can order in a period. This case reflects the lack of material in the production

section when there is stable demand. In this context, managers will be able to make decisions regarding the source of supply and its varieties.

- Assumption 3: This assumption considers both demand and supply to be stochastic, and both impact the levels of demand and supply. This approach provides accurate estimation of the required material to be stored in order to meet the sudden backorders of specific products, enabling managers to take action by storing greater quantities to meet the predicted demand for specific items. Hence, this assumption treats every parameter in the problem as a stochastic parameter, reflecting the unstable and non-continuous behaviour of the production section regarding the demand, supply and backorders. This assumption enables managers to be always ready for any backorders based on tight estimation; therefore, there will always be a quantity of raw materials ready for production based on the type of order and products, as depicted in Figure 4-7. Furthermore, managers will be more accurate regarding the decisions that are taken in the steel manufacturing factory.

Table 4-2. Summary of the Different Model Assumptions

		Demand	Supply	Backorder
Assumption # 1	*Deterministic*		✓	
	Stochastic	✓		✓
Assumption # 2	*Deterministic*	✓		✓
	Stochastic		✓	
Assumption # 3	*Deterministic*			
	Stochastic	✓	✓	✓

In addition, in order to succeed in the complex task of maximising net profit for the steel manufacturing company over the entire 52-week period, several sub-assumptions are made for the developed mode to simplify it. These further assumptions are:

1. The model is based on one market price policy scenario, which means that the fluctuating in the prices of raw materials has almost similar behaviour for all the suppliers.
2. The model is based on the assumption that weekly storage costs are similar.

3. The cost of additional storage is higher than the cost of storage at the company's warehouse.
4. One unit of raw materials is used to produce every unit of production.
5. Fixed parameters for the raw materials include long-term contracts which are unlikely to change.
6. Deterioration rates are dependent on the physical characteristics of the item, can be calculated precisely, and are fixed as long as the storage conditions remain the same.
7. The maximum production capacity is fixed.
8. The raw material fixed costs include transportation costs.
9. The percentage of moderate and major defects is constant throughout the planning horizon, as long as the production lines are not upgraded.
10. The amounts of up credit and down credit to pay weekly are fixed and defined by contracts with banks.
11. The inflation rate is fixed throughout the planning horizon.
12. The cost to produce a single unit is fixed as long as salaries and other production costs do not change.
13. Demand elasticity is fixed because it depends on the product that is produced.
14. Lead time is assumed to be fixed.
15. The initial cost of raw materials includes the sum of the initial expenditures, such as transportation, installation, preparation for service, and other related costs (Parker, 2003). Hence, it is assumed that the initial cost of raw materials in storage is higher than the raw materials purchase price, while the initial cost of production in storage is lower than its selling price.
16. Demand is stochastic and stock-dependent.

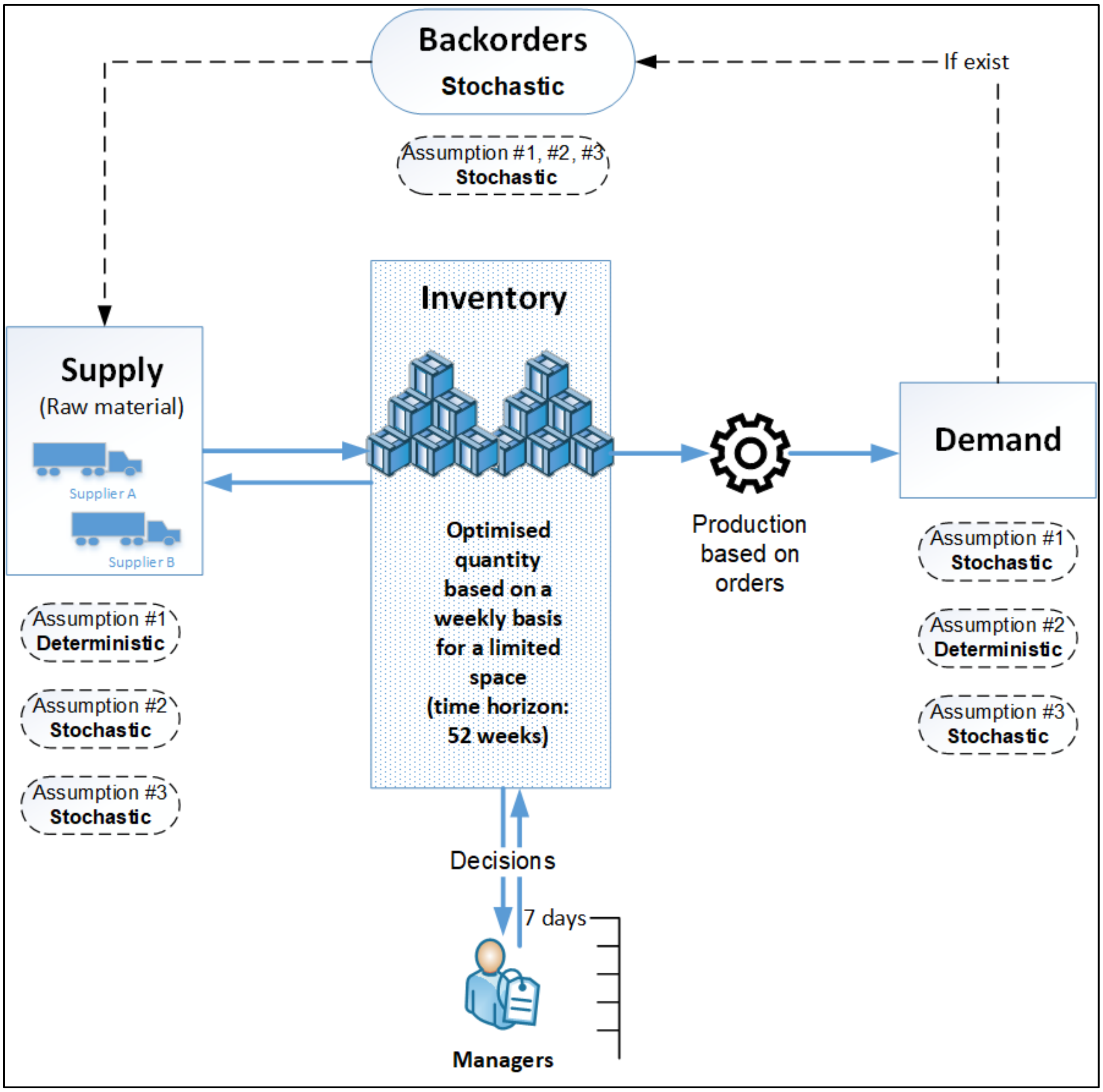

Figure 4-7. Schematic representation of the different model assumptions described in Table 4-2.

The last assumption in the above list establishes that, in the developed model, the demand is considered stochastic and stock-dependent. This assumption deserves a detailed explanation. As discussed in Section 2.3, one of the main assumptions of the classical EOQ model refers to the deterministic and stationary nature of the demand (Kumar, 2016), being usually recommended for environments with steady and predictable demand. Within the context of stochastic demand environments, as is the case of the steel manufacturing industry, the EOQ model is sometimes used to approximate the order quantity in the continuous review inventory system. In this research study, real demand is estimated by linking base demand and market price via an elasticity linker function. Elasticity is generally used to evaluate to what extent individuals change their demand or supply as a result of price or income changes (Hursh, 1984). In particular, elasticity is widely used in the literature to assess the change in consumer demand as a function of a change in a good or service's price. Figure 4-8 shows the demand vs. price curve obtained based on the hypothetical data (generated as described in Section 3.2) for different elasticity values. In this research study, the actual demand has been calculated using the current selling price of the products set by the company $price(t)$, the market price $C_{prod}(t)$ and the demand for this price $D_{prod}(t)$. From Figure 4-8, it can be inferred that high elasticity parameters are associated with goods that customers can easily refuse, whereas low or zero elasticity is characteristic for goods that the consumer must buy regardless of the cost, existing no substitutes for these goods. Note the reader that, in Figure 4-8, all the elasticity curves intersect at the point of market price and the corresponding basic demand, £30 and 70 units, respectively. Based on the demand behaviour depicted in Figure 4-8 and the specific characteristics of the steel manufacturing industry business described in Section 4.2, the elasticity is assumed to be constant and equal to five for the developed model in this paper. This selection means that any decrease in price will lead to a significant increase in demand. This is indeed the real scenario for the steel manufacturing industry since, due to its high competitive nature, any slight decrease in the company's selling price can attract more customers. In addition, as suggested above, high elasticity is often characteristic of unified and standardised products that can be easily replaced by analogue of concurrent companies, as is the case of steel products. In fact, this has particularly been confirmed for such products by a previous investigation of steel production elasticity where it has been found that, for different kinds of steel production factories, the elasticity value ranges between four and six.

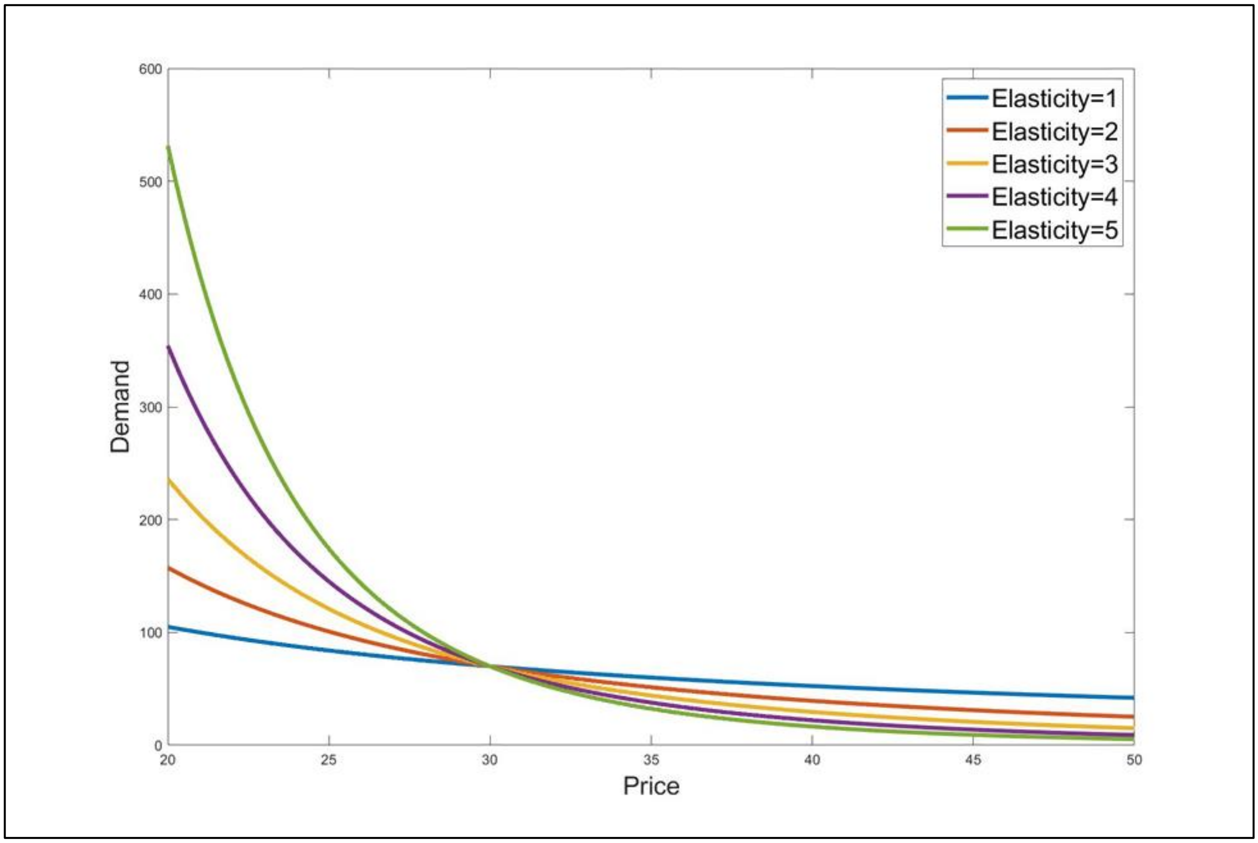

Figure 4-8. Demand plots for different parameters of elasticity (calculated for basic price of £30 per unit and basic demand of 70 units of production per week).

4.3.1.2 Constraints

This section introduces the constraints that are used in the mathematical model to impose limitations on the business actions that can be taken by the factory's management. These constraints can be classified as business controls constraints and raw materials constraints.

A. **Business controls constraints**

First, all of the four controls have relative representation rather than absolute, and their values range from zero to one. Next, the following four constraints correspond to each of the business controls.

$$0 \leq u_{inv}(t) \leq 1, t \in \{1,2,\dots,T\} \tag{4-1}$$

$$0 \leq u_{buy}(t) \leq 1, t \in \{1,2,\dots,T\} \tag{4-2}$$

$$0 \leq u_{order}(t) \leq 1, t \in \{1,2,\dots,T\} \tag{4-3}$$

$$0 \leq u_{price}(t) \leq 1, t \in \{1,2,\ldots,T\} \quad 4\text{-}4$$

The first constraint sets the condition that the company cannot invest more money than it possesses. The second constraint restricts the buying of additional raw materials if the company cannot afford it. Regarding the third constraint, this constraint sets the maximum quantity of final products to be produced to the level of inventory in storage. Finally, the last constraint restricts to set price larger than max price. This constraint is dictated by economic theory about price setting: if price is too high then all customers will switch their choice to competitor factories. We can set max price as maximal observed market price in steel market throughout historical time interval.

B. **Raw materials constraints**

The following constraints are used to impose restrictions on the amount of raw materials to be used in producing the final products, and the quantity of the final products to be sold. These constraints basically restrict the quantity of raw materials to be sent to production to the quantity ordered, and the quantity of the final products sold to the quantity produced.

$$ord(t + t_{lead}) \leq raw(t), t \in \{1,2,\ldots,T\} \quad 4\text{-}5$$

$$sell(t) \leq prod(t), t \in \{1,2,\ldots,T\} \quad 4\text{-}6$$

4.3.2 Control Systems for Optimal Business and Storage Strategy Derivation

In this research study, the proposed EOQ model is extended by using a control system which provides measurements and estimations of storage quantities of both products and raw material from the steel manufacturing company. Having these inputs available will help the whole model to handle timely data about the most important parameters and process them accordingly towards making the best decisions regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon. As introduced in Section 1.4, two different control approaches are proposed in this study, namely the open-loop and the ANN based closed-loop ones. Finally, a PSO technique is employed to solve the developed model.

A control system is an algorithm that generates the optimal decision variables for an entity throughout the planning horizon (Stefano, 1976). There are different types of control systems; however, the decision to use a particular type in the developed model depends on the results obtained from that system in terms of the optimisation objectives. Due to the complex nature of the developed model, the model requires training, which is the process of changing the control system parameters iteratively in order to increase the profit. This process is conducted by feeding a set of predefined scenarios over and over into the control system until an average satisfactory profit for these scenarios is achieved. In order to achieve the research objectives, the proposed control systems are updated on the basis of their respective output parameters regarding:

1. Maximising the factory's profit.
2. Providing sufficient stability, i.e. effective business management under all economic conditions. This condition is especially important because a control system needs to perform well not only on the samples from the training set, but also on any valid testing set of stochastic economic parameters.

During the training process, multiple evaluations of the objective function with different decision variables are performed for the sake of adjusting them so that the objective function is maximised. This objective function in itself can be deterministic, having the same results if the exact same controls are used to derive it; or stochastic, with different results, following a certain probability distribution, even when using the exact same controls. In the case of the steel manufacturing industry addressed in this study, the control parameters, for each week over the entire planning horizon, are:

a. Investment
b. Raw materials purchasing
c. Production ordering
d. Price setting

These parameters are adjust using an external algorithm, the optimiser, which in the case of the developed model is the PSO algorithm, as already discussed in the previous sections (Kirkpatrick et al., 1983; Nazari-Heris et al., 2018).

In this research study, two types of control systems are considered:

1) Open-loop control system.
2) Closed-loop control system.

A schematic diagram for each one of them is shown in Figure 4-9. Figure 4-9(a) shows the scheme behind the open-loop system. As shown from this figure, the control system depends only on the current time, and the entire logic of the control system is hardcoded inside the control block. On the other hand, Figure 4-9(b) shows the scheme behind the closed-loop system, which uses the feedback from the current state of the model and the current external parameters to develop the logic of the system (Berger et al., 2018).

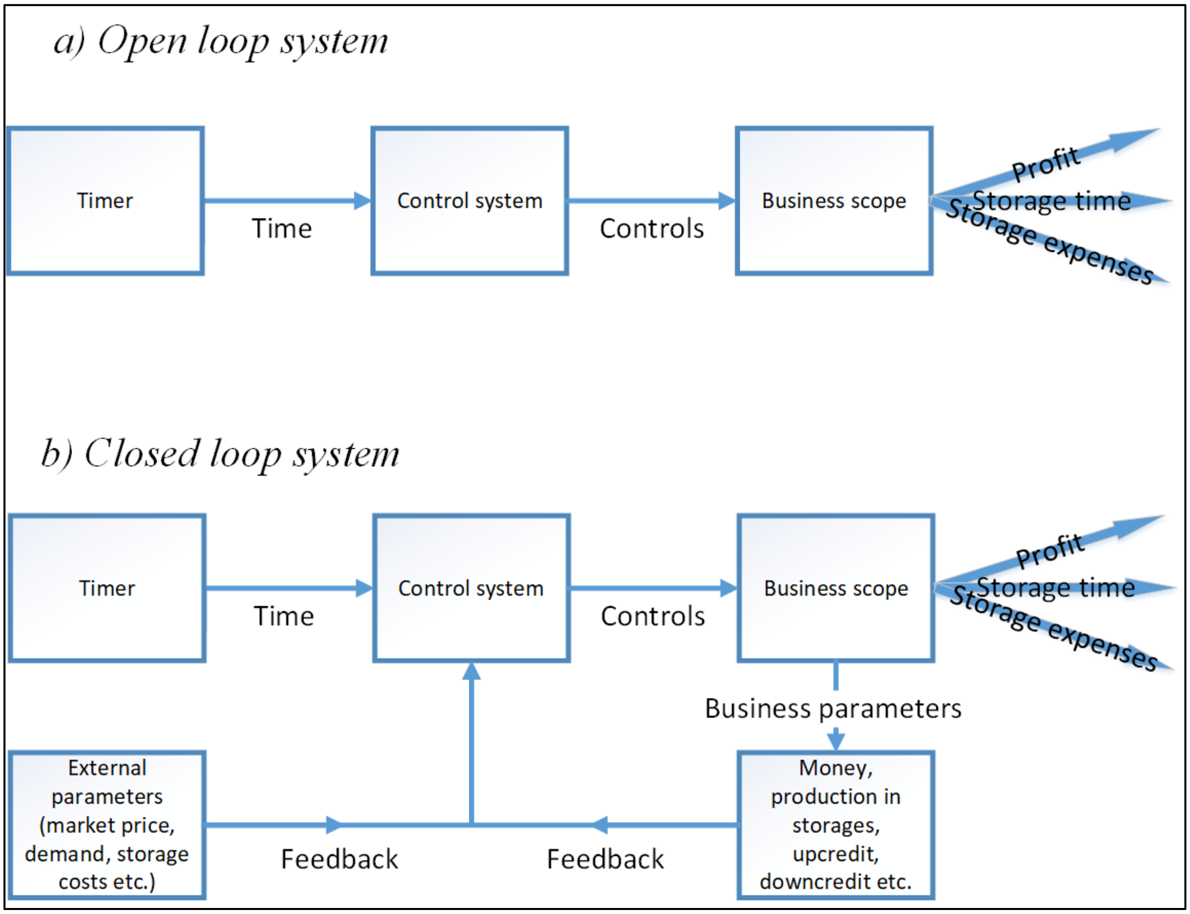

Figure 4-9. Scheme of the open loop and close loop control systems for models that are dealing with optimal inventory problems.

In order to select the most applicable control system for the steel manufacturing factory, three different systems are considered and compared, two of them are based on the open-loop approach, whereas a third one is a closed-loop system based on ANNs.

4.3.2.1 Open-loop Control System

4.3.2.1.1 Direct control optimisation for fixed set of Monte Carlo runs

In this system, five instances of external stochastic factors are generated using the Monte Carlo method (Kroese et al., 2011; 2014). Since the business model of the steel manufacturing factory described in Section 4.2 is dependent on time, the Monte Carlo method is a suitable option to generate the variables. Moreover, this method is chosen over some other methods, such as stochastic approximation (Robbins and Monro, 1951), stochastic gradient descent (Hinton, 2016), finite-difference (Kiefer and Wolfowitz, 1952), simultaneous perturbation stochastic approximation (Spall, 1992), and scenario optimisation (Calafiore and Campi, 2006), which are mainly used to solve algebraic equations with stochastic right-hand side functions. After generating the stochastic variables, the control for the entire planning horizon of 52 weeks that maximises the average profit for all instances is assumed. In other words, five different time series for each stochastic variable are generated, then an attempt to optimise the average profit is conducted by using the following four decision variables for each week:

a. Investment ratio
b. Ratio of funds to spend on purchasing raw materials
c. Ratio of raw materials to send to the production lines
d. Current selling price as a percentage of maximum allowed selling price.

This approach tries to find the one strategy that is not dependent on the actual values of the stochastic variables and, at the same time, provide the maximum possible profit. Therefore, since, in the case of the developed model, there are 52 weeks in the planning horizon, the model returns four fixed numbers each week, i.e. a total of 208 parameters to be adjusted, *viz.*, investment ratio, ratio of funds to spend on purchasing raw materials, ratio of raw materials to send to the production lines, and the current selling price as a percentage of maximum allowed selling price. The main advantage of this control approach is its simplicity. On the other hand, its major drawback is that it does not incorporate feedback about the actual state of the business or the stochastic parameters. That is to say, neither measured or predicted current values of the different model's variables nor measured or predicted values of the current external parameters can be evaluated and used by the model to update its internal logic (Berger et al., 2018). In this scenario, the model cannot adjust its internal parameters to reflect any change in the economic or business environment over the planning horizon. Finally, another disadvantage of this approach is the limited amount of training data available to train it.

4.3.2.1.2 Direct control optimisation for dynamically generated scenarios

This system is very similar to the previous system, except for the fact that stochastic parameters and indicators are regenerated with each training iteration. Therefore, the objective of this system is to find the set of controls that maximises the mathematical expectation of the objective function value for all possible variations of the external stochastic indicators during the planning horizon.

The previously mentioned two open-loop control systems provide one set of controls regardless of the actual economic parameters. For example, even if economic conditions indicate that it is better to decrease the selling price because of low demand, such a control system will blindly follow the final control numbers for all 52 weeks. To overcome such limitations, a novel model is developed in this study, which uses ANNs that dynamically predicts optimal controls.

4.3.2.2 ANN control optimisation with feedback

In Figure 3-3, a scheme of the proposed closed-loop approach where the interaction among the mathematical model based on the EOQ concepts, the system proposed to control the model and the PSO algorithm can be seen in detailed, has been introduced. According to this scheme, in the first place, the inventory management behaviour of the steel manufacturing industry is modelled by a set of equations that extend the EOQ concept by including different parameters to the model that can catch the specific dynamics of the steel manufacturing industry. These parameters are used as inputs of the ANN. Then, the ANN processes these parameters according to the ANN fundaments described in Section 2.4.4.1.1 and provides the set of outputs that maximises the objective function (profit function). This set of outputs, which are used as control variables, is input to the PSO algorithm in order to adjust them. Based on the actual values of the control variables, the PSO calculates an updated set of weights for the ANN. In the literature, 99% of ANNs are optimized using feedforward procedure. However, in the case addressed in this paper there is an objective function at the end of year, rather than a “target” value of controls that is wanted to achieve during each week. Therefore, PSO is the only choice to fit the ANN weights. The training process is then as follows:

1) All weights of ANN are concatenated into one single vector
2) This vector is passed into PSO algorithm

3) After simulation yearly Total Net profit is evaluated, which serves as an objective function for PSO algorithm
4) Weights of ANN are modified according to PSO algorithm
5) Resulting vector is decomposed into layer and bias weights (inverse operation to the step 1)

Finally, the updated weights are used by the ANN to compute the control variables and provide the updated values to the mathematical model so that the corresponding decisions can be made.

As seen from Figure 3-3, the proposed closed-loop model uses an ANN to generate the optimal control parameters (Sanchez-Sanchez & Izzo , 2018) which in the case of a steel manufacturing factory are investment, raw materials purchasing, production ordering and price setting for each week over the entire planning horizon. That is to say, the ANN is fed by stochastic parameters and business state parameters on a weekly basis, and provides an output of four optimal control variables that are used as input for the following week. In particular, the following business parameters and stochastic variables are used as inputs to the neural network:

a. Available funds
b. Down credit amount
c. Up credit amount
d. Quantity of raw materials in storage
e. Level of the final products in storage
f. Final products to be produced during the current and following weeks
g. Cost of one unit of raw materials
h. Fixed ordering cost
i. Market price for the final product
j. Demand for the final product
k. Storage cost for raw materials
l. Storage cost for the final products

In Section 2.4.4.1.1, the basic fundaments of ANN are introduced. In order to use ANN in the developed model, the basic scheme shown in Figure 2-3 is extended to the ANN architecture shown in Figure 4-10. The architecture in Figure 4-10 is defined by a trial and error methodology. Here, the only constraint imposed is the number of weights of the ANN which should be kept as small as possible in order to have acceptable results. As it can be seen from Figure 4-10, a feed-forward topology is selected. This type of ANN are widely used in cases where the target classes are hard to classify. In particular, simple feed-forward neural networks are capable of dealing with noisy data and are also relatively simple to maintain. In order to build the ANN, different parameters should be defined. In the first place, the activation function for the hidden and output layers should be defined. In this research study, the activation function for the hidden layers is a Rectified Linear Unit (ReLU) (Kriesel, 2007) given by Equation 4-7:

$$relu(x) = \max(x, 0), \tag{4-7}$$

whereas the activation function of the output layer is a sigmoid function (Kriesel, 2007) (Equation 4-8) which allows output control to assume the values from zero to one:

$$S(x) = \frac{1}{e^{-x}+1}. \tag{4-8}$$

Activation functions are crucial for ANN, as they add nonlinearity to the modelled process. Without activation functions ANN would be able to model only linear functions. In this reserch, the ReLU activation is chosen for the hidden layers since it is widely used in ANN and outperforms classical activation functions (Nwankpa, 2018). According to the model definition, all four controls should be in the range from 0 to 1. Therefore, activation function of the output layer (sigmoid), it is needed to map output to the [0,1] interval.

Once the activation functions have already been defined, the number of hidden layers and the number of neurons in each layer (input, hidden and output) should be established. The proposed ANN based model assumes the economic indicators and business parameters listed above as inputs. In this way, the input layer consists in 13 neurons, each of them representing one of these parameters. The following step is to decide the number of hidden layers, as well as how many neurons each of them should contain. Here, if few hidden layers are used, high training and generalisation errors may occur due to under fitting. On the other hand, if too many hidden layers are used, the training errors will be low, but the training process will be unnecessarily slow, resulting in poor generalisation because of overfitting. There exists then a trade-off between number of hidden layers and training and generalisation errors. In order to find the optimal number of hidden layers as well as of neurons in each one of them, different tests are performed using different values of such parameters on a separate dataset. In this way, the number of hidden layers and neurons that obtained the best results, in terms of maximising the objective function, on this separate data set are selected to build the ANN for the developed model. In particular, the best results are obtained with one hidden layer containing 10 neurons. The output of the ANN model should provide the control variables that will be used by the model to adjust itself to the current business environment. As discussed in Section 4.2, the control variables are the investment, raw purchase, goods order and price rates, making a total of 4 neurons in the output layer.

Another important ANN parameter that needs tuning are weights. This is done in the same way explained for the number of hidden layers and neurons. The model is evaluated on a separate dataset using different set of parameters and the set that maximises the objective function on this dataset is chosen. Finally, as it can be seen from Figure 4-10, bias is introduced as an additional parameter in the proposed ANN architecture, to adjust the output along with the weighted sum of the inputs. The value of this bias is constant, which helps the model in finding the best fit for the given data. Furthermore, a bias value allows the shifting of the activation function either to the left or right, which is important for the success of the learning process of the system.

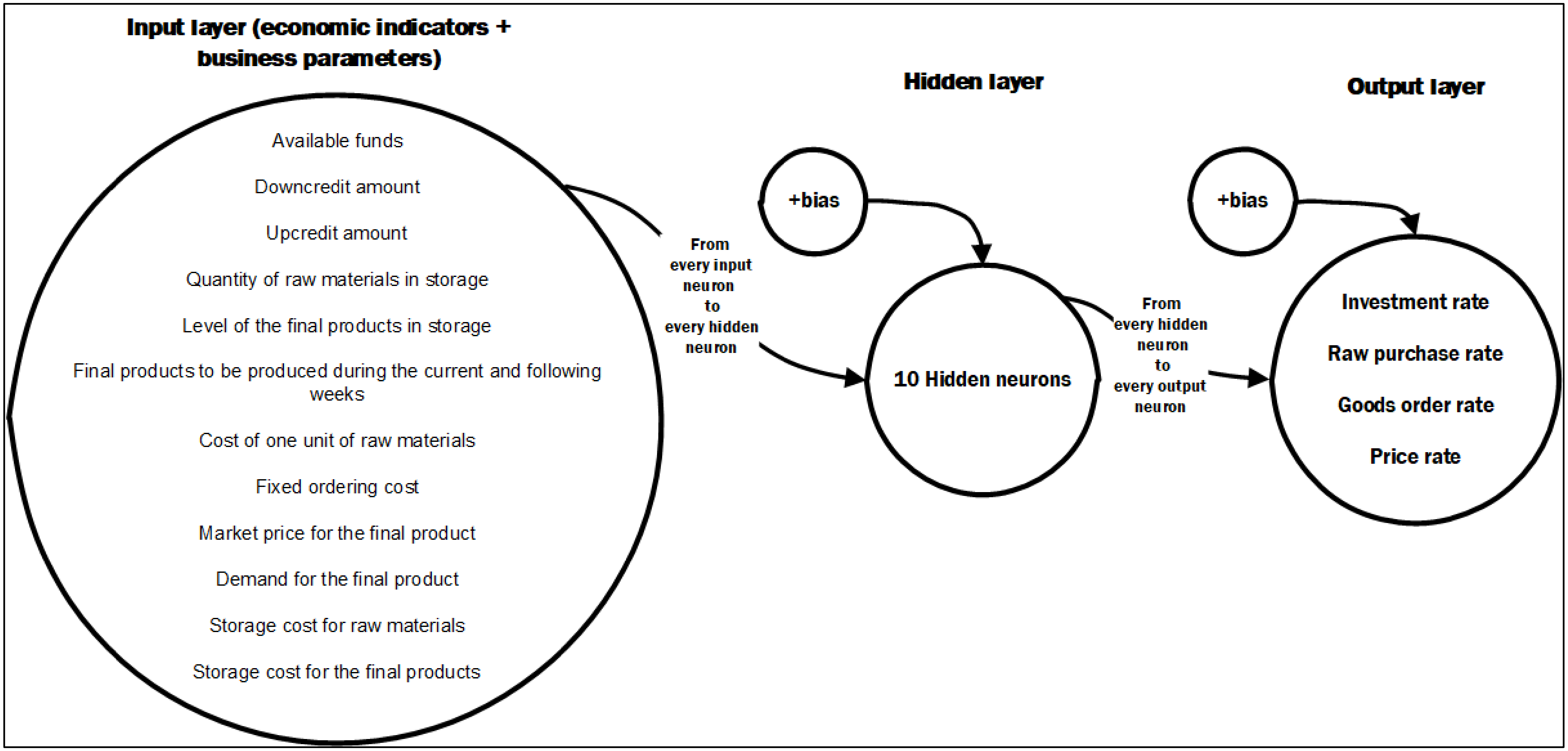

Figure 4-10. Scheme of the proposed closed-loop ANN control system.

The final equation for the calculation of the controls according to the closed-loop ANN model depicted in Figure 4-10 can be derived by combining Equations 4-7 and 4-8 into Equation 2-4 introduced in Section 2.4.4.1.1. In this case, for Equation 2-4, I (vector of inputs) consists of 13 numbers, $f_{hidden}(x) = relu(x)$, $f_{output}(x) = S(x)$, and the vector of controls U is given by $U = \left(u_{inv}(t), u_{raw}(t), u_{order}(t), u_{buy}(t)\right)^T$. In particular, U will be converted to an actual amount of money for investment, raw materials purchase, raw materials to be sent to the production lines, and the price of the final products. For example, if the output $u_{inv}(t) = 0.2$ and $u_{raw}(t) = 0.9$ for time t, then the actual amount of money to invest will be the amount of money available multiplied by 0.2. Moreover, the amount of money to spend on raw materials will be the remaining amount after investment multiplied by 0.9.

As seen from the above structure and mathematical model, the ANN control optimisation with feedback is a suitable model to be used in the case of a steel manufacturing factory due to the following reasons:

1. The number of neural network weights does not increase with the increase in the planning horizon. Therefore, we already need to select more parameters for an open loop algorithm than for a closed loop one for a 52-weeks planning horizon. Increasing this horizon will increase the number of parameters of the open loop algorithms linearly while the number of parameters for the closed-loop system will remain fixed.
2. A closed-loop system can produce controls starting from any week, unlike the open-loop system.
3. The closed-loop system is valid even if we change the amount of initial funds and the production parameters; on the other hand, the open-loop model is likely to produce a much worse result.

Finally, the open-loop static control systems described in Section 4.3.2.1 do not account for the economic conditions nor the business state. Thus, the recommendations derived from these systems do not change as a result of any change in the model's parameters. For instance, in the case of a 52-week planning horizon, the recommendations are adjusted during the training process of the system and saved as 208 numbers, which reflect the presence of four recommendations for each week. In other words, the open-loop static control system is merely 208 numbers that are saved into the model after the training process, and do not change again throughout the entire planning horizon. These systems are simple to implement, as they only need the defined number of weeks as an input, but they do not adjust their recommendations in the case of an irregular business situation or economic event having taken place in previous weeks. On the other hand, the closed-loop ANN based control system uses artificial intelligence to produce recommendations based on the current economic conditions or business situations, being able to react according to any change in these environments. This system is more complex to implement, since it require to train the ANN with a set of input parameters, such as available cash, the quantity of raw materials and/or final products in storage, current market prices, and the demand for the final product, in order to provide a set of accurate recommendations. In particular, the most challenging aspect of this system is to develop algorithms that are capable of producing continuous controls over the planning horizon (Gu et al., 2016).

4.3.3 Mathematical Model for the Steel Manufacturing Inventory Management

In this section, the mathematical model that depicts the business cycle described in Section 4.2 is developed. Therefore, the main goal of this section is to convert the economic model of the steel manufacturing factory into a system of different recurrent equations that describe the dynamics of the business indicators and parameters of the business state. Moreover, such a system must be evaluated automatically in the loop from the first to the 52nd week to get the final profit of the factory. The scheme of the mathematical model is displayed in Figure 4-11.

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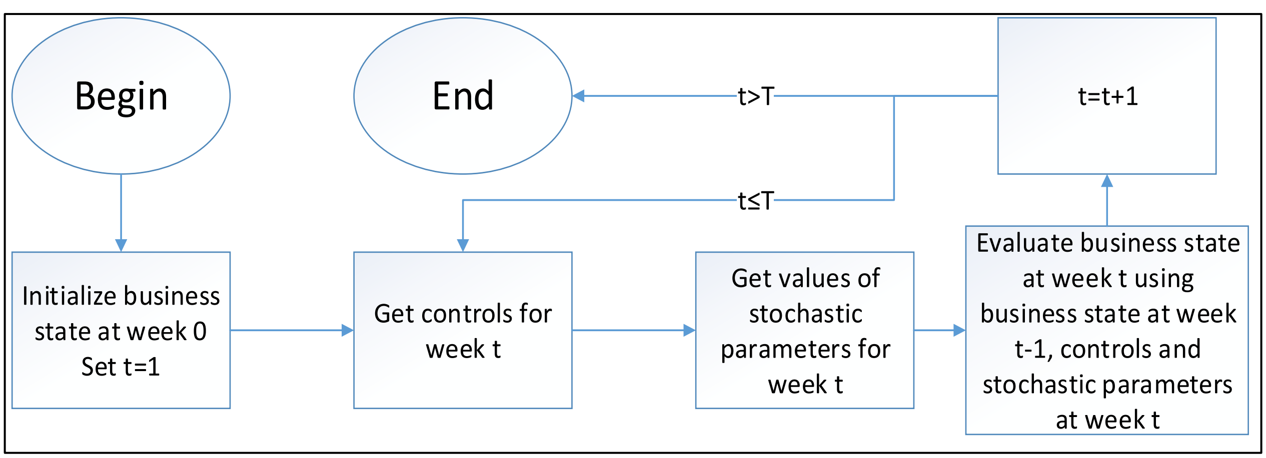

Figure 4-11. Simplified flowchart of the mathematical model of the steel-consuming factory.

The more variables included in the developed model, the more accurate the model will be, better reflecting the real-life process. For instance, similar to Hajiaghaei-Keshteli and Fard's (2018) model, several suppliers could be included, each with different raw material quality, or a logistic subroutine that will influence the delivery cost. Nevertheless, increasing the complexity of the model would lead to tremendous problems in terms of its solvability. Thus, a trade-off exists between the complexity and solvability of the equations. In general, the used equations can be categorised into four groups as follows:

1. Money management group
2. Raw material purchasing group
3. Production ordering and manufacturing group
4. Selling group

Since, during each week, there are four controls from which to choose in order to maximise the company's profit, then let $u_{inv}(t)$ be the percentage of money to invest, $u_{buy}(t)$ the percentage of raw materials to buy, $u_{order}(t)$ the percentage of raw material to send to production, π the inflation rate per period and $u_{price}(t)$ the price for the final product at time t. Consequently, to calculate the annual profit for the company, the equations of the four blocks should be calculated iteratively from the first week to the last. For the first and simplest block, the money management block, Equations 4-9 and 4-10 are used to recalculate the available funds for the following period and adjust the amount of money that was invested, respectively:

$$m(t+1) = m(t) \cdot \left(1 - u_{inv}(t+1)\right) \cdot (1-\pi) + up(t) \cdot i_{up} - down(t) \cdot i_{down} \tag{4-9}$$

$$inv(t+1) = inv(t) + m(t) \cdot u_{inv}(t) \tag{4-10}$$

For the second block, raw materials purchasing and ordering, Equation 4-11 is used to calculate the maximum amount of raw materials that the company can afford to buy.

$$buy(t+1) = \frac{m(t+1) - \overline{C_{raw}}(t)}{\widetilde{C_{raw}}(t)} \cdot u_{buy}(t+1) \tag{4-11}$$

If this maximum amount is less than the minimum order, then the company will cancel the entire order for the current week (Equation 4-12):

$$buy(t+1) = buy(t+1) \cdot H(buy(t+1) - min_{ord}^{raw}) \qquad \text{4-12}$$

Accordingly, to calculate the amount of down credit at the next period, and following the method used by Dordevic et al. (2017), the cost of one unit of raw material is added to the fixed cost of purchasing. However, similar to Farhangi and Mehdizadeh (2016), if the order is more than the discount amount, then the price for the entire order will be decreased by d_{val}^{raw} ; likewise, if the ordered amount of raw materials is less than d_{fix}^{raw}, then the fixed cost is reduced, as seen in Equation 4-13:

$$down(t+1) = down(t) \cdot (1 - i_{down}) + buy(t+1) \cdot \left(\widetilde{C_{raw}}(t)(1 - H(buy(t+1) - d_{am}^{raw}) \cdot d_{val}^{raw})\right) + \min\left(1, \left(\frac{buy(t+1)}{d_{fix}^{raw}}\right)^2\right) \cdot \overline{C_{raw}}(t) \qquad \text{4-13}$$

Finally, Equation 4-14 updates the amount of raw materials in storage according to the amount that has been bought.

$$raw(t+1) = raw(t) + buy(t+1) \qquad \text{4-14}$$

The third block concerns the goods ordering process, and the following equations are derived:

$$ord(t + t_{lead} + 1) = \min\left(raw(t + 1), \frac{\max(m(t+1), 0)}{c^{prod}}, max^{prod} + max_{over}^{prod}\right) \cdot u_{order}(t+1) \quad 4\text{-}15$$

$$raw(t + 1) = raw(t + 1) - ord(t + t_{lead} + 1) \quad 4\text{-}16$$

$$m(t + 1) = m(t + 1) - ord(t + t_{lead} + 1) \cdot c^{prod} - max\big(ord(t + t_{lead} + 1) - max^{prod}, 0\big) \cdot c_{over}^{prod} \quad 4\text{-}17$$

$$ord(t + t_{lead} + 1) = ord(t + t_{lead} + 1) \cdot (1 - p_{cdef}^{prod}) \quad 4\text{-}18$$

$$prod(t + 1) = prod(t) + ord(t + 1) \quad 4\text{-}19$$

Equation 4-15 makes sure that all the constraints of the business case are met by ensuring that the quantity ordered does not exceed the original and extra production capacities of the factory; if there are no funds available, we cannot order the production of any products, and if there are raw materials in storage, no more items can be ordered. At the same time, Equation 4-16 balances the raw materials in storage according to ordered goods, while Equation 4-17 decreases the available funds as a result of the ordered goods; in the case of the order being larger than the factory's production capacity, an extra cost is added for the overtime use. Equation 4-18 takes into account the probability of a critical defect in production which reduces the quantity of goods delivered, albeit they have already been paid for. Finally, Equation 4-19 adds the amount of ordered goods to the available quantity of final products that is ready to be used at the current time.

The final block concerns the sale of the produced goods:

$$price(t + 1) = c_{max}^{prod} \cdot u_{price}(t + 1) \quad 4\text{-}20$$

$$demand(t+1) = D_{prod}(t+1) \cdot \left(\frac{C_{prod}(t+1)}{price(t+1)}\right)^{el_c^{prod}} \quad 4\text{-}21$$

$$sell(t+1) = min\big(prod(t+1), demand(t+1)\big) \quad 4\text{-}22$$

$$up(t+1) = up(t) \cdot (1 - i_{up}) + sell(t+1) \cdot price(t+1) \cdot (1 - i_{tax}) \quad 4\text{-}23$$

$$prod(t+1) = prod(t+1) - sell(t+1) \quad 4\text{-}24$$

$$m(t+1) = m(t+1) - \frac{sell(t+1) \cdot p_{def}^{prod} \cdot c_{def}^{prod}}{1 - p_{cdef}^{prod}} \quad 4\text{-}25$$

Equation 4-20 simply sets the price for the final products using the control fraction $u_{price}(t+1)$. In Equation 4-21, the current demand for production is calculated, while Equation 4-22 is used to calculate the demand for the final products as a function of price. Equation 4-23 adjusts the up credit amount by the amount paid by clients each week, and increases with the purchase of more final products, in credit, by the consumers. Equation 4-24 decreases the amount of final products in storage according to the quantity sold. Equation 4-25 takes into account the moderate and major defects in production; for moderate defects, a repair is needed, while for major defects, a replacement for the product is needed and delivered to the customer for free.

To account for the storage costs and the deterioration of final products over the planning horizon, Equation 4-26 takes into account the storage costs for both raw materials and final products, as well as the fixed costs per period. Equation 4-27 follows the method developed by Sekar et al. (2017), used to adjust the raw material storage costs in case external storage is needed. Finally, Equations 4-28 and 4-29 are required to model the deterioration of products and raw materials over time. This is based on the known air conditioning characteristics which govern the percentage of steel that will deteriorate every week.

$$m(t+1) = m(t+1) - raw(t+1) \cdot S_{raw}(t) \\ - prod(t+1) \cdot S_{prod}(t) \\ - c_{fix} \tag{4-26}$$

$$m(t+1) = m(t+1) \\ - \max(raw(t+1) \\ - max_{stor}^{raw}, 0) \cdot c_{stor}^{raw} \tag{4-27}$$

$$raw(t+1) = raw(t+1) \cdot (1 - frac_{det}^{raw}) \tag{4-28}$$

$$prod(t+1) = prod(t+1) \cdot \left(1 - frac_{det}^{prod}\right) \tag{4-29}$$

Furthermore, as demonstrated in Table 4-1, the profit function includes a backorder block which is calculated at the end of the last modelled week. In this block, an iteration for all weeks is performed and checks whether the reserves for each week are sufficient to cover the orders for a number of future weeks, defined by the Supply Fail Duration parameter. Thus, if there are any orders that are not covered by the reserves, a backorder cost is calculated for the extra quantity needed, which is defined by the parameter "Supply Fail Probability" and backorder extra cost for one item. For instance, if we have 100 items that are not covered by reserves and the risk of backorder is 5% with an additional cost of £10K, then, on average, we expect to have 100*0.05*£10 000=£50K additional losses because of backorders; these calculations are performed using Equations 4-30 and 4-31.

$$shortage(t) = max\left(\sum_{i=t}^{t+t_{df}^{raw}} sent(i) - raw(t), 0\right) \tag{4-30}$$

$$BO_{loss} = \sum_{t=1}^{T-t_{df}^{raw}} shortage(t) \cdot P_{df}(t) \cdot C_{df}(t) \tag{4-31}$$

As seen from the above equations, Equation 4-31 is used to calculate the possible raw material shortage for week t by calculating the difference between the present and future amounts of raw materials sent to production lines until week $t + t_{df}^{raw}$, and the amount of raw materials at week t , which results in the maximum amount of shortage in case all raw materials deliveries during the next t_{df}^{raw} weeks fail. Moreover, Equation 4-32 estimates the annual backorder loss based on the supply delay frequency and the extra losses per delayed item of raw materials. Consequently, all the assets of the factory at time t can be calculated as:

$$Total\ worth(t) = inv(T) + m(T) - down(T) + up(T) + raw(T) \cdot c_{inv}^{raw} \qquad \text{4-32}$$

$$+ \left(prod(T) + \sum_{t=T+1}^{T+t_{lead}-1} ord(t) \right) \cdot c_{inv}^{prod}$$

According to Equation 4-32, the total worth of the company includes free and invested funds, the initial cost of raw materials, and the final products in storage, as well as the initial cost of all future paid final product orders, which are defined by the fixed variables c_{inv}^{raw} and c_{inv}^{prod}. Finally, the net profit is given by Equation 4-33:

$$Net\ profit = Total\ worth(T) - Total\ worth(0) - BO_{loss} \qquad \text{4-33}$$

Equation 4-33 represents the objective function which should be maximised by the proposed model, as follows:

$$\max Net\ profit \qquad \text{4-34}$$

From Equations 4-32 and 4-33, it can be seen that the objective function is the sum of investments, current cash and up credit, minus the down credit and backorder loss at the end of the year. This clearly shows that the objective function in the developed model is not a classical objective function optimising just one parameter. On the contrary, by linking different important parameters on the net profit final calculation shown in Equation 4-33, the optimisation of the objective function allows simultaneously optimising all of these parameters. This demonstrates the multi-dimensional nature of the objective function used in the developed model. This is in fact one of the most important contributions of this research study, since allows the model to depict the real-life scenario of the manufacturing industry, and the steel one in particular, more accurately. This is achieved by not only focusing in minimising costs, as the majority of the models available in the literature does, but also considering the influence of other crucial parameters, such as investments, current cash, up credit, down credit and backorder loss in the objective function to be optimised.

4.3.4 Model Optimisation Algorithm

The specific parameters described in Section 4.3.1 included in the developed model, make it not possible to find an analytical optimal strategy that leads to maximum profit through the use of linear equations. Consequently, the developed model becomes a nonlinear model, as demonstrated from the mathematical framework presented in Section 4.3.3 deriving the model consisting in 25 discrete recurrent equations. Note the reader that the developed model results even more complex than the well-known dynamic lot-size model, which although assuming stochastic demand, it does only assume one stochastic parameter, disregarding producing, selling or investment policies (Wagner and Whitin, 1958). Figure 4-12 presents the developed model in detail which aims at maximising the objective function represented by the net profit in Equation 4-33.

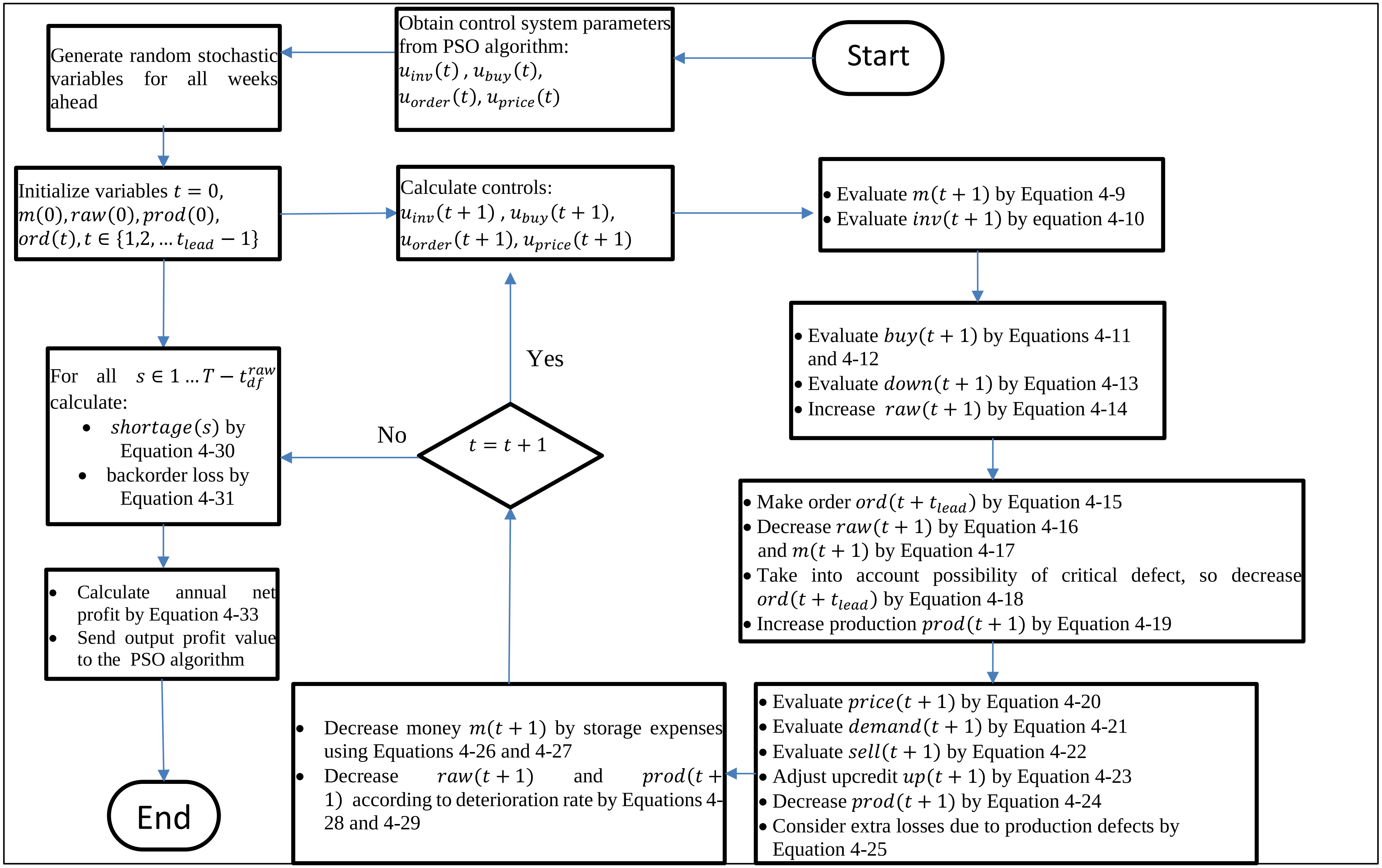

Figure 4-12. Detailed flowchart of the developed model of the steel-consuming factory.

As seen from the flowchart of Figure 4-12, the developed model consists of six main blocks. Five of them are presented via the equations developed in Section 4.3.3, whereas the sixth corresponds to the control system described in detail in Section 4.3.2. According to the flowchart of Figure 4-12, the first step in the model is to compute the optimal values for the set of variables included in the control system. Here it is important to highlight that, in the case of using the control system based on the open loop approach, the control variables will be computed without taking into account the current state of the business, whereas in the case of using the ANN closed-loop approach, the current state of the business can be taken into account to calculate the control variables by taking advantage of the provided feedback. Due to the complexity of the developed model, linear programming algorithms cannot be used to calculate the control variables. In this context, as discussed in the previous chapters, these variables are adjusted using the PSO algorithm which fundaments have been introduced in Section 2.4.4.1.2. In particular, PSO algorithms allow models to find the maximum of the objective function even if it is non-differentiable by all the control parameters, or is a discontinuous function (Yin, 2004), as in the case analysed in this research study. For each of the control system approaches proposed in Section 4.3.2, namely the open and closed-loop ones, experiments running the model repeatedly on the hypothetical data generated as described in Section 3.2 with the same fixed PSO parameters have been performed to optimise the model's variables in terms of the objective function (the net profit function in Equation 4-33). The number of model launches is equal to the product of the swarm size, the maximum number of iterations, and the number of Monte Carlo runs, with randomly generated instances of stochastic parameters for all planning horizons. For this research study, the swarm size is set to 1000, the maximum number of iterations is set to its default value of 1000, and the number of Monte Carlo runs is set to 5, which is a commonly used number of runs when performing Monte Carlo based experiments.

Figure 4-13 shows the optimisation algorithm for the case of the ANN closed loop control approach. As seen from Figure 4-13, each PSO particle is represented by a neural network. For each neural network, the corresponding weights define the control decisions represented by the following four variables:

- Percentage of money to invest
- Percentage of money to spend on raw materials
- Percentage of raw materials to send to production lines

- Price

In this context, each particle can be seen as a vector containing all the neural network weights, *i.e.* a vector in a multidimensional space. Then, the optimisation process is conducted as follows. The first step consists in generating as much neural networks as particles are considered in the PSO algorithm. As mentioned above, the number of particles is set to 1000, so 1000 neural networks are generated with random weights. Then, the objective function of the developed model (net profit function in Equation 4-33), is evaluated using each of the 1000 neural networks generated in the first step. The next step consists in changing the weights of each neural network towards making them closer to the best fitted neural network in terms of the objective function. This is done by adjusting the neural network weights according to Equations 2-7 and 2-8. This process is repeated 1000 times according to the selected number of iterations for the PSO algorithm. In the final step in Figure 4-13, the best suited neural network, which is the one that achieves the largest value for the objective function (net profit), is selected. Finally, the selected best suited neural network computes the control variables (neural network outputs) in terms of the current business parameters (neural network inputs). These optimised control variables will then be the inputs for the mathematical model as depicted in Figure 4-12.

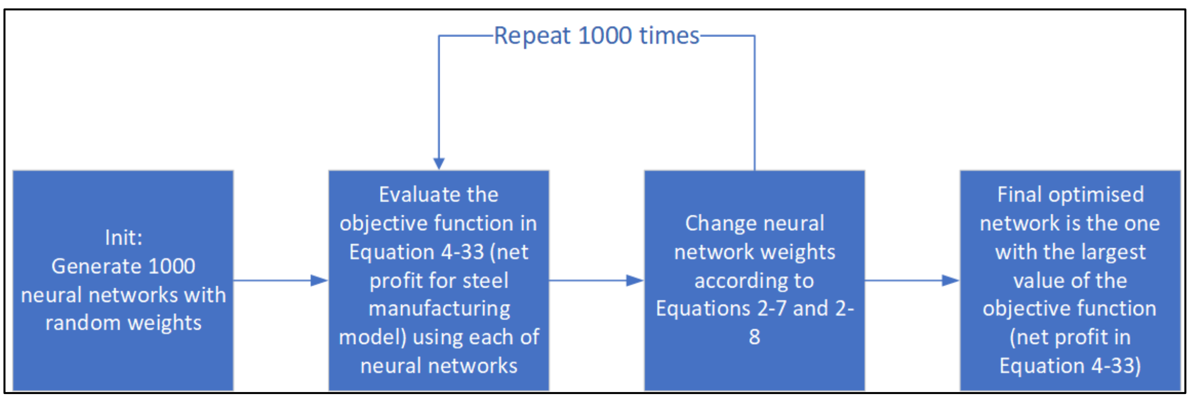

Figure 4-13. Optimisation algorithm for the ANN closed-loop control approach.

When implementing the model following the flowchart depicted in Figure 4-12, the experimental results have demonstrated that the proposed PSO algorithm converge to sub-optimum and does not change significantly when adding more particles into the PSO model. In this line, the proposed PSO approach gives a nearly optimal strategy for the addressed application, demonstrating that the chosen internal parameters of the PSO algorithm, namely the swarm size and the maximum number of iterations are well suited for solving the developed model.

Finally, analysing inventory usage and economic order quantity is the crucial part of the developed model. With the exception of analysis of the quantity of raw material in storage, a more complex analysis is required to answer the question of how long each raw material batch waits in storage until being consumed and moved to the production lines. Therefore, given $buy(t)$ is the quantity of inventory bought by the company and $out(t) = ord(t + t_{lead})$ is the quantity of inventory moved to the production lines at time *t,* the following auxiliary algorithm for raw materials maturity analysis in storage is proposed:

Program starts
Input variables: vector[T] buy, vector[T] out
Start loop variable i from 1 to T
Assign $Input(i) = buy(i)$
Assign $OUT = out(\mathrm{i})$
Start loop variable s from 1 to i

Assign $Consume = \max\left(\min\big(OUT, Input(s)\big), 0\right)$

Assign $Input(s) = (Input(s) - Consume) \cdot (1 - frac_{det}^{raw})$

Assign $Maturity(i - s + 1) = Maturity(i - s + 1) + Consume$Assign

Assign $OUT = OUT - Consume$

End loop for variable s

End loop for variable i

Output: Maturity vector

Program ends

The proposed pseudo-code gives the maturity vector of consumed steel. Here, it is important to highlight that this algorithm is not related to the PSO optimization algorithm, being an auxiliary algorithm used to evaluate maturity of raw materials and production. In particular, this algorithm is only used during the analysis of performance of main closed loop control system. This means of analysis is a powerful tool for comparing business storage and ordering policy, which is independent of business scale. The idea of this pseudo-code is that during the outer loop an iteration is done over week number and raw materials are input to the cell which corresponds to the week of raw materials arrival. The goal of inner loop is to move materials from these cells into production lines according to out vector

4.4 Chapter Summary

In this chapter, the development of the inventory management model proposed in this research study has been presented. In order to do so, first the model was described from the economic and business cycle points of view. Through this step, the relevant economic variables were defined, assuming as stochastic the most critical ones the probability of delivery failure, the extra charge per unit of raw material in case of delivery failure, the ordering cost of raw materials, the basic cost of one unit of material, the raw material storage costs per period, the final product storage costs per period, the selling price, and the production demand. After this step, the model was converted into a system of 25 different recurrent equations in order to implement the model in Matlab.

In addition, since one of the main challenges of implementing the extended EOQ model is not only modelling the business cycle, but also finding the optimal strategies of the business cycle that maximise profit while minimising storage costs, two control systems were developed and described in Section 4.3.2:

1. A simple control system that adjusts its control parameters during the training phase regardless of the current state of business and external market parameters.

2. A neural network system that uses the most important business measures and economic parameters as inputs to provide output in terms of the recommended controls for each week in the planning horizon.

Finally, the PSO algorithm implemented to adjust the control system's parameters has also been introduced in this chapter. Here, it is important to highlight that, although many heuristic algorithms can be applied to solve the model, PSO was chosen since it has the ability to simultaneously find the minimum of the objective function in many points of the search space.

In conclusion, in this chapter the developed model has been fully explained and its unique complexity has been discussed. In fact, to the best of the author's knowledge, the developed model in this research study is one of the most complete models regarding the problems of optimal storage. In this line, it has been shown that, with such a complex model that deals with a three-fold problem of storing high-volume materials in a limited storage space, reducing the energy required to preserve the product from deterioration, and maximising the company's profit, it is not possible to use classical algorithms for EOQ, EPQ and dynamic lot size model problems. In this context, the developed model extends the EOQ classical equations to accommodate the stochastic nature of the input parameters through three main assumptions, as well as a set of sub-assumptions, in order to decrease storage period of raw materials in inventory and, at the same time, retain the company's turnover and net annual profit at the highest possible level. In addition, due to the difficulty in estimating errors as a result of the lack of actual controls, the PSO optimisation technique was used in the developed model, since it is capable of finding the set of neural network weights that provide the factory with maximum profit while minimising storage costs.

Finally, the developed model is robust in the sense of having the ability of performing precise long-term business simulation and optimisation. As discussed in the SLR conducted in Chapter 2, most of the previously developed models in the literature proposed solutions on a monthly basis. The developed model provides a superior solution being capable of providing weekly results in a 52-week planning horizon. This is due to the fact that the ANN closed-loop approach can provide feedback in terms of optimal control variables computed based on current parameters of the business environment, such as market price of raw production and steel price. Moreover, the developed model is capable of capturing both the supply and production activities of the steel manufacturing industry, as well as its consumer demand and price sensitivity.

5 Model Validation and Results

5.1 Introduction

In this chapter, the developed model is implemented and applied numerically for the case of the steel manufacturing factory under study. This type of factory produces steel structures according to different designs through a well-defined production process, from various raw materials. In addition, to manufacture a final product that fits customers' needs, several cutting and fitting activities take place within the factory. Finally, the steel manufacturing factory must have a warehouse to store both raw materials and final products. The developed model contemplates three different business scenarios under which a steel manufacturing factory might operate:

- fixed demand
- fixed supply
- fully stochastic scenario (stochastic demand and supply)

The model results are analysed, and proof of the robustness of the model is provided through the validation of the model's steps and results.

The chapter is organised into six main sections. Section 5.2 outlines the steps of implementing the developed model within the Matlab software context, as well as of validating it. Section 5.3 describes the conducted experiments. In particular, Section 5.3.1 details the comparison between the different control systems proposed in this research study. In this section, different aspects of these models' results are compared in order to determine the best suited model for the steel manufacturing factory. Then, in Section 5.3.2 the most robust model is applied to a scenario in which demand is assumed to be fixed, and the performance results are discussed. Similarly, in Section 5.3.3, the same model is applied to a scenario in which supply is assumed to be fixed, and the performance of the model under such a condition is analysed, whereas in Section 5.3.4, a fully stochastic scenario is assumed and the results of the model's application are presented. After applying the model under these three scenarios, the model's performance in each one of them is compared in Section 5.3.5. Finally, Section 5.4 summarises the findings from the most robust model's implementation and provides conclusions and recommendations.

5.2 Model Implementation

After developing the model in Chapter 4, the application of the model and control systems on the steel manufacturing factory case is performed in this chapter. As discussed in previous chapters, one of the most challenging issues of inventory management in steel manufacturing applications is to account for the whole production process. In order to fill this research gap, the inventory management process developed in this research study is a two-fold process. On one hand, the inventory management is focused on dealing with the required storage of raw materials towards producing the final products/structures. This management should be performed before the raw materials are actually used in the production process. On the other hand, the inventory management is focused on handling the storage of the final products/structures manufactured. This management should take place before the produced goods are actually sold. Practically, the described inventory management process is conducted based on the widely used Matlab software, taking into account three main steps represented by three modules, viz., the data processing, the training and the analysis module, as shown in Figure 5-1. Further details of each one of the modules can be seen in Figure 5-2. From the flowchart depicted in Figure 5-2, it can be seen that the developed inventory management process which objective function is focused on managing both raw materials and final product storage volumes and time, includes a crucial decision-making regarding whether the result of the process is acceptable or not. This decision can be subjected to different factors depending of the particular company's management policies. As such, it is the company's management the one responsible to answer such question by assessing the actual and current business objectives, such as increase profits, reduce bankruptcy risk, increase market share, etc.

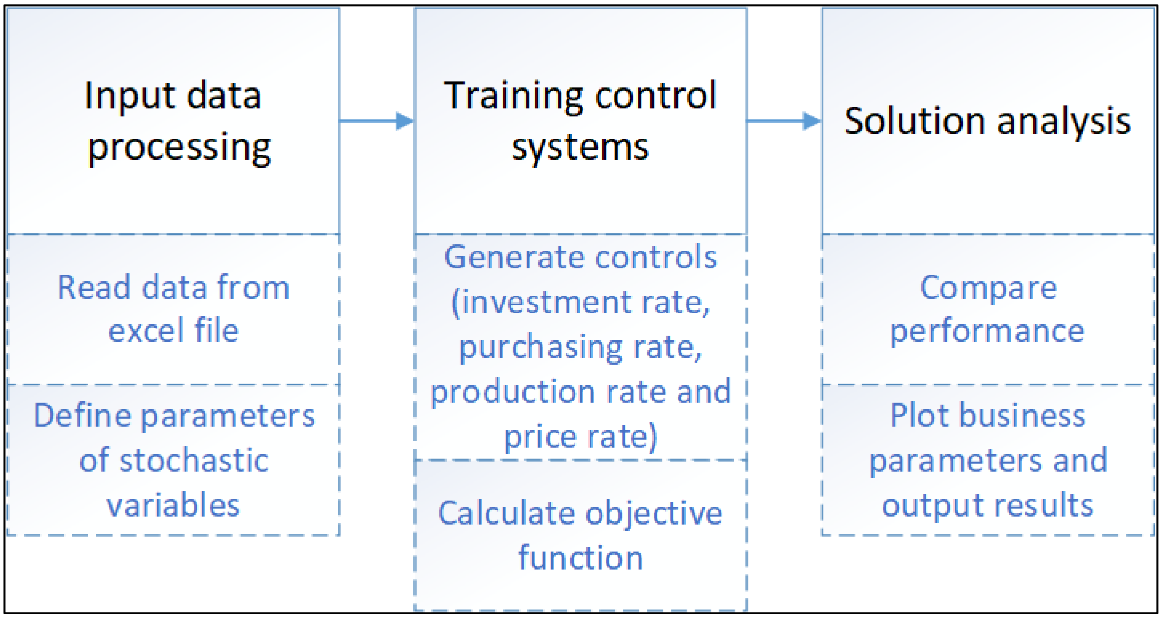

Figure 5-1. Scheme of model implementation in Matlab.

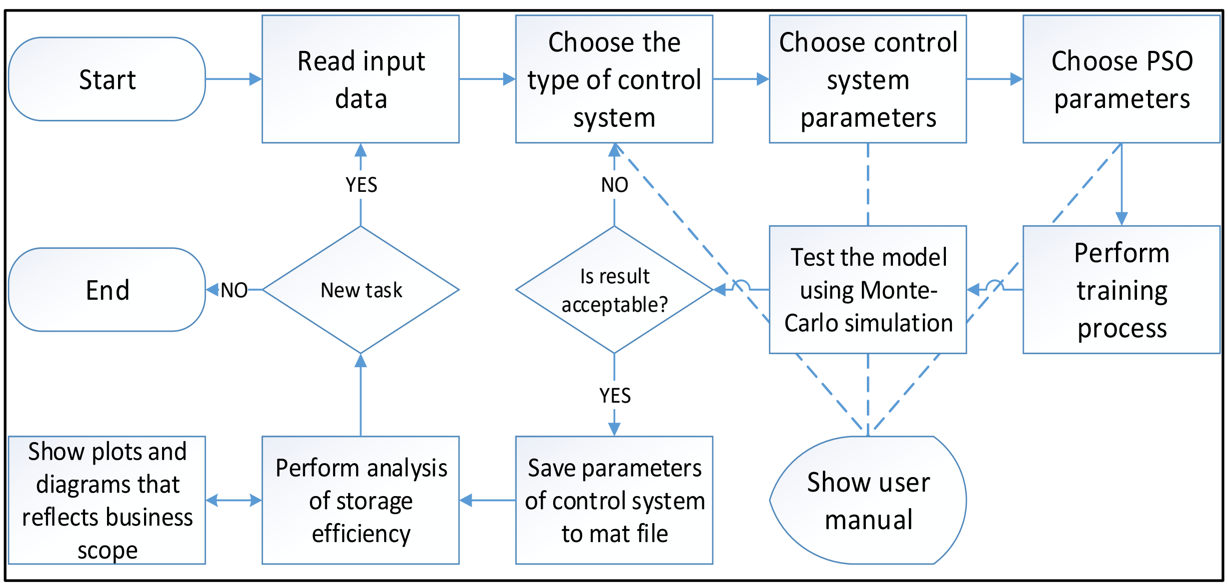

Figure 5-2. Detailed flowchart of the scheme of model implementation in Matlab.

In Appendix C, the main Matlab codes used to implement the developed model are provided. In addition, Appendix D describes the user interface developed in the GUI of for the proposed model implementation.

5.2.1 Data Processing Module

In this module, the data from the Excel file is fed into the software, through which all the stochastic variables are identified, as well as their mean values and standard deviations are set (see Appendix C). Since all the stochastic variables in the Excel data file are named in the form of X_VarName, where X is the first letter of distribution type and VarName is the name of a stochastic variable, Matlab identifies these variables along with their respective mean values $\mu(t)$ and standard deviations $\sigma(t)$ for each week *t*. Furthermore, the Matlab model supports three types of the stochastic variable distribution functions, which are:

1. Normal distribution: The normal distribution is saved in Matlab by defining the mean values and standard deviations. In the case under study, the economic variables usually have normal distributions, thus this distribution is used for all the stochastic variables.

2. Uniform distribution: This is a distribution that has a constant probability and is defined by the lower bound $l_b(t)$ and upper bound $u_b(t)$, which are calculated using Equation 5-1:

$$l_b(t) = \mu(t) - \frac{\sigma(t)}{\sqrt{3}};\ u_b(t) = \mu(t) + \frac{\sigma(t)}{\sqrt{3}}; \qquad \text{5-1}$$

3. Beta distribution: In order to use this type of distribution in Matlab, the parameters $\alpha(t) and\ \beta(t)$ must first be calculated using Equations 5-2 and 5-3 below:

$$\alpha(t) = \frac{\mu(t) \cdot \left(\sigma^2(t) + \mu^2(t) - \mu(t)\right)}{\sigma^2(t)} \qquad \text{5-2}$$

$$\beta(t) = \frac{\left(1 - \mu(t)\right) \cdot \left(\sigma^2(t) + \mu^2(t) - \mu(t)\right)}{\sigma^2(t)} \qquad \text{5-3}$$

This type of distribution is often used to describe the variables that fall within the range of zero to one, such as the probability of delivery failure and the deterioration rate. This latter variable is defined as the percentage of raw materials or final products whose physical properties deteriorate during their storage in the warehouses, and which become obsolete and unsuitable for either manufacturing or selling, respectively, during a one-week period. For example, if the deterioration rate is 1% and we have 100 tons of raw materials at week 1, then the usable quantity of raw materials will only be 99 tons at week 2.

Within the context of the steel manufacturing application addressed research, most of the economic variables behave according to the normal distribution. In this line, this distribution is the one selected to model all the considered stochastic variables.

5.2.2 Training Module

This module performs the training process for both the open-loop static control system and closed-loop neural network control system. Both control systems are trained with the same set of PSO parameters, which are:

1. Maximum number of iterations equal to 1000: This is the default value and it is usually the best option.
2. Swarm size is equal to 1000.
3. Matrix calculations is set to be true: This means that all swarm is calculated once in a time inside one objective function which saves a lot of computational time.
4. Maximum number of stall iterations is equal to 100: This means that the algorithm will stop if the results from 100 consecutive iterations are not changing.

In the particular case of the ANN based closed-loop system, the training process depends on the objective function itself and on the neural network processing module. In addition, in this case the processing module should convert the vector of optimised control variables by the PSO algorithm into ANN weights so that the business and economic parameters can be input into the ANN towards obtaining the current controls. As introduced in Section 4.3.2.2, the ANN architecture used in the developed model (shown in Figure 4-10) consists in an input layer of 13 neurons, a hidden layer of 10 neurons and an output layer of 4 neurons. In order to apply the generic Equation 2-4 to the used ANN architecture and convert the vector of control variables into neural network weights to launch the neural network, the following steps are followed:

1. Given that the input layer is 13 neurons and the hidden layer is 10 neurons, hence, we need 13*10=130 weights. Thus, the first 130 parameters of a vector of control variables are reshaped into a matrix with dimensions of 13×10.
2. The matrix is multiplied by the vector of scaled economic variables and business parameters listed in Section 4.3.2.2.
3. Since the hidden layer has 10 neurons and each layer has a bias weight, 10 numbers are extracted from the vector of control variables and added to the hidden layer's output result.

4. The hidden layer's output is then transformed using the transfer function presented by Equation 4-7.
5. Consequently, the output layer has four neurons; hence, the weight matrix for converting the hidden layer to the output layer has 10*4 = 40 weights. Therefore, the next 40 parameters of a vector of control variables are reshaped into a matrix with dimensions of 10×4.
6. Next, the above matrix is multiplied by the vector obtained in step 4.
7. Finally, the results are transformed using the transfer function represented by Equation 4-8, and the output is saved as the business controls of the current week.

As seen from the above procedure, in total, there are 13 neurons in the input layer, 10 in the hidden layer and four in the output layer; therefore, the number of weights to optimise is $(13 * 10) + (10 * 4) + 10 + 4 = 184$, which is less than the number found in the open loop model, thus the optimisation process will converge faster.

5.2.3 Analysis Module

This module plots both the business parameters and the output results, as well as performing a comparison of the different control systems under various scenarios. Moreover, this module performs a profit analysis to realise the most efficient control system. In general, the output of the model consists of the following:

1. Saved values of the control system's variables in the form of raw vector of control system parameters. In the case of an open loop control system, the vector has a length of 208 numbers, while in the case of the closed-loop control system it has a length of 184 numbers; and the results are saved in a *.mat file.
2. Results of the model testing on the five Monte Carlo runs with different scenarios of stochastic random variables.
3. Analysis of the storage efficiency (maturity analysis). This analysis involves the evaluation of the maturity distribution for both raw materials and final products, as well as determining the percentages of raw materials and final products that are lost due to deterioration.
4. Diagrams and plots that display the performance of the factory, averaged over five Monte Carlo runs. This includes plots related to money management, storage

efficiency, raw materials management, final product management and price setting policy.

5.3 Model Validation

The main aim of the validation process conducted in this research study is to prove the economic and technical feasibility of the developed model for inventory management within the context of the steel manufacturing industry. As introduced in Section 3.4, the unique and complex assumptions made when developing the proposed model makes this validation process to be not straightforward. This is mainly due to two reasons. On one hand, the parameters that maximise the profits of a particular steel manufacturing factory are highly dependent of each factory's business and operating conditions. In this context, there are no benchmark results available in the literature to compare the obtained results with the developed model. On the other hand, the unique, complex and model-oriented assumptions made when developing the proposed model makes it difficult to compare the obtained results with the ones obtained with similar models published in the literature. Then, in this research study, the two different control approaches proposed to solve the developed model are compared against each other for the sake of model validation. In addition, this comparison will allow to determine which control system is better suited for the application under study in terms of robustness and accuracy.

In this research study, the comparison is conducted within the context of different real-life scenarios, involving different sets of variables covering the most frequent behaviours of raw materials and final product costs and demand. In particular, the different economic scenarios with different assumptions regarding the stochastic variables, are considered:

1. All stochastic variables remain stochastic.
2. All variables that are related to demand are fixed.
3. All variables that are related to supply are fixed.

In all the above three scenarios, the backorder variables remain stochastic; hence, the aim of the validation process is to:

1. Prove the economic feasibility of the mathematical model of the steel manufacturing factory.
2. Outline the strengths of the closed-loop neural network control system over the classical open loop one.
3. Analyse the obtained results and deduce the relevant conclusions.

These aims are achieved through a numerical experiment used to calculate the following parameters:

1. Amount of funds.
2. Amount of invested funds.
3. The quantity of raw materials in store.
4. The quantity of final products in store.
5. The level of ordered final products.
6. The final product's selling price and demand's stochastic variables.

Moreover, the superiority of the neural network control system is proven through the storage business parameters, which are the average storage time, the distribution of storage time, and the area of storage used. Furthermore, another measure that proved the model's robustness is the company's annual profit.

5.3.1 Control System Comparisons

As detailed in Chapter 4, the first proposed control system for the developed model is direct control optimisation for a fixed set of Monte Carlo runs. Unfortunately, the experimental results obtained for this control approach show a poor performance. This underperformance is mainly due to the fact that the training process has been performed on a limited number of Monte Carlo runs, being the control system highly susceptible to overfitting. In particular, the algorithm demonstrated to be not capable of adjusting itself to changing business variables, in the sense that if a testing variable value deviates from their average training value, the algorithm fails in fitting it. A possible solution for the overfitting issue could be increasing the number of Monte Carlo runs when training the model. In this line, further experimental tests have been performed increasing the number of Monte Carlo runs during the training phase. Nevertheless, increasing the number of Monte Carlo runs has only led to a significant increase in the training time, rather than to an improvement in the testing results. Then, this control approach will not be considered for the final implementation of the developed model.

The validation of the developed model will then be conducted based on the comparison of the performances of the direct open-loop control optimisation for dynamically generated scenarios and the ANN control optimisation with feedback. In order to make such comparison, the values for the fixed variables as well as the distribution parameters for the stochastic variables listed in Section 4.3.1 should be defined. As discussed in Section 3.2, one of the most challenging tasks in the steel manufacturing application addressed here is to deal with the lack of available real-life business data. In this line, hypothetical data (Gasior and Recchia, 2019) is generated based on different average indicators of the steel industry available in the literature (Pardipto and Lussy, 2019; Tseng and Yu, 2019; Tavakoli and Taleizadeh, 2017; Rabieh et al, 2016) as well as on historical trends and publicly available business reports, such as the ones in (OECD, 2017; World Steel, 2018). The generated hypothetical data and its set values to test and compare the performance of both control systems, are shown in Table 5-1 and Table 5-2, for the fixed variables and the distribution parameters, in terms of mean values and standard deviation, for the stochastic variables, respectively.

Table 5-1. Values of important fixed variables that are listed in the input data file of the fully stochastic scenario

Variable name	Value
Number of samples in Monte Carlo method	5
Planning horizon	52 weeks
Leading time	5 weeks
Production orders before leading time	100 for each week
Initial level of final products in inventory	100 units
Initial funds available	£5M
Initial quantity of raw materials	100 units
Minimum order of raw materials	30 units
Discounted order of raw materials	100 units
Discount percentage	5%
Deterioration rate of raw materials and final products	5% per week
Basic production capacity	100 units per week.
Overtime production capacity	50 units per week.
Overtime extra cost per unit	£2500
Up credit and down credit interest rates	20% per week
Inflation rate	0.8% per week
Tax rate	5%
Demand elasticity	5

Table 5-2. Parameters of stochastic variables that are listed in the input data file of the fully stochastic scenario

Variable name	Distribution type	Mean	Standard deviation
Probability of delivery failure	Normal	0.1	0.05
Average extra charge per unit of raw material whose delivery failed	Normal	5	0.5
Ordering cost of raw materials	Normal	575	75
Purchase cost of one unit of raw materials	Normal	4	0.05
Storage costs of raw materials	Normal	0.5	0.05
Storage costs of final products	Normal	0.5	0.05
Market price of final products	Normal	30	5
Basic demand for final products	Normal	70	2.5

As seen from Table 5-1, the most important fixed variables for the steel manufacturing factory are defined. Some of these variables describe general economic conditions, such as inflation and tax rates. In research, a weekly inflation of 0.8% and an annual inflation of 34% is assumed. Some other variables represent more specific production aspects, such as leading time. In this research, the leading time is set at 5 weeks, since it usually takes 5 weeks to produce final products from the raw materials. In this case, whenever there is a production queue, the quantities for weeks 0–4 need to be defined. In addition, methodological variables, such as the number of testing Monte Carlo runs, are also defined in Table 5-1. In this research study, the number of testing Monte Carlo runs is set to 5, since 5 runs are usually considered to be a good trade-off between the obtained accuracy and the time consumption, in the sense of being enough to estimate the average profit and other business indicators with acceptable precision, while being not so time-consuming. The experimental results have indeed confirmed that 5 testing Monte Carlo runs are enough to obtain accurate results since the coefficient of variance of the company's profit estimated for the different testing runs has resulted to be 3.8% and 4.4% for the closed and open loop systems, respectively, which are below the generally accepted value of 5%.

Furthermore, to analyse how the presence of uncertainty affects the performance of the two systems, two additional scenarios were created:

1. Deterministic demand: This means that demand for final production is constant over time. In this scenario, all the parameters are assumed to be the same as those of the fully stochastic scenario, except for the demand stochastic variables. Hence, the

standard deviation of the final products' market price and the basic demand for final products are set to zero, which makes them fixed.

2. Deterministic supply: This means that supply for raw materials is constant over time. In this scenario, all the parameters are assumed to be the same as those of the fully stochastic scenario, except for the supply stochastic variables. Thus, the standard deviation of the ordering costs of raw materials and the purchase cost of one unit of raw material are set to zero, which makes them fixed.

Nevertheless, for all three scenarios, the storage costs for both raw materials and final products, as well as the backorder-related variables, remain stochastic. As the main objective of the research study is the storage of raw materials and final products, setting the storage costs to be constant do not depict the real-life scenarios of the steel manufacturing industry, and will lead to inaccurate results. At the same time, the main reasons behind keeping the backorder-related variables stochastic concern backordering, by nature, being a stochastic process, and the additional cost that the factory incurs for shortages depending on the final products' market price.

After setting the values of all the variables, the first step in validating the two systems is to examine how each system optimises the storage of raw materials and final products. Consequently, if a certain volume of raw materials $V_{raw}(m)$ is stored for m weeks, then after this period the losses are calculated as follows:

$$Loss(V_{raw}(m)) = V_{raw}(m) \cdot (1 - (1 - frac_{det}^{raw})^m) \quad \text{5-4}$$

For the final product losses, a similar equation holds:

$$Loss\left(V_{prod}(m)\right) = V_{prod}(m) \cdot \left(1 - \left(1 - frac_{det}^{prod}\right)^m\right) \quad \text{5-5}$$

Hence, the total deterioration rates of all raw materials and final products are computed using Equations 5-6 and 5-7, respectively.

$$D_{raw} = \sum_{m=0}^{M_{raw}} Loss(V_{raw}(m)) \quad \text{5-6}$$

$$D_{prod} = \sum_{m=0}^{M_{prod}} Loss\left(V_{prod}(m)\right) \quad \text{5-7}$$

where M_{raw} and M_{prod} are the maximun observed maturity for raw materials and final products, respectively.

Next, to make the losses for the two control systems comparable, the impacts of the total volumes of raw materials and final products need to be neutralised; hence, the equations can be modified in order to calculate the deterioration losses as percentages of the volumes. Therefore, first, let $P_{raw}(m)$ and $P_{prod}(m)$ be the proportions of raw materials and final products that were stored during m weeks; then:

$$\sum_{m=0}^{M_{raw}} P_{raw}(m) = \sum_{m=0}^{M_{prod}} P_{prod}(m) = 1 \quad \text{5-8}$$

This implies that we can split all the raw materials and final products into batches, with $P_{raw}(m)$ and $P_{prod}(m)$ being the probabilities of a unit being in an m-th batch. Then, the probabilities that a raw material unit or a final product unit is in this batch and deteriorated are given by Equations 5-9 and 5-10, respectively:

$$Ploss\big(P_{raw}(m)\big) = P_{raw}(m) \cdot (1 - (1 - frac_{det}^{raw})^m) \quad \text{5-9}$$

$$Ploss\left(P_{prod}(m)\right) = P_{prod}(m) \cdot \left(1 - \left(1 - frac_{det}^{prod}\right)^m\right) \quad \text{5-10}$$

Consequently, the total probabilities of deterioration for both raw materials and final products are calculated using Equations 5-11 and 5-12, respectively:

$$PD_{raw} = \sum_{m=0}^{M_{raw}} Ploss\big(P_{raw}(m)\big) \quad \text{5-11}$$

$$PD_{prod} = \sum_{m=0}^{M_{prod}} Ploss\left(P_{prod}(m)\right) \quad \text{5-12}$$

Moreover, Equations 5-11 and 5-12 can also be used to determine the percentages of raw materials and final products that will deteriorate over the entire planning horizon, respectively.

After laying out the mathematical foundation for the comparison between the closed-loop and open loop systems, six different parameters are used to compare the performance of these two systems, as detailed in the following sections. These parameters are: maturity and distribution rates, generated profits per storage unit used, profit generated by each Monte Carlo run, investment strategy, money management, and learning progress.

5.3.1.1 Maturity and Deterioration Rates

As described in Chapter 1, in order to compare the two systems in real-life settings, the two most common real-life scenarios of dealing with manufactured goods are considered, and the results of each system, in each scenario, are compared. These two scenarios are:

1. **Make to order**: where products are manufactured based on the orders received; hence, they are shipped directly to customers without being stored.
2. **Make to stock**: where products are manufactured in excess of the orders received to cover any emergency orders; hence, these products will be kept in storage until they are sold.

Moreover, since managing the storage of these high-volume materials is one of the main objectives of the developed model, measuring the duration for which raw materials are waiting in the store before being sent to the production lines, and the duration for which final products are kept in the store before being sold, will determine the degree of effectiveness of the developed model. Hence, the results of implementing the two control systems under each of the above scenarios, for both raw materials and final products, are depicted in Figure 5-3(a) and (b), respectively. These figures depict the percentages of raw materials and final products that were stored during each week of the planning horizon until used. In addition, through these plots, the maturity distributions for each system is analysed using the average maturity values and deterioration rates.

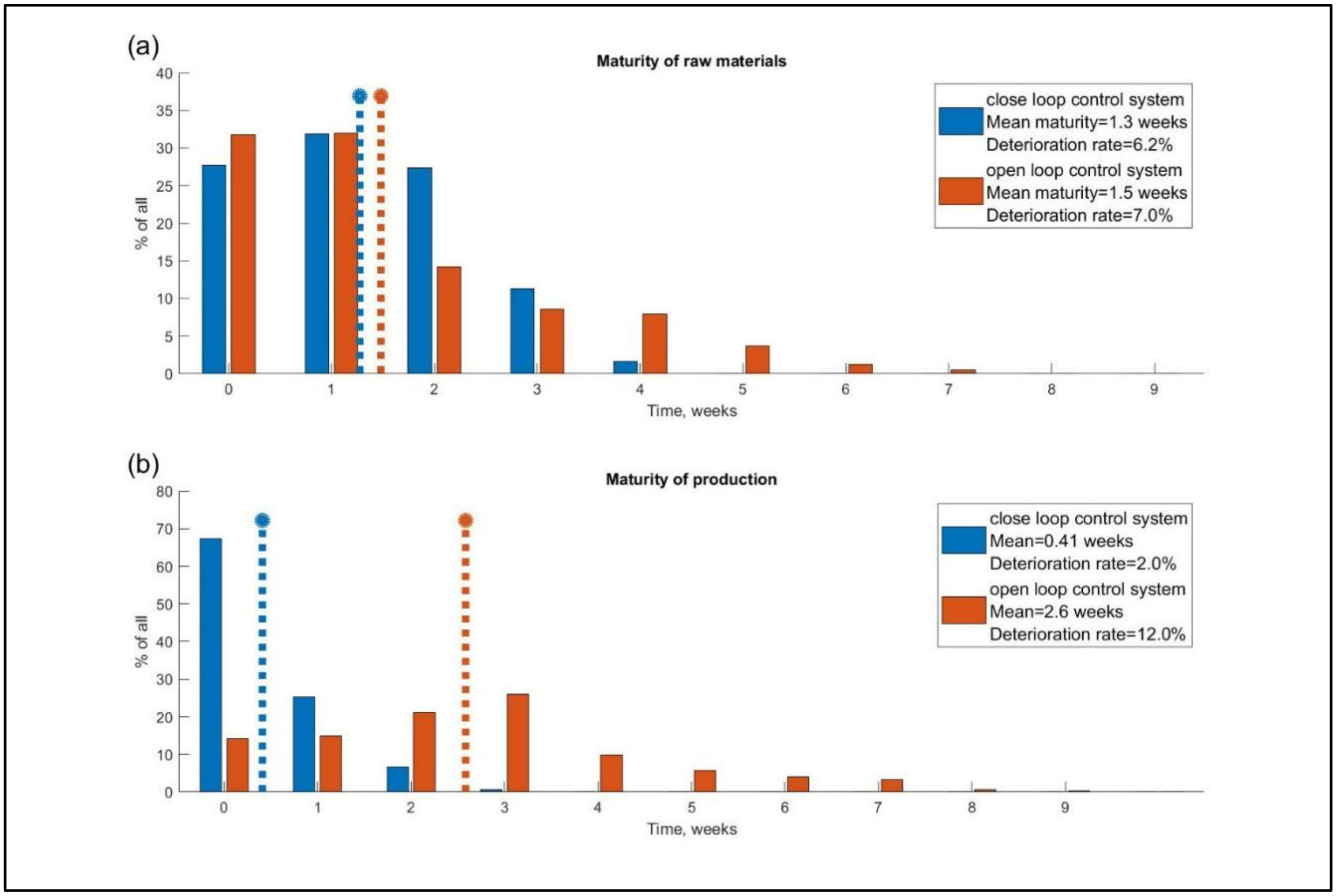

Figure 5-1: Maturity distribution comparison of closed-loop control system and open-loop control system: (a) raw materials maturity, and (b) production maturity.

As seen from the above figure, the neural network closed-loop control system has lower mean maturity, which is the number of weeks that raw materials or final products are stored without going to the production lines or being sold, and deterioration rate values for both raw materials and final products. This means that the neural network model optimises the inventory ordering process and the production process better, as it utilises almost all the available inventory, which reduces the associated storage costs. Regarding the former, the better performance of the neural network closed-loop system is reflected by lower mean maturity and deterioration rate than their respective values for the open loop system, by approximately 13% and 0.8%, respectively. This difference becomes even larger in the case of final products, as the differences between the two systems regarding the mean maturity and deterioration rate are approximately 84% and 10%, respectively. In other words, the neural network system generated a business strategy that enabled the factory to store most of the raw materials for two weeks or less, and sell the majority of final products instantly, i.e. make to order policy. On the other hand, according to the open-loop control system, some of the raw materials and final products are stored for seven and nine weeks, respectively, i.e. make to stock policy. This latest observation is especially important in the case under study; the maturity factor is important for the quality of steel, as the longer the steel is stored, the more likely it is to corrode; thus, clients strongly prefer to buy final products that have been stored for no longer than a few weeks. Therefore, the entire quantity of final products with maturity of more than M weeks (with large corrosion) will be very difficult to sell. Nonetheless, this scenario is not reflected in the business model, as the entire quantity of final products have equal chances to be sold.

5.3.1.2 Generated Profits per Storage Unit Used

Another parameter used to compare the performances of the two systems is the storage volume utilised. This parameter is calculated for each system over the entire planning horizon, and their performances are compared as shown in Figure 5-4.

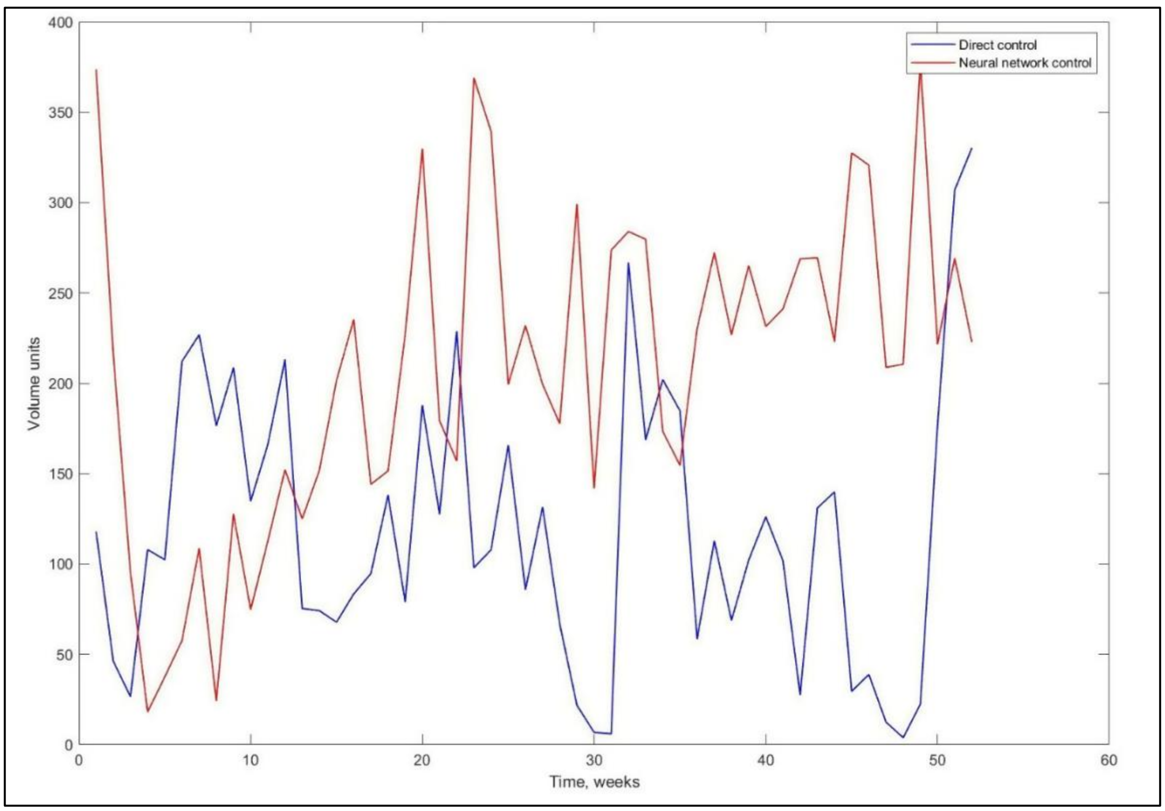

Figure 5-4. Comparison of the used storage volume for the two control systems.

From Figure 5-4, it can be seen that the volume of storage utilised is highly fluctuating over the entire planning zone, for the two tested control approaches. Although the dynamic is similar for both systems, there are some periods, especially the first weeks of the cycle, where the open-loop uses more storage units, whereas from the rest of the weeks the closed-loop approach requires more storage space. As already discussed throughout the whole thesis, the objective function of the developed model is profit maximisation. In this context, no conclusions regarding which system is better can be reached based only on the comparison of the used storage volume. Then, in order to be able to compare both approaches more efficiently, the information plotted in Figure 5-4 regarding the storage volume utilised is normalised based on the generated profits. In this way, both approaches can be more efficiently compared in terms of how much profit (in pounds) is realised from using one unit of storage. The results of such normalisation are shown in Table 5-3.

Table 5-3. Profit generated per unit of storage volume

#	Indicator	Storage Costs		Increase in Storage Cost Scenario	
		Direct control system	**Neural network control system**	**Direct control system**	**Neural network control system**
A1	Total storage volume units used (sum for all weeks) (units)	6,196	10,811	2,837	3,462
A2	Annual profit (£)	47,387,000	79,473,000	38,001,000	55,797,000
A3	All purchased raw materials (number)	4,752	8,136	3,479	6,360
A4	Profit per storage volume unit (A2/A1) (£/unit)	7,648	7,351.12	13,394.78	16,116.98
A5	Storage volume usage of one raw material unit (A1/A3)	1.3	1.33	0.82	0.54

From Table 5-3, it can be seen that one unit of storage volume for the ANN closed-loop system generates £7,351 of profit, whereas the direct control open-loop system generates £7,648. This can be attributed to the fact that the closed-loop approach allows the factory to produce and sell many more products, assuming limited storage availability. In this way, the more items that are stored the greater the storage costs, and thus, the less the profits. In addition, for both systems, the volume used by one unit of purchased raw material is about 1.3 volume units of storage. These results suggest that the normalised storage characteristics are very similar for both control systems. Nevertheless, the difference between the two systems becomes clearer when the storage costs increase, as in the case of the ANN closed-loop approach. In fact, from Table 5-3 it can be also seen that the profit generated by one storage volume unit through the ANN closed-loop system is significantly higher than the profit generated by the direct control open-loop system when the storage costs increase (£16,116.98 vs. £13,394.78). Moreover, this profit improvement is achieved with less storage volume utilised. Finally, the results shown in Table 5-3, provides useful tools that can be used in the practice towards helping the steel manufacturing management to estimate the extra profit it will be possible to achieve if they buy extra storage with some capacity.

5.3.1.3 Profit Generated by each Monte Carlo Run

Another comparison conducted between the two systems concerned the total profit generated by each system during each Monte Carlo run, which is depicted in Figure 5-5.

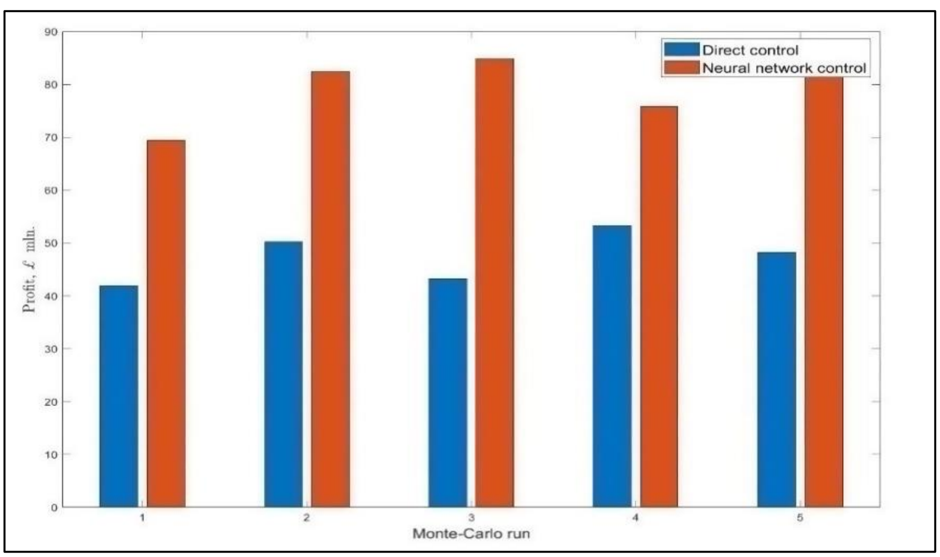

Figure 5-5. Profit comparison of the neural network control system and the direct control system.

As seen from the above chart, on average, the neural network closed-loop system results in 43% higher profit than the direct control system. The main reason for such a difference in performance is the fact that the neural network system uses the actual observations of business parameters at each run as inputs, from which it produces the weekly business decisions; on the contrary, the direct control model uses one strategy from the first week that cannot be changed in response to any external changes in the business environment.

5.3.1.4 Investment Strategy

Another important parameter used to compare the two systems is the way by which each system managed the investment of funds into the business. Figure 5-6 shows the dynamics of the investments made for both systems over the entire planning horizon.

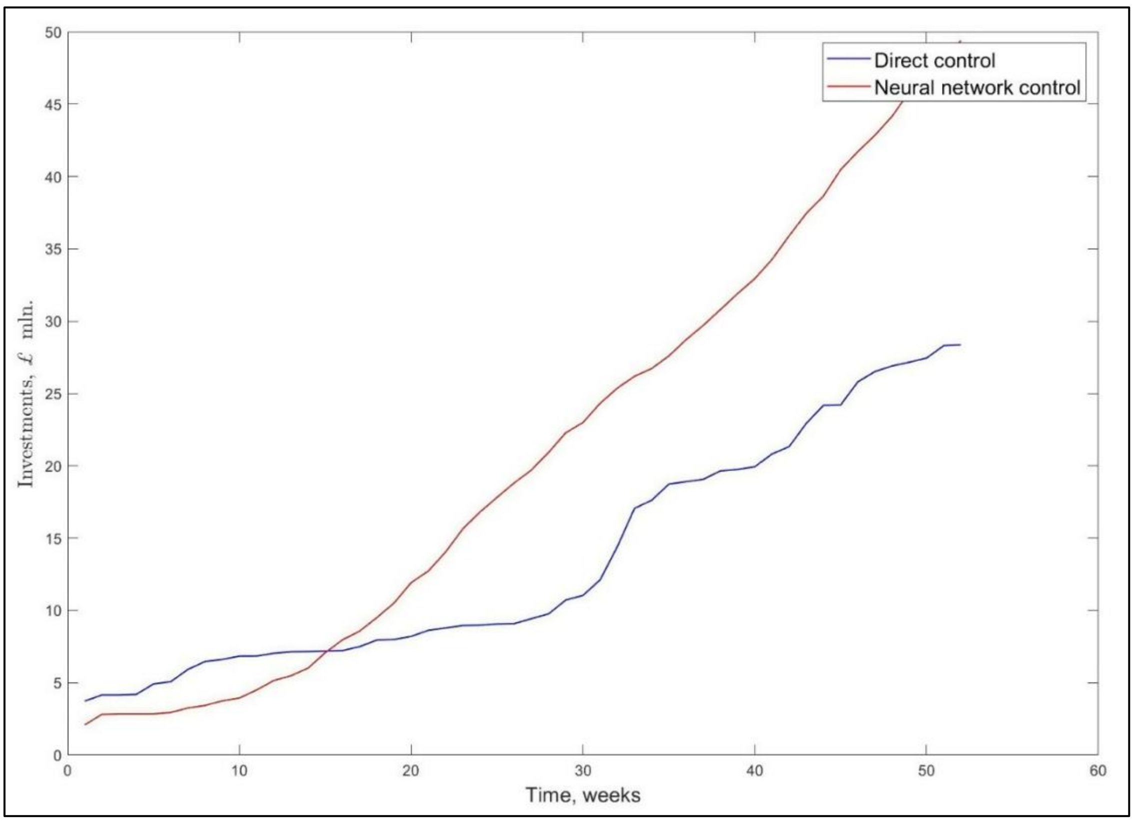

Figure 5-6. Investment comparison of the neural network control system and the direct control system.

As indicated by the above figure, the direct control open loop system tends to invest more money at the beginning of the planning horizon; however, as we move further in the planning horizon, the company will run out of money, forcing it to scale down its production activity and investment rate, which is evident by the lower levels of investments made at the end of the planning horizon. On the contrary, the neural network closed-loop system results in a more balanced strategy, in which a small investment is made at the beginning of the planning horizon, then the amount of this investment increases as we move forward in the planning horizon. This strategy is more beneficial for the steel manufacturing factory, as it ensures that there will be sufficient funds available throughout the planning horizon to cover any additional demand.

5.3.1.5 Money Management

The decision of whether to invest the money in the business or hold it in stock is an important decision that must be made by the management of the steel manufacturing factory. From Figure 5-7, it is clear that the neural network system tends to have more funds in stock rather than moving it to investment, as the y-axis represents the funds available for the factory. On the other hand, the direct control system has much less money over the entire planning horizon. This latter strategy decreases the ability of the company to increase the quantity of raw materials purchased as a reaction to any increased demand, which might hinder the company's operations and prevent it from maximising its profits.

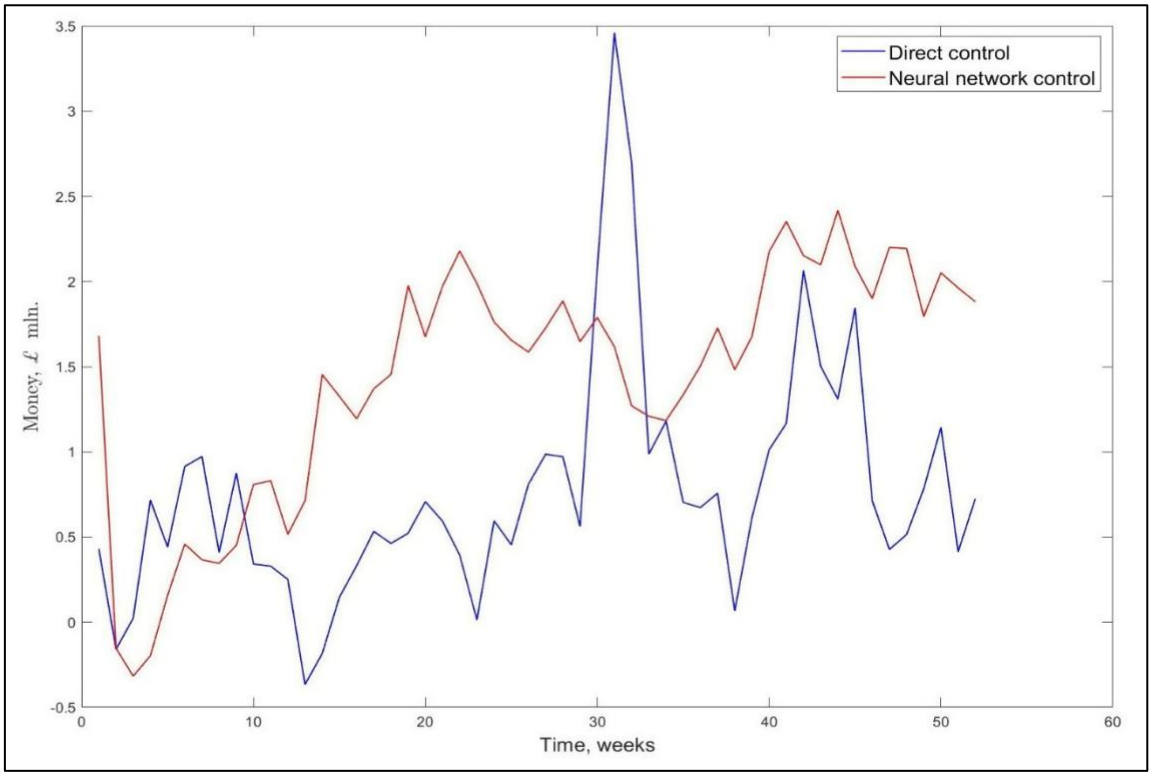

Figure 5-7. Money management comparison of the neural network control system and the direct control system

5.3.1.6 Learning Progress

Finally, during the optimisation process, different scenarios are assumed in order to determine which system, under which scenario, leads to profit maximisation for the company. Six different scenarios are used, three for each system, and the amount of profit generated through each scenario is depicted in Figure 5-8.

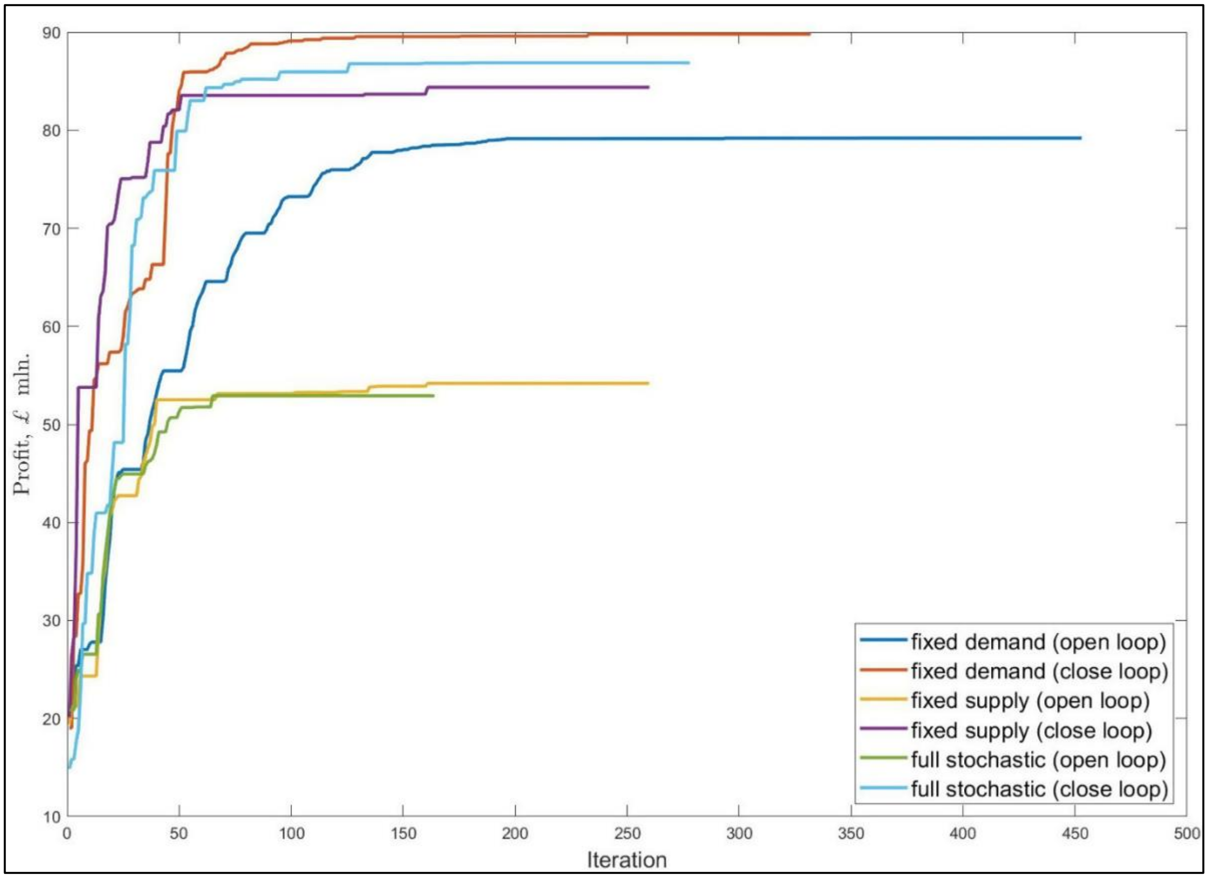

Figure 5-8. Learning progress for all models (shows the best achieved profit for each model).

As seen from the above figure, the highest profit realised resulted from the fixed demand scenario model based on neural network closed-loop optimal control generation. In fact, the three scenarios that used the closed-loop system generated the three highest profits, and, for most cases, the learning process lasted fewer than 300 iterations before converging. On the other hand, the only open loop control system that came close to the neural network close loop system is the fixed demand one, while the other two scenarios showed much worse performance. Finally, the shapes of the learning plots depicted above are typical of global optimisation techniques, as they first exhibit a sharp increase in the target function, and then this increase slows down. Hence, from the above figure, it is evident that 500 iterations are sufficient to find the solution for the model, which is close to optimal.

In conclusion, as evidenced by all six comparable parameters, the performance of the neural network system is much better for all parameters than the performance of the direct system. In particular, the conducted experiments and the results analysed above have shown that the ANN closed-loop model:

1. Has shorter maturity rates for both raw materials and final products
2. Generates more profit per unit of storage when the storage costs are high
3. Generates more profit per Monte Carlo run
4. Provides a more sound investment strategy
5. Manages the available funds more efficiently
6. Has a better learning progress.

Hence, this control system is chosen for the developed model, and will be applied for the three basic scenarios described earlier. The results of each scenario are discussed in detail in the following sections.

5.3.2 Fixed Demand Scenario

In this scenario, all variables related to demand for the final product are set to be fixed, while the storage and supply variables remain stochastic. After adjusting the variables as detailed, the neural network closed-loop system is implemented, and the performance of this system is analysed in terms of:

- Maturity and deterioration rate
- Money management
- Final product management
- Raw materials management

5.3.2.1 Maturity and Deterioration Rates

Two of the most important outputs of the implemented model are the maturity and deterioration rates of both raw materials and final products. Therefore, when applying the neural network closed-loop model under the fixed demand scenario, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 5-9(a) and (b) for raw materials and final products, respectively.

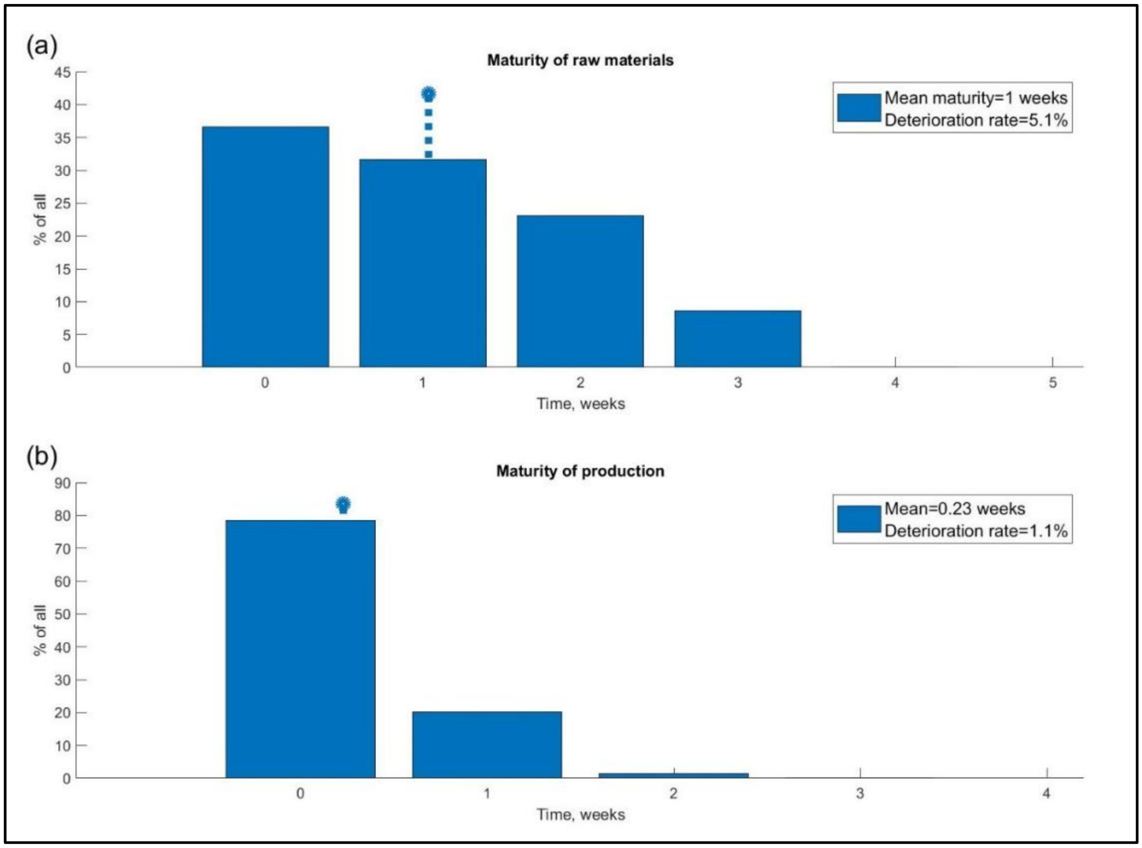

Figure 5-9. Maturity plots for the fixed demand scenario: (a) raw materials maturity, and (b) production maturity.

In the above figure, subplot (Figure 5-9(a)) shows how long the raw materials have to wait in the store before being sent to the production lines, while subplot (Figure 5-9(b)) shows how long the final products are kept in storage before being sold or sent to the buyer, and the dashed blue lines with asterisks at the top of each plot show the mean of the maturity distribution. From these plots, the average maturity for the raw materials is one week, which means that the majority of the raw materials was consumed in a one-week interval. Moreover, the average maturity of the final products is approximately 0.23 weeks, which is drastically lower than the corresponding value for raw materials. This can be explained by the fact that the factory needs to keep extra raw materials in the store for the case of an increase in demand. In addition, more than 35% of all purchased raw materials and 78% of final products are made to order.

5.3.2.2 Money Management

Under this parameter, four different indicators are used to assess the performance of the developed model under the fixed demand scenario. These indicators are the dynamics of available funds (Figure 5-10(a)), the dynamics of the amount of money invested in the business (Figure 5-10(b)), the change in the selling price over the planning horizon (Figure 5-10(c)), and the amounts of up credit and down credit (Figure 5-10(d)).

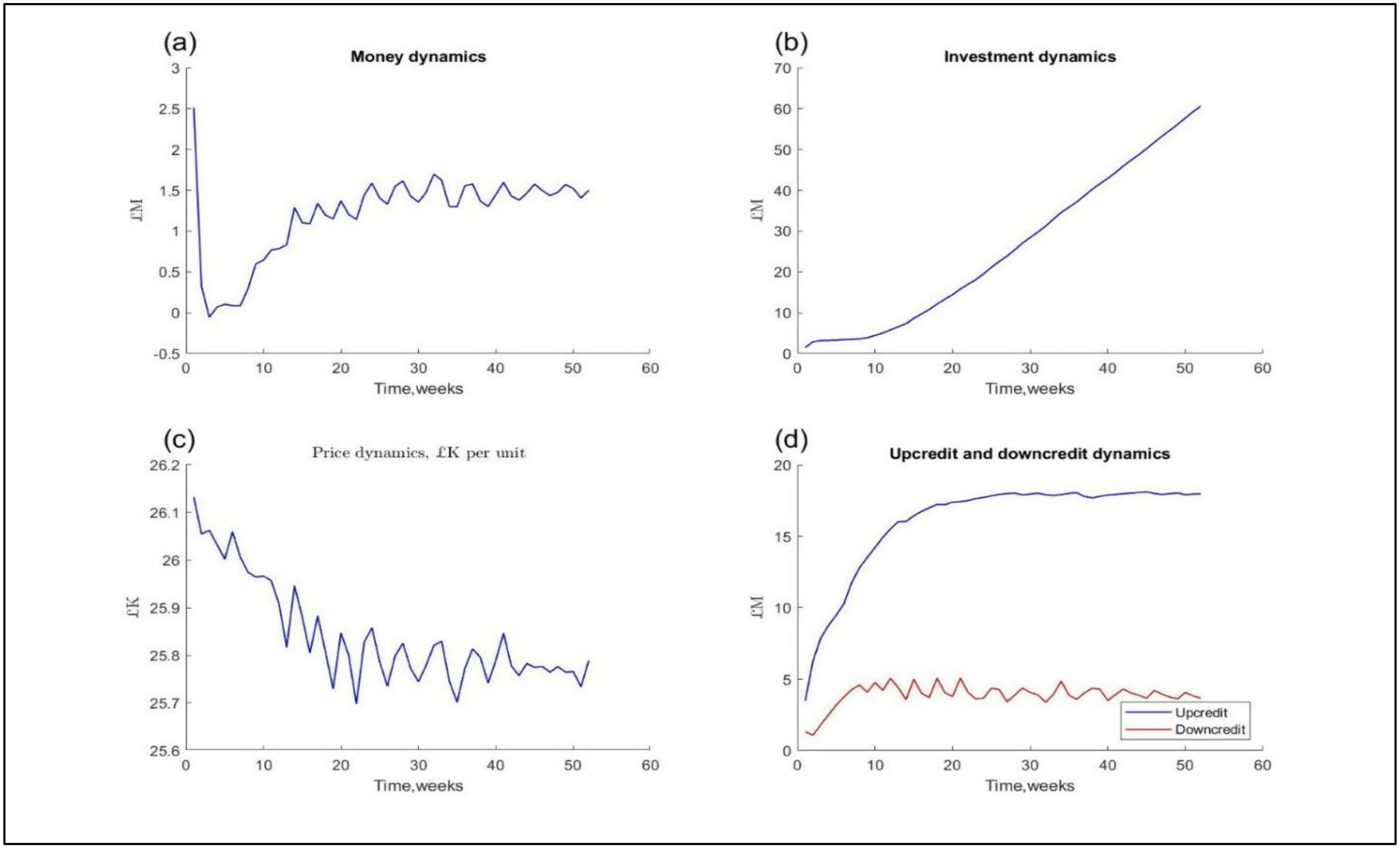

Figure 5-10. Money management for the fixed demand scenario: (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics.

As seen from Figure 5-10(a), the plot starts with the amount of initial funds available before the start of the planning horizon, then it decreases dramatically to almost zero by the third week. This initial decrease is normal, as there is a need during that period to purchase more raw materials and increase their level in the store in order to produce more goods. Furthermore, at the beginning of the planning horizon, the initial quantity of final products produced is only 100 units, as per the model assumptions, which do not generate much income to offset the initial costs of the raw materials. However, as we move forward in the planning horizon and the production quantity is increased, the amount of available funds starts to increase, gradually, until it reaches almost £1.5M in the 52nd week. Unlike the above trend, the amount of investment follows a continuously increasing pattern over the planning horizon, as seen in Figure 5-10(b), until it reaches more than £60M, which shows that the model ensures the continuous running of the factory's operations. Moreover, Figure 5-10(c) shows the dynamics of the final product selling price over the entire planning horizon. As seen from that figure, the selling price tends to decrease over time due to the increase in the quantity of final products in stock, from an initial peak of £26.1K to as low as £25.7K. However, this decrease is not continuous over the entire planning horizon, as there are periods of fluctuation between an increase and decrease in the selling price based on the quantity of final products in the store, i.e. as the quantity of final products in the store increases, there is a need to sell the products as fast as possible, thus price reduction becomes necessary. Finally, in Figure 5-10(d), both the up credit and down credit dynamics are shown. As demonstrated for both indicators, the amounts start to increase at the beginning of the planning horizon, and then stabilise and remain at approximately the same level until the end of the planning period. Another observation from this figure is that the down credit amount is always much lower than the up credit amount, because the purchase price of raw materials is lower than the selling price of final products.

From the four plots analysed here, it is evident that the factory increases the amount of available funds at the end of the period. In particular, investments have shown to increased continuously, showing a higher increasing rate towards the end of the period to mitigate the impact of inflation. In addition, the amount of up credit grows until week 20 and remains constant then. This is due to the fact that the factory cannot sell more than what it produces, which has a maximum of 150 units.

5.3.2.3 Final Products Management

To assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of final products in storage (Figure 5-11(a)), the quantity of final products produced (Figure 5-11(b)), the quantity of final products sold (Figure 5-11(c)), and the percentage of produced goods sold (Figure 5-11(d)).

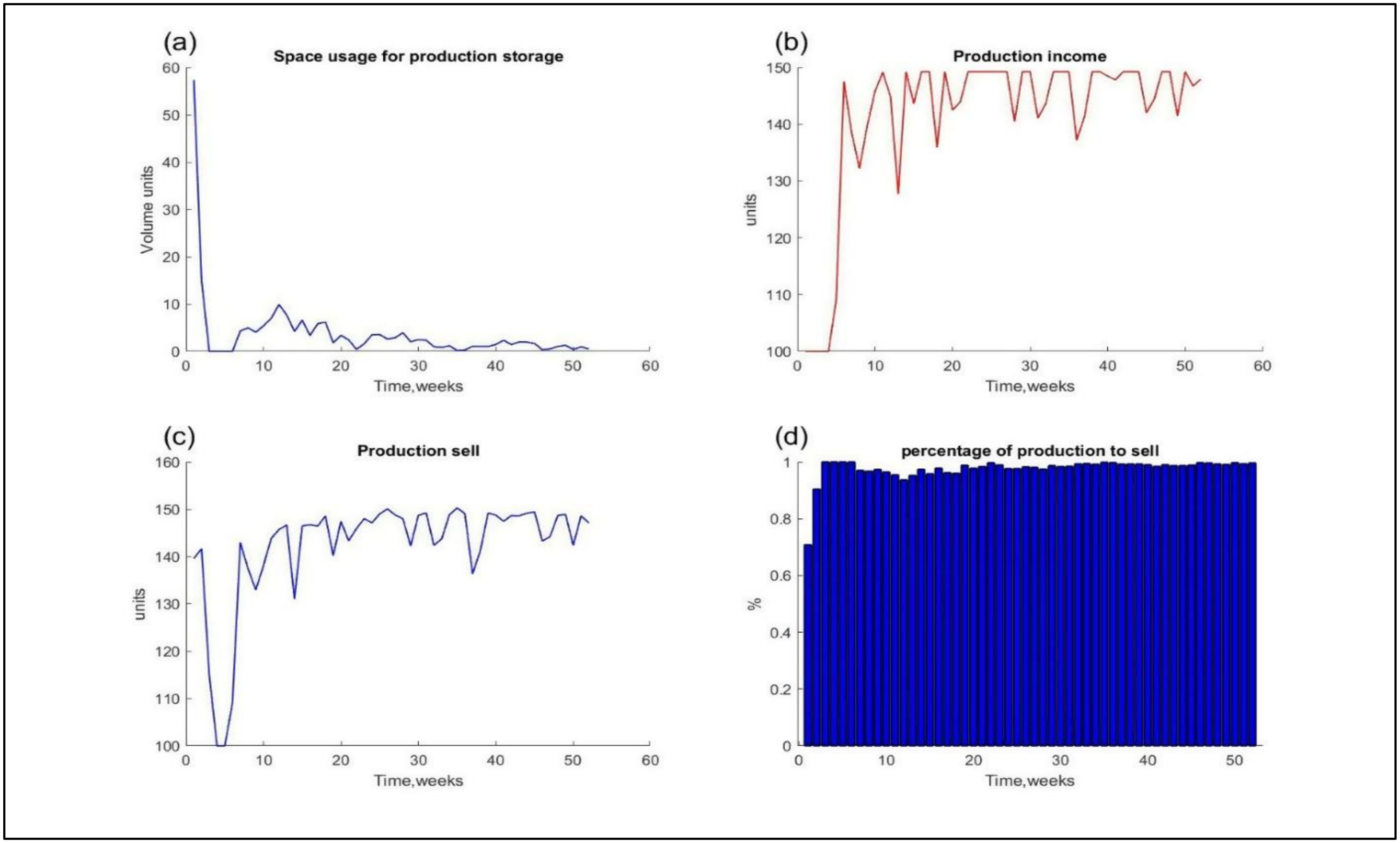

Figure 5-11. Production management for the fixed demand scenario: (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold.

From Figure 5-11(a), the quantity of final products in the store starts with the initial quantity held before the start of the planning period, then falls dramatically until it reaches zero after four weeks. Then, throughout the rest of the planning horizon, the quantity of final products in the store did not exceed 10 units, as most of the final products that were produced are sold, which demonstrates the effectiveness of the developed model in minimising the store of the final products. However, it is worth noting that in this scenario, demand is static, and future production can be easily forecasted with great precision. Figure 5-11(b) depicts the trend of the quantity of final products produced over the planning horizon. As seen from this plot, the production level starts at zero, and then the factory almost reaches its maximum capacity (150 units) within five weeks. After this point, the factory continues to utilise its maximum capacity over most of the planning horizon. Similarly, Figure 5-11(c) shows the quantity of final products that were sold. As can be seen from this figure, the quantity of sold products is almost identical to the quantity of produced products over the entire planning horizon, as it can also be inferred from Figure 5-11(a), where the quantity of products in storage has shown to be low. Finally, Figure 5-11(d) further confirms the previous conclusion from Figure 5-11(a) and (c), showing that the percentage of sold products relative to produced products is almost 100% over the entire planning horizon.

5.3.2.4 Raw Materials Management

As with final products management, to assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of raw materials in storage (Figure 5-12(a)), the quantity of raw materials purchased (Figure 5-12(b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 5-12(c)), and the percentage of raw materials that went into production (Figure 5-12(d)).

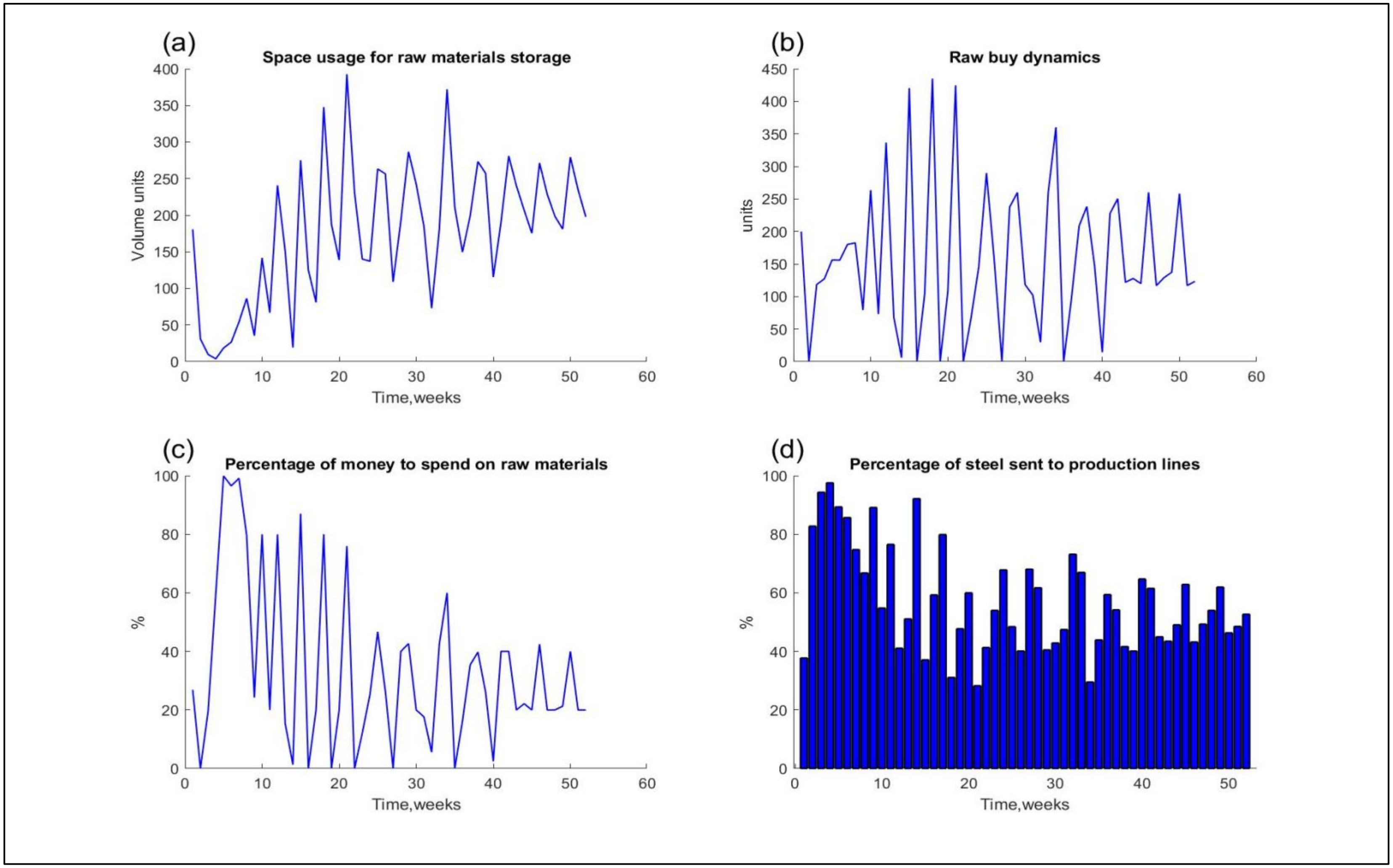

Figure 5-12. Raw materials management for the fixed demand scenario: (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) the percentage of raw materials sent to the production lines.

From Figure 5-12(a), after the initial accumulation of raw materials, their level in storage oscillates around 250 items to ensure that there are sufficient raw materials to cover any increase in demand. Moreover, Figure 5-12(b) shows the quantity of raw materials purchased over the planning horizon, which highly oscillates around 150 items per week. In some weeks, the factory does not even buy any raw materials due to the high quantity present in the store. The main reason behind such oscillation is that it is more profitable for the company to buy bulk amounts of raw materials, in a given week, to benefit from discounts, and then not buy at all in the next week. On average, the factory buys the same level of raw materials required to produce and sell goods, which is no more than 150 items of final products per week. Similarly, Figure 5-12(c) shows the percentage of money that was spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased. Finally, Figure 5-12(d) shows the percentage of raw materials that moves from the store to the production lines. As seen from these two figures, the optimal strategy for the company is to accumulate funds over two to three weeks, and then make a bulk order of raw materials that covers multiple weeks. After making this order, the company immediately sends these raw materials to production. In conclusion, the raw materials dynamic analysed in Figure 5-12, shows that, although during some periods there is a large quantity of raw materials in the store, this quantity is likely to be consumed in the next one or two weeks.

5.3.3 Fixed Supply Scenario

In this scenario, all variables related to the supply of raw materials (probability of delivery failure, average extra cost per unit of failed raw material, ordering cost, and unit cost of raw materials) are set to be fixed, while the storage and demand variables remain stochastic. After adjusting the variables as detailed, the neural network closed-loop system is applied, and the performance of this system is analysed in terms of:

- Maturity and deterioration rate
- Money management
- Final product management
- Raw materials management, is analysed

5.3.3.1 Maturity and Deterioration Rates

When applying the neural network closed-loop system under the fixed supply scenario, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 5-13(a) and (b) for raw materials and final products, respectively.

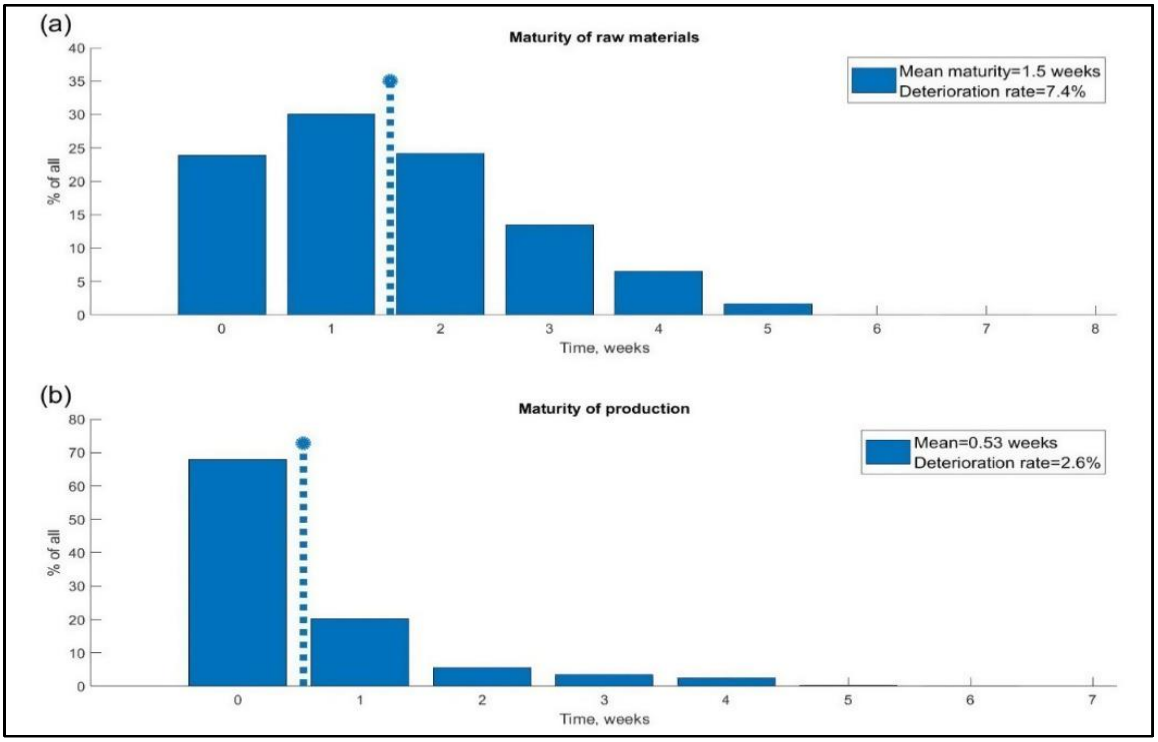

Figure 5-13. Maturity management for the fixed supply scenario: (a) raw materials maturity, and (b) production maturity.

From the above plots, the average maturity for raw materials is one and half weeks, which means that the majority of raw materials are consumed in that time interval. Moreover, the average maturity of final products is approximately four days, which is drastically lower than the corresponding value for raw materials. This can be explained by the fact that the factory needs to keep extra raw materials in storage, in case of an increase in demand.

5.3.3.2 Money Management

Under this parameter, four different indicators are used to assess the performance of the developed model under the fixed demand scenario. These indicators are the dynamics of available funds (Figure 5-14(a)), the dynamics of the amount of money invested in the business (Figure 5-14(b)), the change in the selling price over the planning horizon (Figure 5-14(c)), and the amounts of up credit and down credit (Figure 5-14(d)).

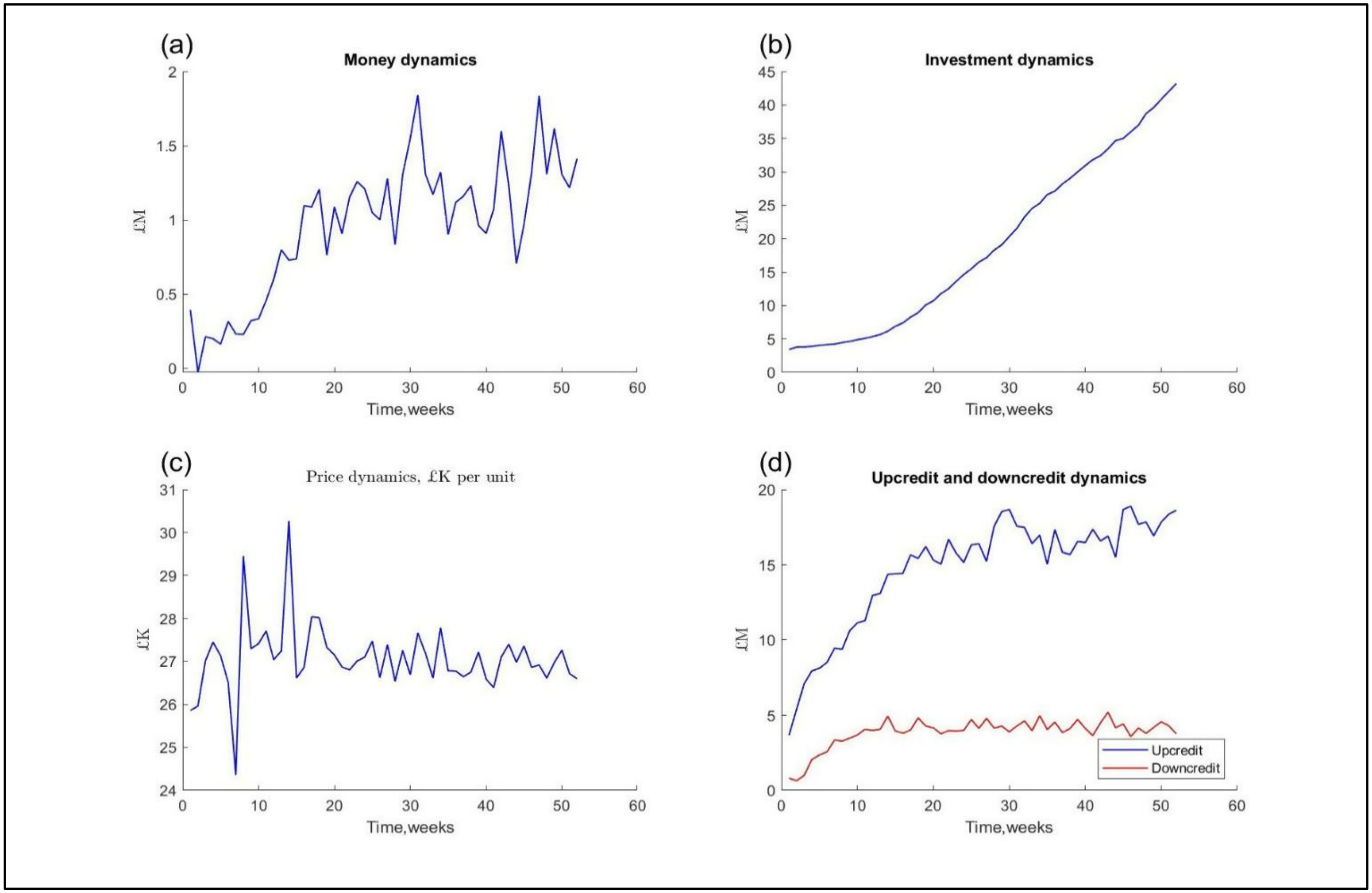

Figure 5-2. Money management for the fixed supply scenario: (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics.

As seen from Figure 5-14(a), the plot starts with the amount of initial funds available before the start of the planning horizon, then it decreases to zero by the second week. However, as we move forward in the planning horizon, the level of available funds starts to grow over time until the 25th week, then it starts to oscillate around the £1.5M. This oscillation is not observed in the fixed demand scenario, as in that scenario the demand is fixed, so there is no potential for an increase in demand. However, in the fixed supply scenario, demand is stochastic; hence, the probability of an increase in demand becomes higher, and the company will need to keep extra funds to cover any potential increase in demand. Unlike the trend in Figure 5-14(a), the amount of investment follows a continuously increasing pattern over the planning horizon, as seen in Figure 5-14(b), until it reaches almost £45M. Figure 5-14(c) shows the dynamics of the final product selling price over the entire planning horizon. From this figure, during the first weeks of the planning horizon, there is a lot of change in the selling price before it reaches equilibrium at around the £27K mark. After this point in the planning horizon, the price management becomes similar to the fixed demand scenario, except that the average price is a bit higher. Finally, in Figure 5-14(d), both the up credit and down credit dynamics are observed. As demonstrated for both indicators, the amounts start to increase at the beginning of the planning horizon, as the factory starts buying more raw materials, hence the down credit increases. On the other hand, the up credit amount increases as the factory starts to sell more final products, hence, the down credit increases, and then these amounts stabilise and remain at approximately the same level until the end of the planning period. Moreover, another observation from this figure is that the down credit amount is always much lower than the up credit amount, because the purchase price of raw materials is lower than the selling price of final products, which is similar to the trends observed in the fixed demand scenario. In addition, from this figure, the amount of cash available for the factory can be computed as the difference between the up credit and down credit in any given week.

5.3.3.3 Final Products Management

To assess the performance of the developed model under this parameter in the fixed supply scenario, four different indicators are used. These indicators are the quantity of final products in the store (Figure 5-15(a)), the quantity of final products produced (Figure 5-15(b)), the quantity of final products sold (Figure 5-15(c)), and the percentage of produced goods sold (Figure 5-15(d)).

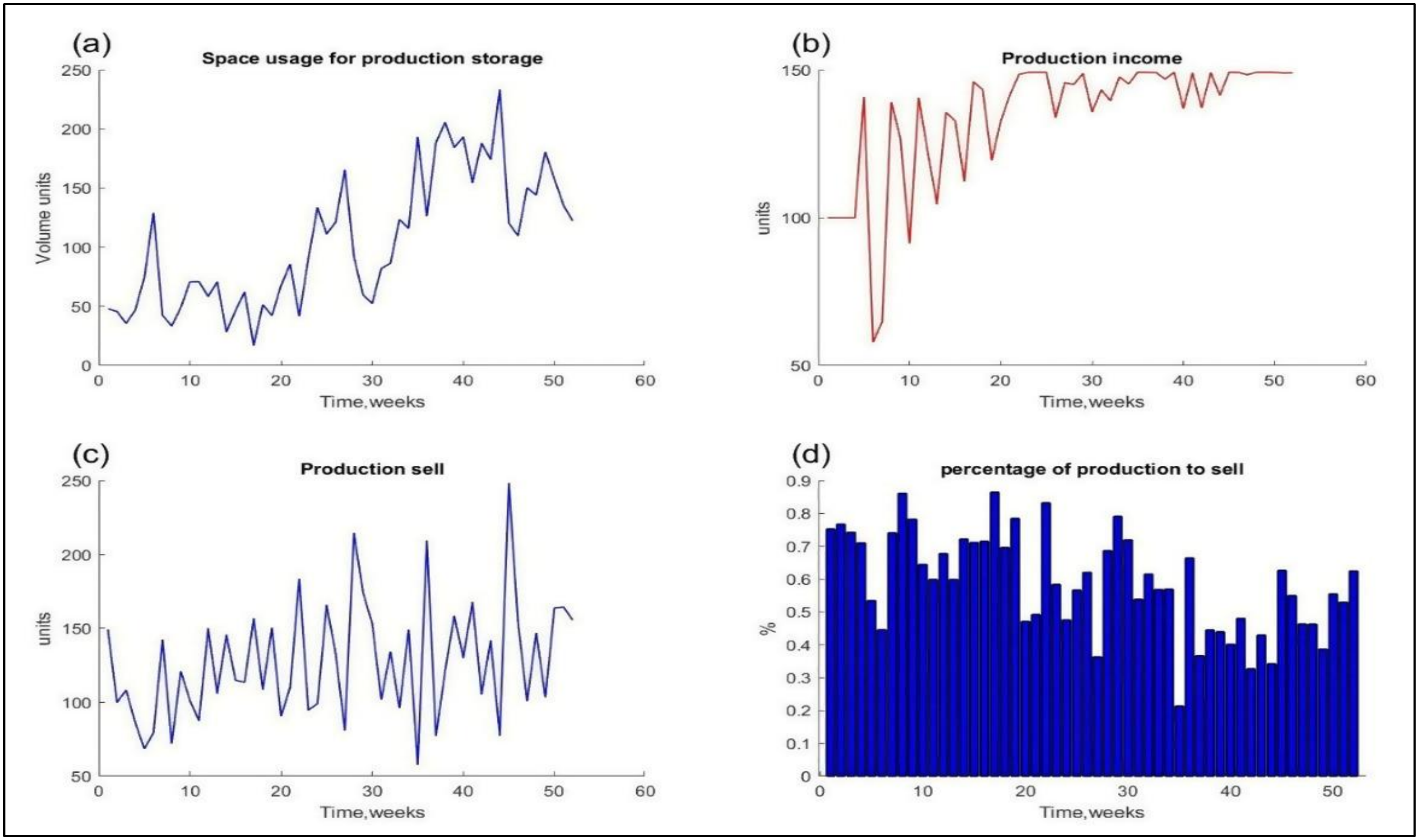

Figure 5-15. Production management for the fixed demand scenario: (a) the storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced sold.

In Figure 5-15(a), the quantity of final products in the store varies drastically over the entire planning horizon, going as low as 20 units in the 18th week, and as high as 230 units in the 44th week. This high fluctuation, especially when compared to the fixed demand scenario, is due the fact that demand is stochastic and the accuracy of its forecast is not as high as in the fixed demand scenario; hence, the factory needs to store extra products in order to be able to satisfy any increase in demand due to its stochastic nature. Furthermore, Figure 5-15(b) depicts the trend of the quantity of final products produced over the planning horizon. As seen from this plot, the production level starts at 100 units and then it fluctuates drastically between 55 and 140 units, until the factory almost reaches its maximum capacity (150 units) within 21 weeks. After this point, the factory operates at maximum capacity over most of the remaining planning horizon. Similarly, Figure 5-15(c) shows the quantity of final products sold. As initially explained, there is a need to produce additional goods to cover any increase in demand; thus, in some weeks the factory had excess quantity of final products that were not sold. Finally, this above conclusion is depicted more clearly in Figure 5-15(d), as the percentage of sold products to produced products ranged between 40% and 80% over the entire planning horizon.

5.3.3.4 Raw Materials Management

As with final product management, to assess the performance of the developed model under this parameter in the fixed supply scenario, four different indicators are used. These indicators are the quantity of raw materials in the store (Figure 5-16(a)), the quantity of raw materials purchased (Figure 5-16(b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 5-16(c)), and the percentage of raw materials that went into production (Figure 5-16(d)).

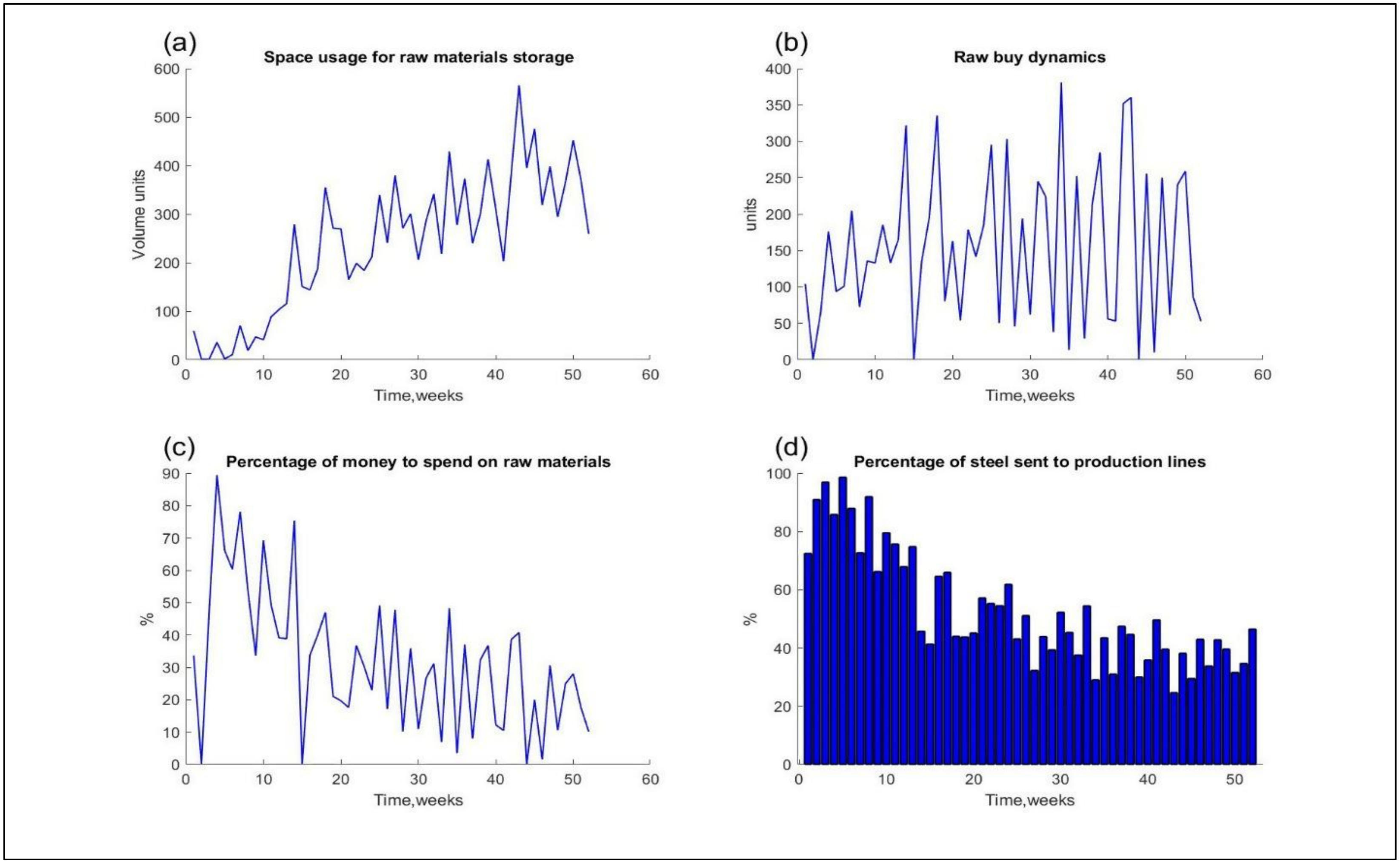

Figure 5-16. Raw materials management for the fixed supply scenario: (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) the percentage of raw materials sent to the production lines.

From Figure 5-16(a), after the initial drop in the quantity of raw materials, their level in the store starts to increase gradually over the entire planning horizon to cover any additional production needed. This indicates that the factory buys more raw materials as the production rate increases, to cover the demand. Moreover, Figure 5-16(b) shows the quantity of raw materials purchased over the planning horizon, which highly oscillates around 150 items per week due to the same reasons as those explained in the fixed demand scenario. Similarly, Figure 5-16(c) shows the percentage of funds spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased, i.e. the factory spends money to cover the purchase of raw materials. Finally, Figure 5-16(d) shows the percentage of raw materials that moves from the store to the production lines, which reached 100% in the first few weeks, then started to decrease gradually to as low as 30%. This indicates that, at the beginning of the planning horizon, the model is more effective in managing raw materials, as almost all of them were used in production, whereas towards the end of the planning horizon there were extra raw materials in store that were not used in production.

In conclusion, compared to the previous scenario where fixed demand has been considered, the factory uses more space to store the final products since in the scenario analysed in this section the demand is stochastic needing the factory to keep more raw materials in storage in case of shortage in production which is caused by sudden increase in demand. However, since, for the closed-loop control system, increasing the company's profit is a higher priority than minimising the level of raw materials in storage, allowing slightly more raw materials in the store to achieve much more profit is an acceptable outcome.

5.3.4 Fully Stochastic Scenario

The last scenario in which the developed model is implemented is the fully stochastic scenario. In this scenario, the stochastic and fixed business parameters listed in Table 5-1 and Table 5-2 are used to reflect all the complexity of the business scope. To model this scenario, the demand is assumed to occur at random, i.e. the demand changes at each week, and is characterised by having independent and identically distributed times during each week. In addition, the supply is also considered to be stochastic and normally distributed with known distribution parameters. After adjusting the variables as detailed, the neural network closed-loop system is applied under this scenario, and the performance of this system is analysed, in terms of maturity and deterioration rate, money management, final product management, and raw materials management.

5.3.4.1 Maturity and Deterioration Rates

When applying the neural network closed-loop model under the fully stochastic scenario, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 5-17(a) and (b) for raw materials and final products, respectively.

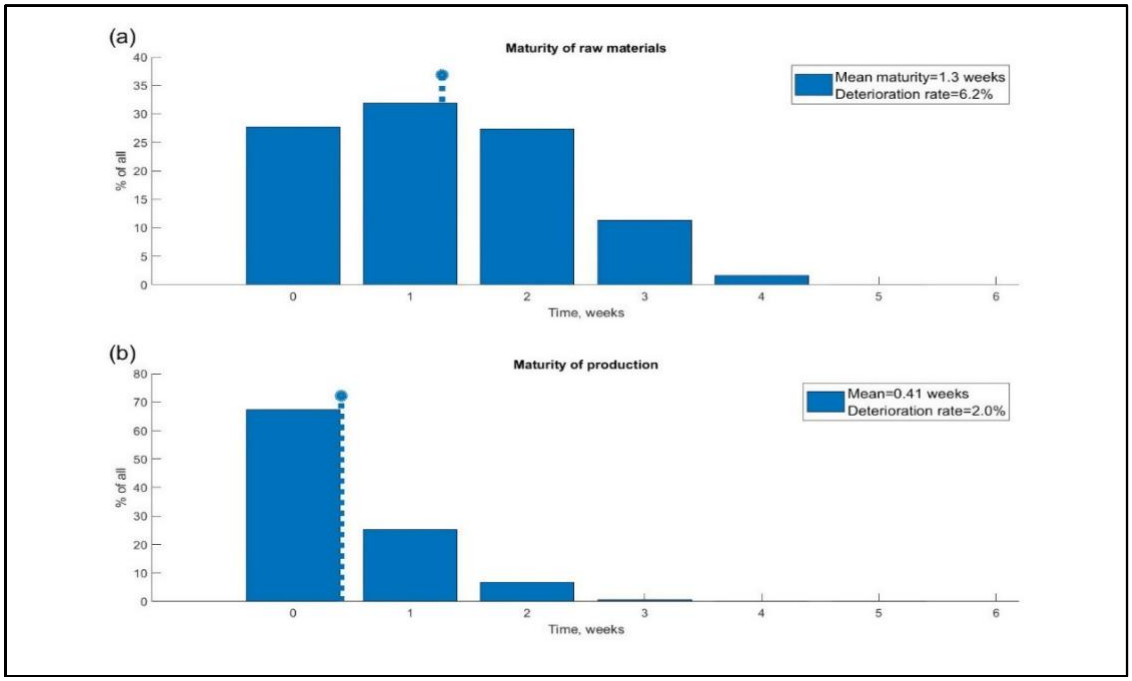

Figure 5-17. Maturity management for the fully stochastic scenario: (a) raw materials maturity, and (b) production maturity

From the above plots, the average maturity for raw materials is nine days, which means that the majority of raw materials is consumed in that time interval. Moreover, the average maturity of final products is approximately 0.57 weeks, which means that the control system sets an optimal selling price so almost all goods in the store are sold immediately. Furthermore, it can be seen that the storage of final products is more optimal than that of raw materials, as it is much easier to dispose of any excess quantity of final products, by assigning a lower price, than to reduce the quantity of raw materials.

5.3.4.2 Money Management

As with the previous scenarios, four different indicators are used to assess the performance of the developed model, in terms of money management, under the fully stochastic scenario. These indicators are the dynamics of available funds (Figure 5-18 (a)), the dynamics of the amount of money invested in the business (Figure 5-18(b)), the change in the selling price over the planning horizon (Figure 5-18(c)), and the amounts of up credit and down credit (Figure 5-18(d)).

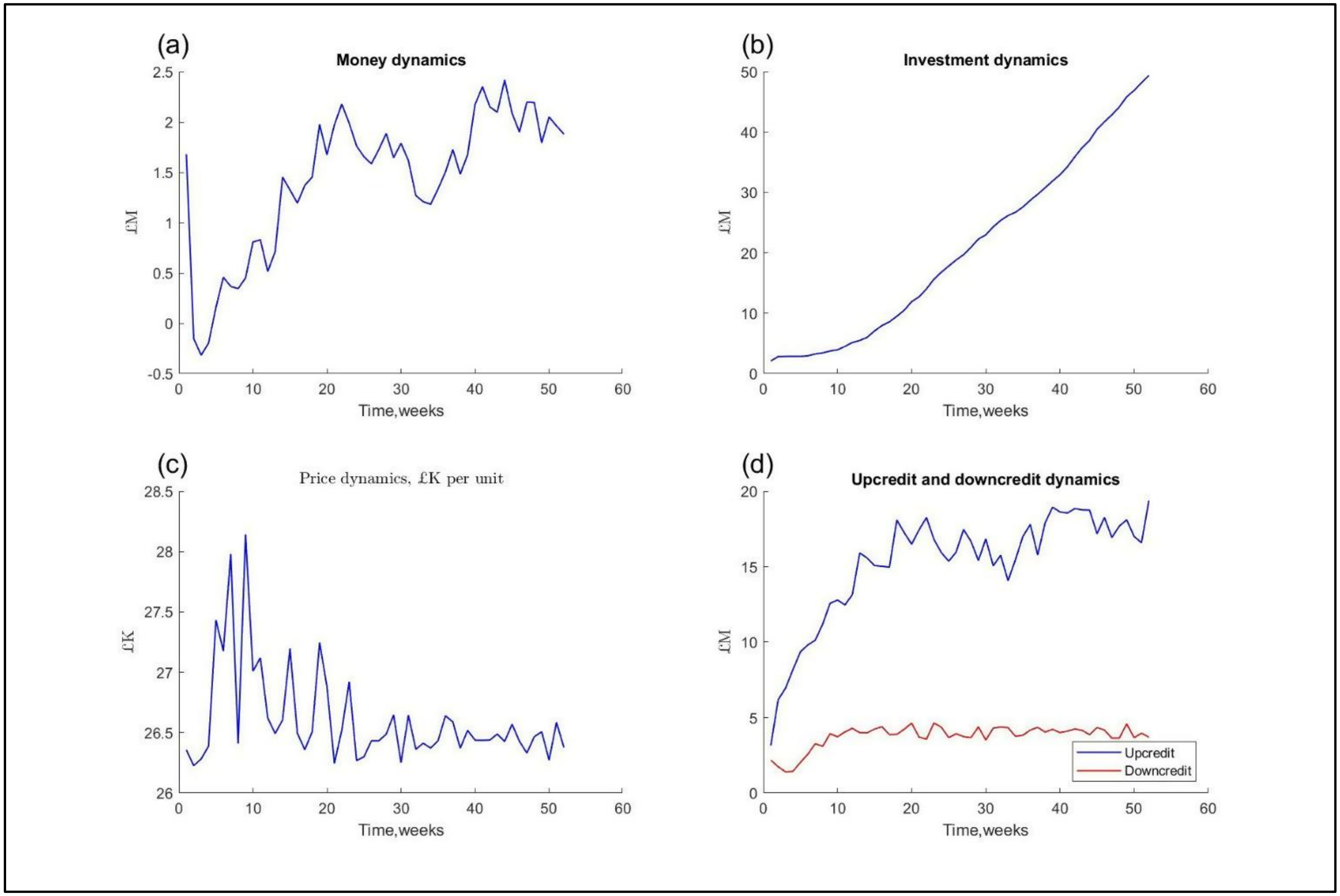

Figure 5-3: Money management for the fully stochastic scenario: (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics**.**

As seen from Figure 5-18(a), and similar to the previous scenarios, the plot starts with the amount of initial funds available before the start of the planning horizon, then it decreases dramatically until it becomes negative by the second week, meaning that the factory ran short of money. However, as we move forward in the planning horizon, the level of available funds starts to grow over time, reaching £2.5M. This means that, compared to the previous two scenarios, the amount of funds kept in the business is higher. Unlike the trend observed in Figure 5-18(a), the amount of investment follows a continuously increasing pattern over the planning horizon, as seen in Figure 5-18(b), until it reaches almost £50M. This means that the final level of investments is lower than that of the fixed demand scenario and higher than that of the fixed supply scenario, as the full stochastic scenario has an overall higher profit than the fixed supply scenario, but lower profit than the fixed demand scenario. Figure 5-18(c) shows the dynamics of the final product selling price over the entire planning horizon. From this figure, during the first weeks of the planning horizon, there is a lot of change in the selling price before it reaches £28K in week 10. After this point in the planning horizon, the price tends to decrease over time as the production volumes increase. Therefore, selling at a discount becomes more profitable for the company than having the final products stuck in the store. Finally, in Figure 5-18(d), both the up credit and down credit dynamics are observed. As demonstrated for both indicators, the amounts start to increase at the beginning of the planning horizon, and then stabilise and remain at approximately the same level until the end of the planning period. Moreover, the up credit curve has more frequent and larger oscillations than the previous two scenarios, which are caused by the stochastically changing demand and price.

5.3.4.3 Final Product Management

As explained in the previous two scenarios, to assess the performance of the developed model under this parameter in the fully stochastic scenario, four different indicators are used. These indicators are the quantity of final products in the store (Figure 5-19(a)), the quantity of final products produced (Figure 5-19(b)), the quantity of final products sold (Figure 5-19(c)), and the percentage of produced goods sold (Figure 5-19(d)).

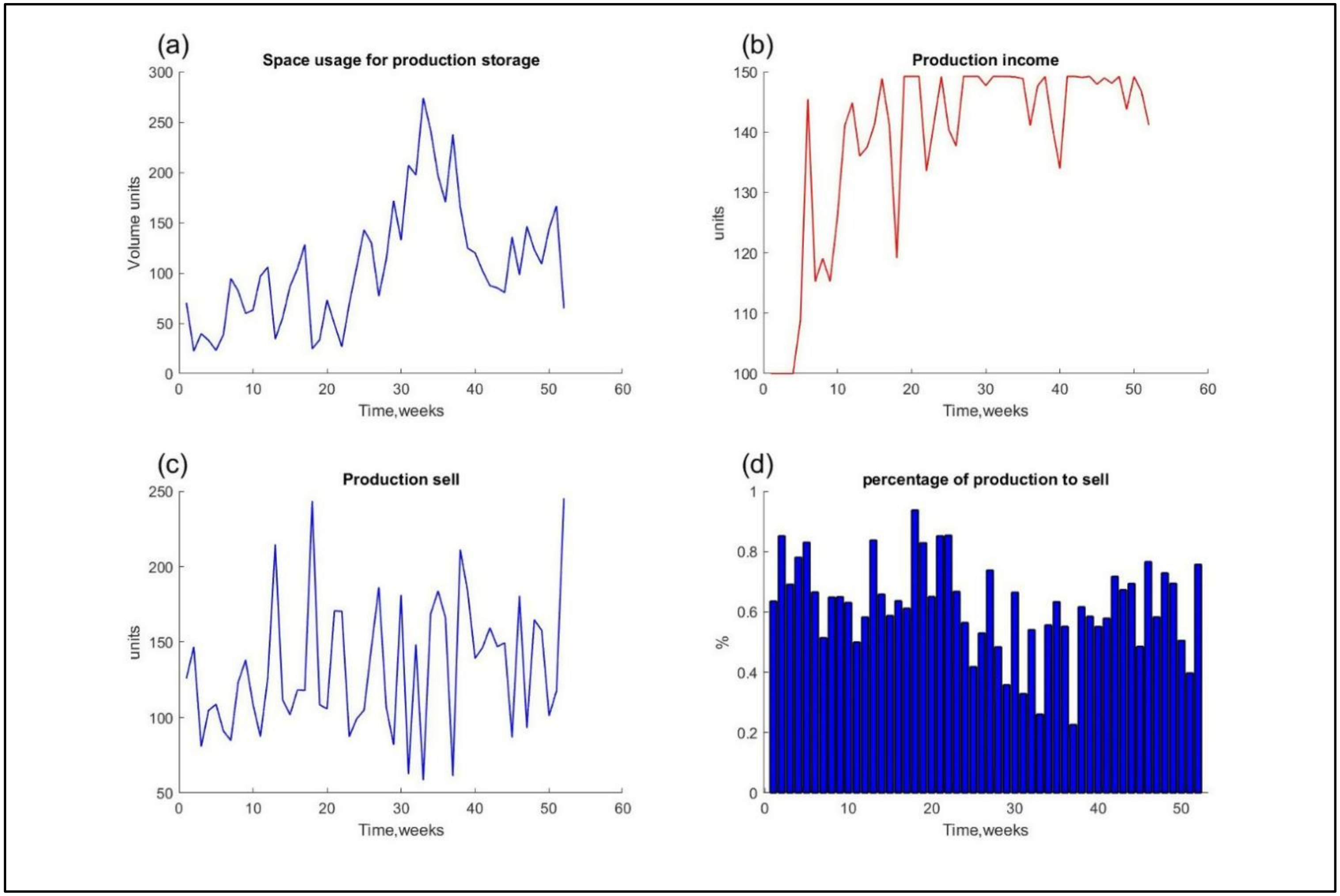

Figure 5-4: Production management for the fully stochastic scenario: (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold.

In Figure 5-19(a), the quantity of final products in the store varies drastically over the entire planning horizon, going as low as 25 units in the 2nd week and as high as 275 units in the 34th week. This high fluctuation, especially when compared to the fixed demand scenario, is due to the fact that demand is stochastic, and the accuracy of its forecast is not high, hence the need to store extra products in case of demand increase. Figure 5-19(b) depicts the trend of the quantity of final products produced over the planning horizon. As seen from this plot, the production level starts at 100 units, then increases sharply to 145 units, and then starts to fluctuate drastically between 120 and 140 units until the factory almost reaches its maximum capacity (150 units) within 20 weeks. After this point, the factory continues to operate at its maximum capacity over most of the remaining planning horizon. Similarly, Figure 5-19(c) shows the quantity of final products that were sold. As initially explained, there is a need to produce additional goods to cover any increase in demand, as demand is stochastic; thus, in some weeks the factory had excess quantity of final products that were not sold. Finally, this above conclusion is depicted more clearly in Figure 5-19(d), as the percentage of sold products to produced products ranged between 40% and 80% over the entire planning horizon.

5.3.4.4 Raw Materials Management

As with final product management, to assess the performance of the developed model under this parameter in the fully stochastic scenario, four different indicators are used. These indicators are the quantity of raw materials in the store (Figure 5-20(a)), the quantity of raw materials purchased (Figure 5-20(b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 5-20(c)), and the percentage of raw materials that went into production (Figure 5-20(d)).

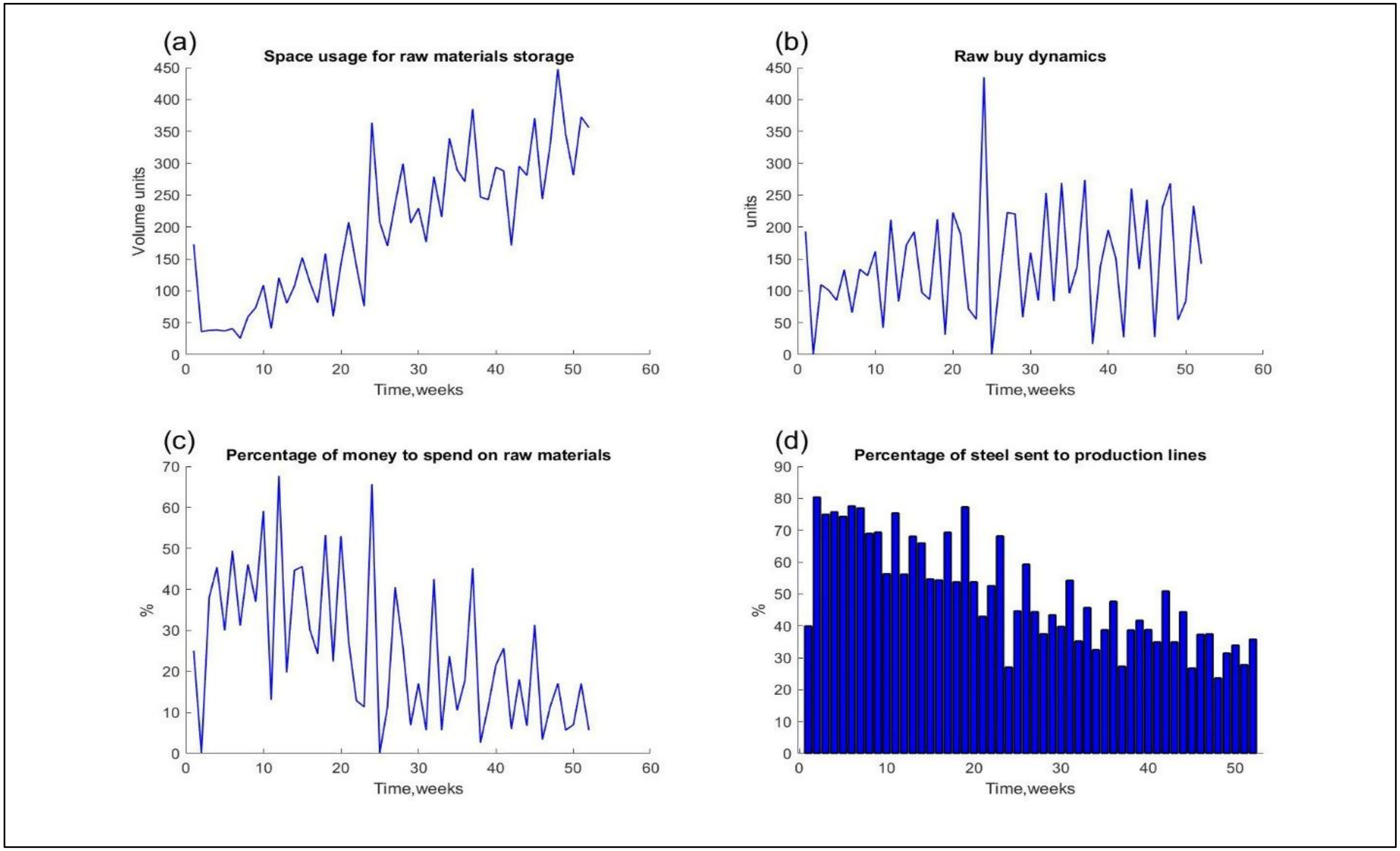

Figure 5-20: Raw materials management for the fully stochastic scenario: (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) the percentage of raw materials sent to the production lines.

From Figure 5-20(a), after the initial drop in the quantity of raw materials, their level in the store starts to increase gradually over the entire planning horizon to cover any additional production required, reaching as high as 350 units after 25 weeks. Moreover, Figure 5-20(b) shows the quantity of raw materials purchased over the entire planning horizon, which highly oscillates around 150 items per week, due to the same reasons as those explained in the fixed demand scenario. Similarly, Figure 5-20(c) shows the percentage of money spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased. Finally, Figure 5-20(d) shows the percentage of raw materials moved from the store to the production lines, which started at only 20% and reached 90% in the fourth week, before stabilising around the 50% mark over the rest of the planning horizon.

5.3.5 Scenarios Comparison

After applying the neural network closed-loop model in the three scenarios explained in the previous sections, the performances of this model under these three scenarios are compared against each other. The first observation from the analysis of the three performances is that they all showed similar trends regarding the following measures:

1. The selling price tends to decrease over time as the quantity of the final products which are in store increased.
2. The quantity of final products that the company produces increases over time.
3. The trends of purchasing raw material and producing final products follow a spike-shaped curve. This is normal for the business, as there is a collection period for money earned from sold products before being able to order a bulk amount of raw materials and immediately send them to the production lines.
4. Raw materials spend more time in storage than final products, as it is more efficient for the company to store extra raw materials and have the opportunity to produce extra quantities of final products.

Despite having a similar behaviour for some measures, there are several differences between the performances of the model under the three scenarios. One of these differences is the maturity of raw materials and final products. In Figure 5-21 the maturity of raw materials for all scenarios under the closed-loop neural network model is observed. As it can be observed, the optimal storage process for raw materials corresponds to the fully stochastic scenario, while the worst case is the fixed supply scenario, as it resulted in the highest maturity rates. This is also true in the case of the maturity of final products.

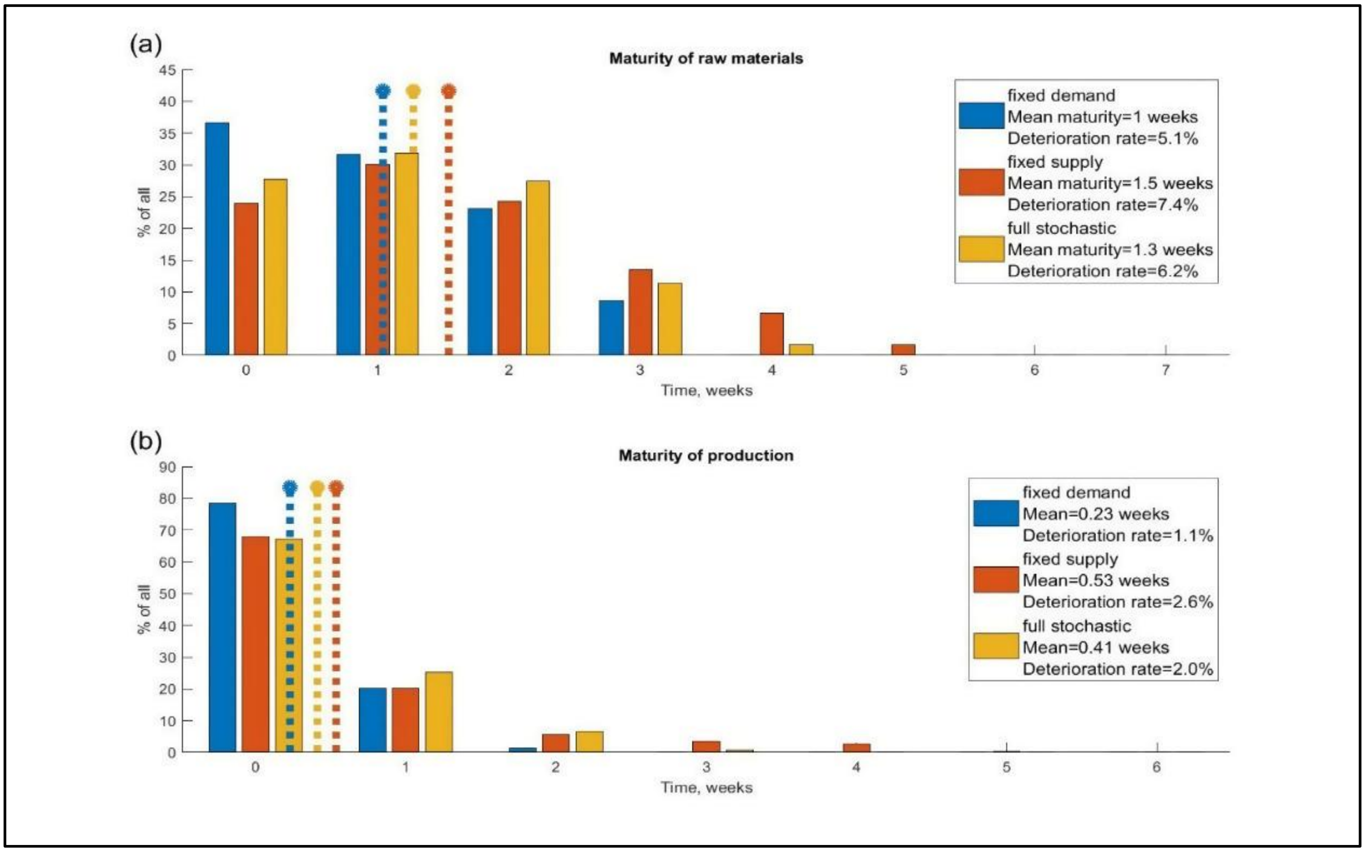

Figure 5-21. Raw materials deterioration rate comparison for all three scenarios (closed-loop neural network control system): (a) raw materials maturity, and (b) production maturity.

Next, a comparison between the performance of the neural network system and the open loop system, in terms of deterioration rate, is performed. As seen from Figure 5-22, under all three scenarios there are quantities of raw materials and final products consumed in as much as five or more weeks under the open loop system, which is an inefficient way to manage this type of inventory due to its deteriorating nature. Furthermore, it is evident that the deterioration results differ a lot between the different scenarios. Therefore, it can be concluded that the open loop system is not sufficiently robust, and that the neural network control system showed better performance than the open-loop system. This is highlighted by the fact that the best storage indicators for the open loop control system are reached in the fixed demand scenario, which is the simplest of the three scenarios.

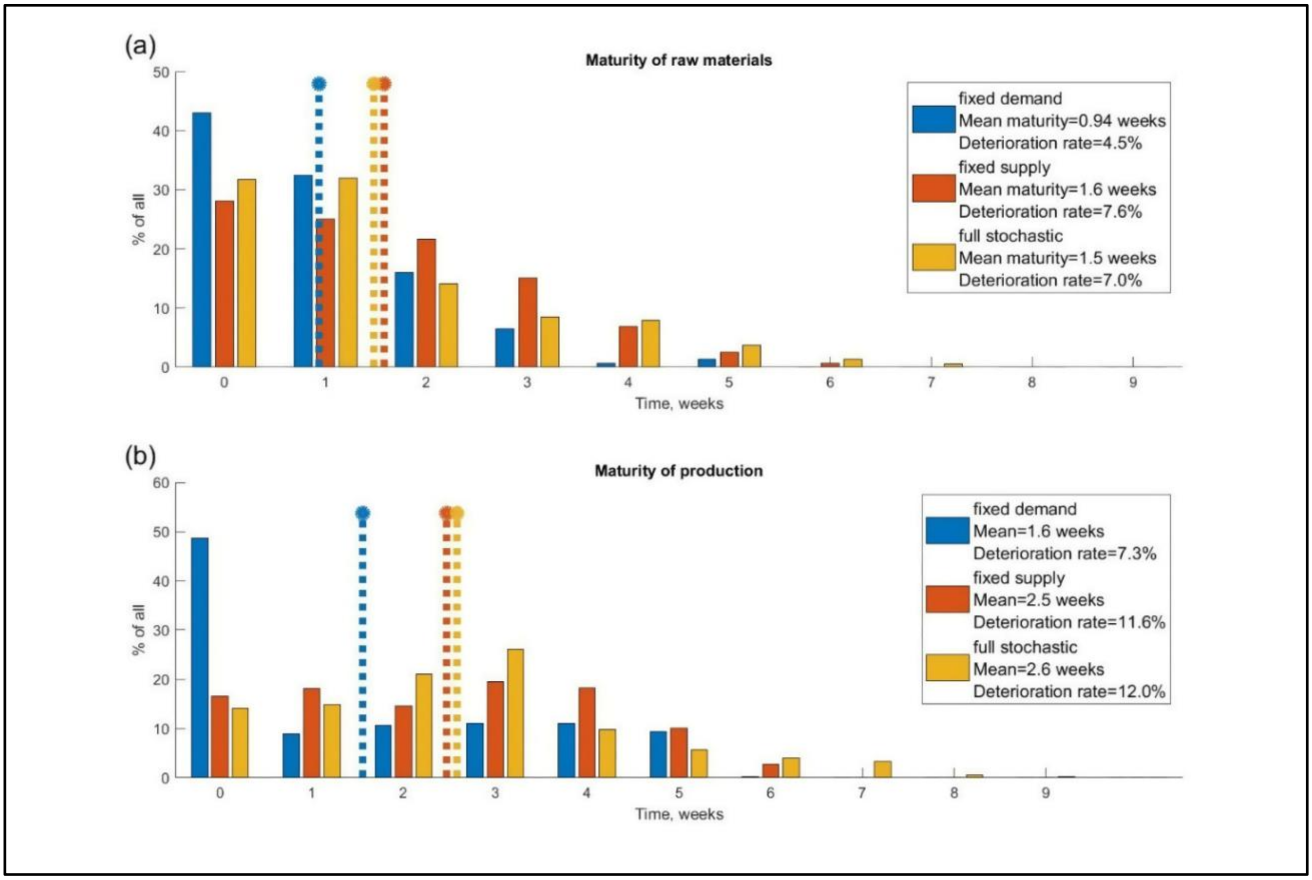

Figure 5-22. Raw materials maturity comparison for all three scenarios (open loop static control system): (a) raw materials maturity, and (b) production maturity.

To further reinforce the above conclusion, Table 5-4 shows a comparison between the final maturity performance for all three scenarios, and their improvement over the entire planning horizon, which is calculated using Equation 5-13:

$$Improvement = \frac{Open\ loop\ indicator - Close\ loop\ indicator}{Open\ loop\ indicator} \quad (5\text{-}13)$$

Table 5-4. Average maturity comprehensive table.

	Fixed demand scenario	**Fixed supply scenario**	**Fully stochastic scenario**
Raw materials (open loop)	0.94	1.58	1.48
Raw materials (neural network)	1.04	1.54	1.27
Improvement (%)	-10.93%	2.75%	14.05%
Production (open loop)	1.56	2.47	2.58
Production (neural network)	0.23	0.53	0.41
Improvement (%)	85.39%	78.32%	84.02%

From the above table, it is evident that the closed-loop system's performance is better in all scenarios, with only one exception, where the closed-loop system showed slightly worse results than the open loop system. This exception concerns raw material storage under the fixed demand scenario. However, as this is the simplest scenario, in which the factory needs to order each week almost the same amounts of final products, and since the open loop control system's strategy is skewed towards production over ordering, it leads to good results for raw material storage, but much worse results for final product storage. Therefore, this one parameter will not optimise storage for the factory, which includes the storage of both raw materials and final products. On the other hand, regarding the final product's storage policy, the closed-loop system fully dominates the open loop system, with approximately 80% average improvement.

Finally, when comparing the performance of the two systems in terms of the deterioration rates, Table 5-5 shows that the improvement trends for the two systems are similar to the maturity trends. As seen from the table, the open loop system allows up to 12% of the final products to be lost to deterioration, which is completely inefficient for the business. On the other hand, under the neural network control system, only 2% of the final products were lost to deterioration, which is a sizable 10% improvement over the open-loop system.

Table 5-5. Deterioration rate comprehensive table.

	Fixed demand scenario	Fixed supply scenario	Fully stochastic scenario
Raw materials (open loop)	4.55	7.57	7.03
Raw materials (neural network)	5.07	7.41	6.19
Improvement (%)	-11.44%	2.14%	11.96%
Final products (open loop)	7.28	11.55	12.03
Final products (neural network)	1.13	2.59	2.04
Improvement (%)	84.44%	77.56%	83.04%

As demonstrated by the two tables above, the neural network control system yielded better results for both the maturity and deterioration rates of the raw materials and inventory. Hence, the use of this system will help the factory's managers to better manage their operations, allow them to better utilise their resources, reduce the storage and deterioration costs of the inventory, increase the factory's profit by reducing these costs, and improve the sustainability of the factory. Furthermore, the application of the neural network system will allow managers to sell their products at a more competitive price (as their costs are reduced) to overcome competition and gain more market share. Finally, the scheduling of production is another informed decision that the factory's managers will be able to make when they apply the neural network model. Through knowing the average maturity and deterioration rates of the raw materials, these managers can schedule production in a way to utilise the oldest items first, and prevent the total loss of these items.

5.4 Chapter Summary

In this chapter, the model implementation has been described. In particular, the performances of the open and closed-loop control systems have been compared in order to select the best suited approach, in terms of robustness and accuracy. To achieve this goal, the performance of each model, under six main parameters, *viz.*, maturity and distribution rates, generated profits per storage unit used, profit generated by each Monte Carlo run, investment strategy, money management, and learning progress, were analysed. From the performed comparisons, it was shown that the neural network closed-loop control system slightly outperforms the open loop control system in raw materials storage management, and drastically outperforms the open loop system in final product storage management. Moreover, a similar trend was deduced for the deterioration rates of raw materials and final products. Hence, through this comparison, it was concluded that the neural network closed-loop control system is the most beneficial system to maximise profits in the case of a steel manufacturing factory.

Consequently, to fully examine the performance of the neural network closed-loop system, this system was implemented under all three different business scenarios, *viz.*, the fixed demand scenario, the fixed supply scenario, and, the most complex, the fully stochastic scenario. The corresponding performances were compared, using the maturity and deterioration rates, money management, final product management, and raw materials management, to deduce the business patterns associated with each scenario. From this comparison, it was observed that the highest profit was achieved in the fixed demand scenario. This was an expected result since the fixed demand scenario is the simplest one. Nevertheless, promising results have also been obtained for the fully stochastic scenario. In this line, the developed model has shown to be a useful tool for factory's managers allowing them to better manage their operations and better utilising their resources by reducing the storage and deterioration costs of the inventory, increasing the factory's profit, and improving the sustainability of the factory. Furthermore, using the ANN based model will allow managers to sell their products at a more competitive price (as their costs are reduced) to overcome competition and gain more market share. Finally, knowing the average maturity and deterioration rates of the raw materials, managers will be able to schedule production in a more efficient way, utilising the oldest items first, and preventing the total loss of these items.

To further illustrate the robustness of the neural network closed-loop control system, in the following chapter, various macroeconomic conditions, under which the steel manufacturing factory might operate, are assumed, and the model is implemented under each one of them. In particular, the model's performance is analysed under five different conditions in order to assess its flexibility and adaptability.

6 Cases for Various Macroeconomic Situations

6.1 Introduction

After validating the developed model, a sensitivity analysis is conducted in this chapter applying the developed model to different real-life scenarios to which the steel-manufacturing industry might be subjected. In this way, the robustness of the model and its ability to handle these extreme scenarios is explored. In this type of analysis, several controlling parameters in the model's environment are changed in order to examine their impact on the model's performance; this assists in analysing how sensitive the developed model is to the changes in these parameters. Each of these scenarios reflects either irregular economic patterns or worst-case scenarios, caused by financial crises, political instability, or trade wars, which can take place either on the macro or micro levels.

Any developed model should have the ability to adjust to the described kind of scenarios and provide outcomes that enable the company to adapt to the new business situations and avoid bankruptcy. Therefore, the goal of this chapter is to assess how the closed-loop neural network control system adjusts to the different economic scenarios, and compare the profit generated in those situations with the one obtained for the fully stochastic scenario discussed in Section 5.3.4. In order to achieve this goal, the assumptions of the fully stochastic scenario outlined in Chapter 5 are used, and five cases reflecting the above mentioned scenarios are considered. The performance of the closed-loop neural network model, in all these cases, is assessed in terms of maturity and deterioration rates, money management, final product management, and raw materials management. The considered cases in this analysis are:

1) Increase in storage costs as a result of higher electricity and rent expenses.
2) Seasonal change in the purchasing price of raw materials. Although such a situation is characteristic of mainly fruit and vegetable-dependent industries, in steel manufacturing, the prices of raw materials can also change according to the season, as a result of several factors, such as demand and availability.
3) Seasonal change in demand due to economic cyclicality.
4) Sudden and complete loss of demand, which instantly falls to zero as a result of a severe financial crisis or trade wars.
5) Sudden interruption to the supply channels, as a result of supplier bankruptcy or trade embargos, which leads to cutting off supply or raw materials completely.

This chapter is organised into five main sections. The first ones, from Section 6.2 to Section 6.6, assess the model's performance under each of the above described five scenarios. Finally, Section 6.7 summarises the conclusions drawn from the obtained results, and provide future recommendations in order to further enhance the model's performance with respect to the steel manufacturing industry.

6.2 Increase in Storage Costs

In this case, the storage costs of raw materials and final products increase by four times, from £500 to £2000 per week, on average. In addition, since increasing the absolute values of the stochastic variables also leads to an increase in their respective variances, the standard deviations of the storage costs will also increase four-fold, from 0.05 to 0.2. In the context of the steel manufacturing factory, this case takes place when there is high humidity in the storage area, presenting the requirement of more energy to maintain suitable storage conditions for storing steel.

The effect of the increase in storage costs on the maturity and deterioration rates, money management, final product management, and raw materials management are illustrated in Sections 6.2.1-6.2.4.

6.2.1 Maturity and Deterioration Rates

When applying the neural network closed-loop system after increasing storage costs, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 6-1(a) and (b) for raw materials and final products, respectively.

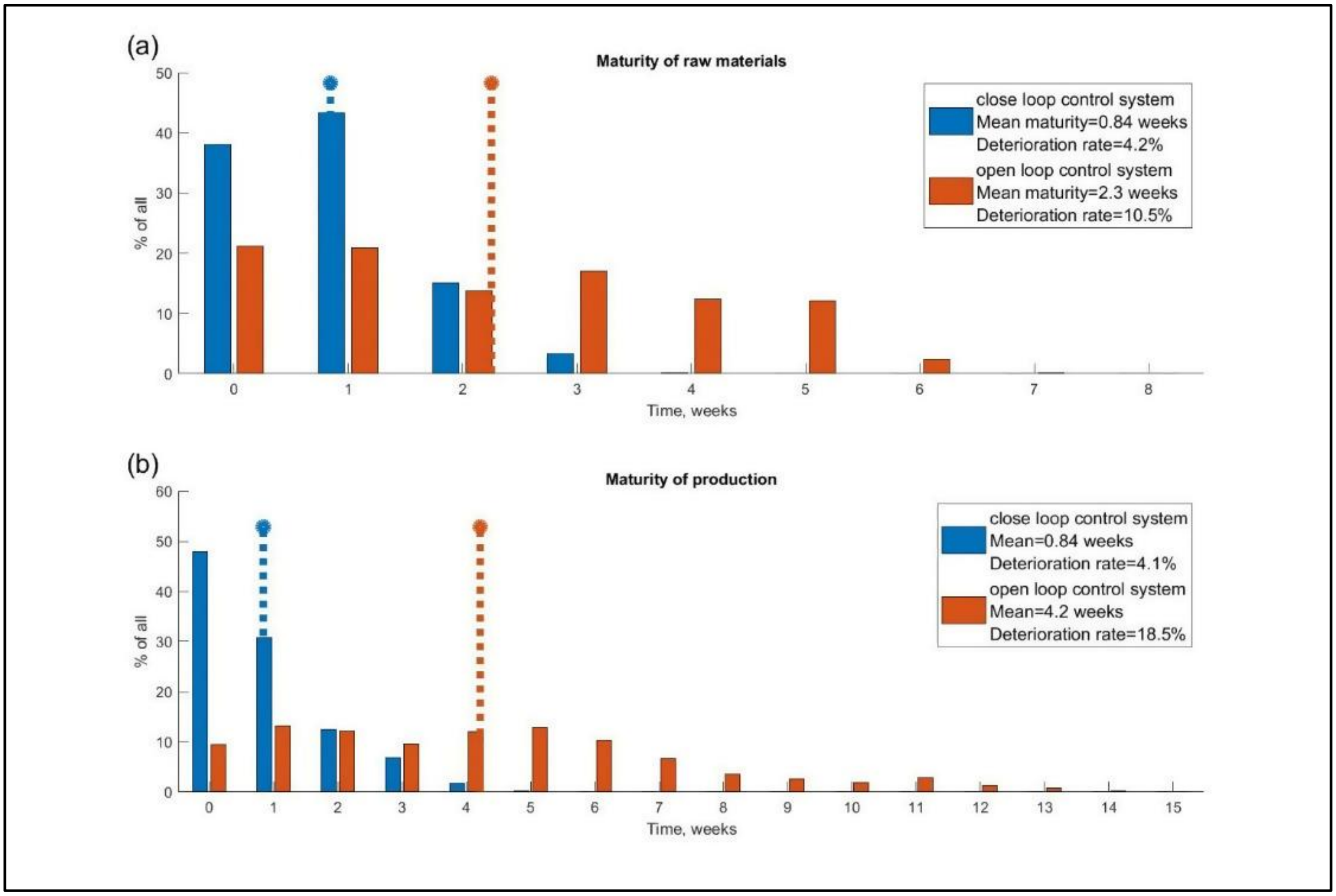

Figure 6-1. Maturity management for the increased storage cost case (fully stochastic scenario): (a) raw materials maturity, and (b) production maturity.

From the above figure, it can be observed that the maturity distributions for raw materials and final products are much better for the closed-loop control system, as they are both stored for fewer weeks before being used in production or sold, at an average of 0.84 weeks. Moreover, the maximum storage time is reduced from seven to three weeks in the case of raw materials, and from fourteen to five weeks in the case of final products. Furthermore, the factory's losses due to deterioration decreased from 10.5% to 4.2%, and from 18.5% to 4.1% for raw materials and final product storage, respectively.

In addition, when comparing the above performance with the fully stochastic scenario (see Figure 5-17), the maturity distribution for raw materials becomes much better, as, starting from week 2, the percentage of raw materials in storage is reduced significantly, and the maximum storage time decreases from four to three weeks. On the other hand, the fully stochastic scenario has a better performance in terms of the final product storage time, as this was only 0.41 weeks versus the 0.84 weeks achieved when the storage costs increased.

6.2.2 Money Management

Under this parameter, four indicators are used to assess the performance of the developed model in the case of increased storage costs. These indicators are the dynamics of available funds (Figure 6-2 (a)), the dynamics of the amount of money invested in the business (Figure 6-2(b)), the change in the selling price over the planning horizon (Figure 6-2(c)), and the amounts of up credit and down credit (Figure 6-2(d)).

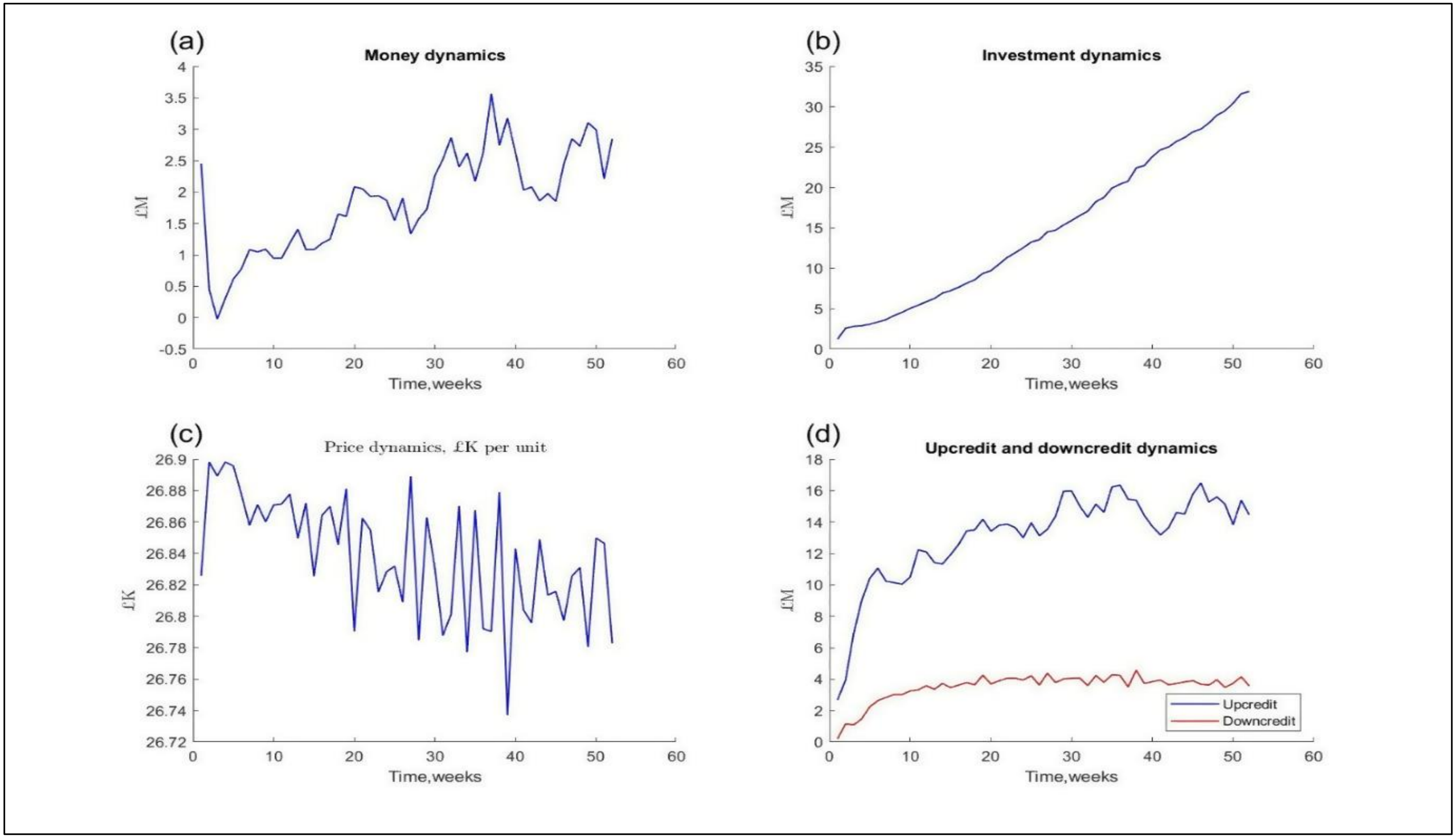

Figure 6-2. Money management for the increased storage cost case (fully stochastic scenario): (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics.

As seen from Figure 6-2(a), the plot starts with the amount of initial funds available before the start of the planning horizon, then it decreases dramatically to almost zero, before it begins to increase towards reaching the level of £3M. This dynamic is similar to the one corresponding to the fully stochastic scenario with ordinary storage analysed in Section 5.3.4, where the amount of funds reach the level of £1.5M instead of £3M. This indicates that when storage costs increase, the implementation of the model resulted in increasing the amount of funds available at the end of the planning period. On the contrary, the amount of funds invested (Figure 6-2(b)) decreases, when compared to the fully stochastic scenario, to reach only £32M at the end of the planning horizon versus £50M. Figure 6-2(c) shows the dynamics of the final product selling price over the entire planning horizon. As seen from that figure, the selling price tends to decrease over time, albeit marginally, from an initial peak at the first week of £26.9K to £26.8K at week 52. This means that the selling price of the final product initially increased, as there were small quantities of final products available, yet with the increase in this quantity over time, the price started to decrease. However, this decrease is not continuous over the entire planning horizon, as there are periods of fluctuation in the selling price based on the quantity of final products available in storage. Finally, in Figure 6-2(d), both the up credit and down credit dynamics are observed. Regarding up credit, the amount increases sharply during the first five weeks, as the company starts selling its final products, then the increase slows down until week 30, as the quantity of final products sold does not increase significantly from one week to another, when it stabilises around the value of £14M. As for down credit, its curve depicts the shape of the raw materials price, since it reflects the money owed by the company to the suppliers, as it first increases and then stabilises at the level of £4M. Again, the observed trends in Figure 6-2(d) are similar to those observed in the fully stochastic scenario.

6.2.3 Final Product Management

To assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of final products in storage (Figure 6-3 (a)), the quantity of final products produced (Figure 6-3(b)), the quantity of final products sold (Figure 6-3(c)), and the percentage of produced goods that were sold (Figure 6-3(d)).

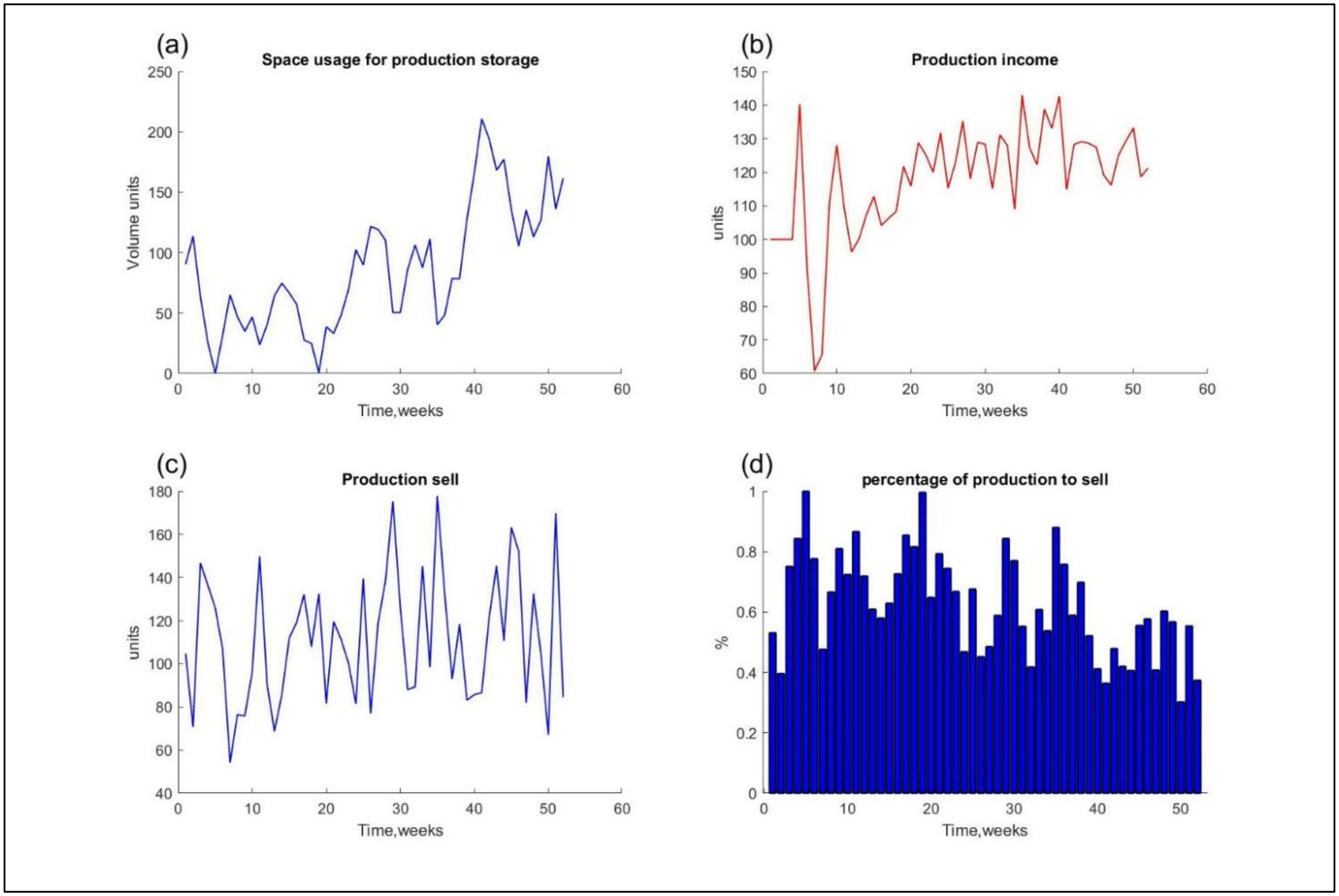

Figure 6-3: Production management for the increased storage cost case (fully stochastic scenario): (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold.

From Figure 6-3(a), the quantity of final products in storage varies drastically over the entire planning horizon, going as low as zero units in the 5th week and as high as 200 units in the 40th week. This high fluctuation is due to the fact that demand is stochastic, and the accuracy of its forecast is not high, generating the need to store extra products in case of demand increase. This trend is almost the same as the one observed under the fully stochastic scenario in Figure 5-19(a). Moreover, Figure 6-3(b) depicts the trend of the quantity of final products produced over the planning horizon. As seen from this plot, compared to the fully stochastic scenario, Figure 5-19(b), the production quantity is lower and never goes higher than the maximum production power of 150 units per week. This is due to the fact that, being stochastic, the demand never reaches the maximum, being no need to produce more products. This is confirmed by Figure 6-3(c), where it can be seen that the quantity of final products that are sold oscillates around 120 units per week, being not necessary to produce more than this quantity. Finally, from Figure 6-3(d), it can be seen that in most weeks along the planning horizon, the factory sold more than 40% of the final products in storage. Even more, it reached a 100% in two different weeks. This means that it was able to sell most of its final products, i.e. it had high production efficiency. Furthermore, when compared to the fully stochastic scenario, the overall trend is similar, albeit in this case the percentage of final products sold has a more uniform distribution along the planning horizon, showing a higher production efficiency.

6.2.4 Raw Materials Management

As with final product management, to assess the performance of the developed model under this parameter in the case of an increase in storage costs, four different indicators are used. These indicators are the quantity of raw materials in storage (Figure 6-4 (a)), the quantity of raw materials purchased (Figure 6-4(b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 6-4(c)), and the percentage of raw materials that went into production (Figure 6-4(d)).

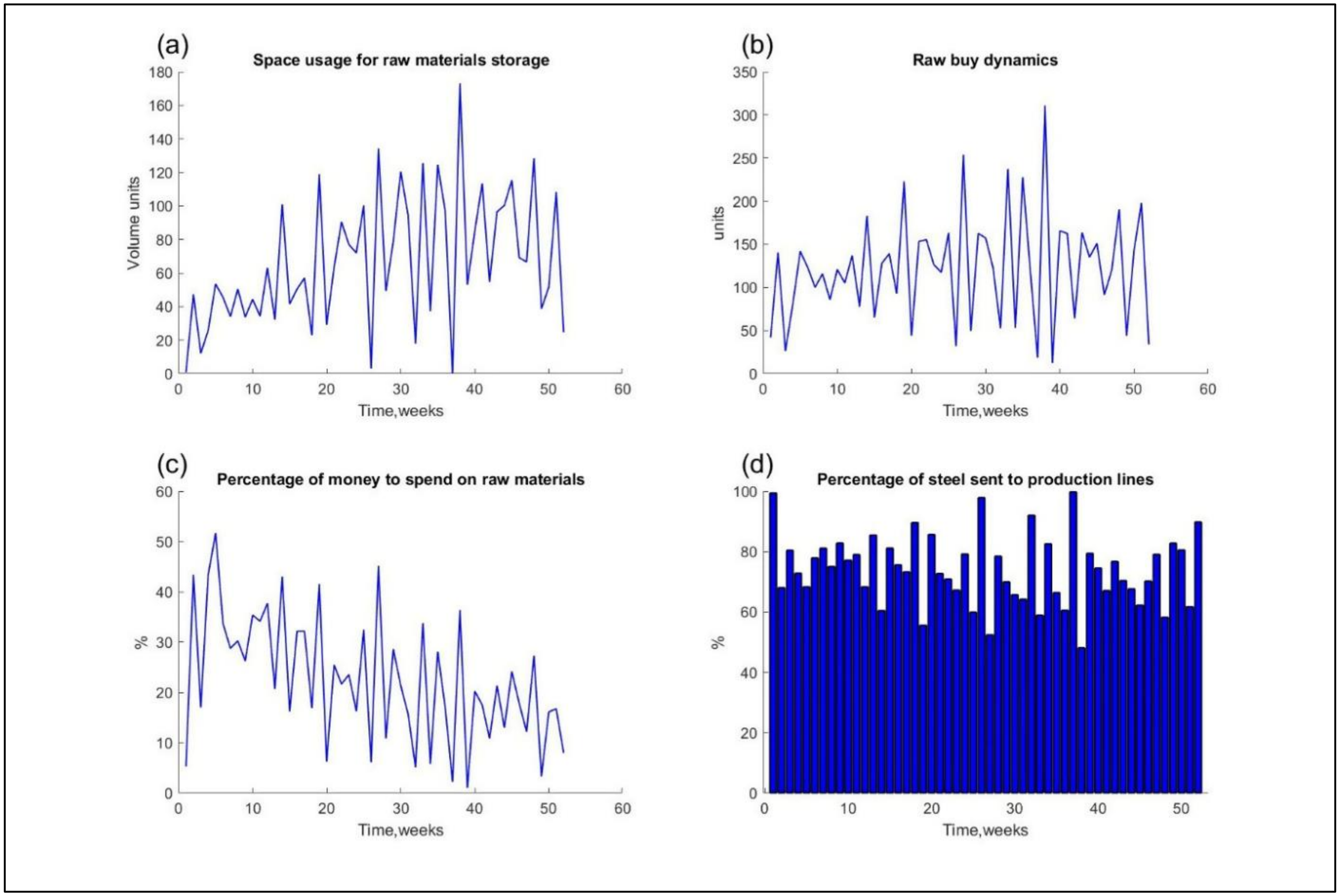

Figure 6-4: Raw materials management for the increased storage cost case (fully stochastic scenario): (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) percentage of raw materials sent to the production lines.

From Figure 6-4(a), after the initial accumulation of raw materials, their level continue to oscillate around the 80-item level (mean value of the space usage), which is much lower than the observed level in the fully stochastic scenario. This shows that the increase in storage cost impacts the quantity of raw materials purchased, as with greater quantities, the cost becomes much higher. This, in turn, is reflected in the quantity of final products produced and sold. Moreover, Figure 6-4(b) shows the quantity of raw materials purchased over the entire planning horizon, which highly oscillates around 100 items per week, and, in some weeks, the factory even buys very small amounts of raw materials because of the high volume in storage, unlike the fully stochastic scenario with ordinary where, at some points, the factory did not buy any raw materials at all. Figure 6-4(c) shows the percentage of money that was spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased, and is less than the amount spent under the fully stochastic scenario, as more funds are allocated to investment and production. Finally, Figure 6-4(d) shows the percentage of raw materials that moves from storage to the production lines, which, unlike final products sold, never goes below 50%, which is much more efficient than the case of the fully stochastic scenario, as when the storage costs increased, the model had to be more efficient in terms of purchasing and storing raw materials in order to reduce the storage cost and maximise profits.

6.3 Seasonal Change in the Purchasing Price of Raw Materials

In this case, the purchasing price of raw materials is assumed to change weekly, as demand increases during certain periods of the year, which is reflected in the price of these raw materials. Hence, the simplest periodical function is used to model this change, with the standard deviation of the purchasing price changing with the changes in the purchasing price. Therefore, the price mean value is derived from the following equation:

$$\begin{cases} \overline{\widetilde{C_{raw}}(t)} = 4 + 3 \cdot \sin\left(\dfrac{(t-1)\cdot\pi}{26}\right) \\ \sigma\left(\widetilde{C_{raw}}(t)\right) = \dfrac{\overline{\widetilde{C_{raw}}(t)}}{8} \end{cases} \qquad \text{6-1}$$

This system of equations is artificially generated to model the simplest seasonal change in the raw materials price, which is a sin wave; the second equation makes the standard deviation of raw materials proportional to its mean value. From these equations, as a result of the seasonality effect, it is seen that the raw materials price can go as low as £1000 per unit and as high as £7000 per unit, in one year, as shown in Figure 6-5.

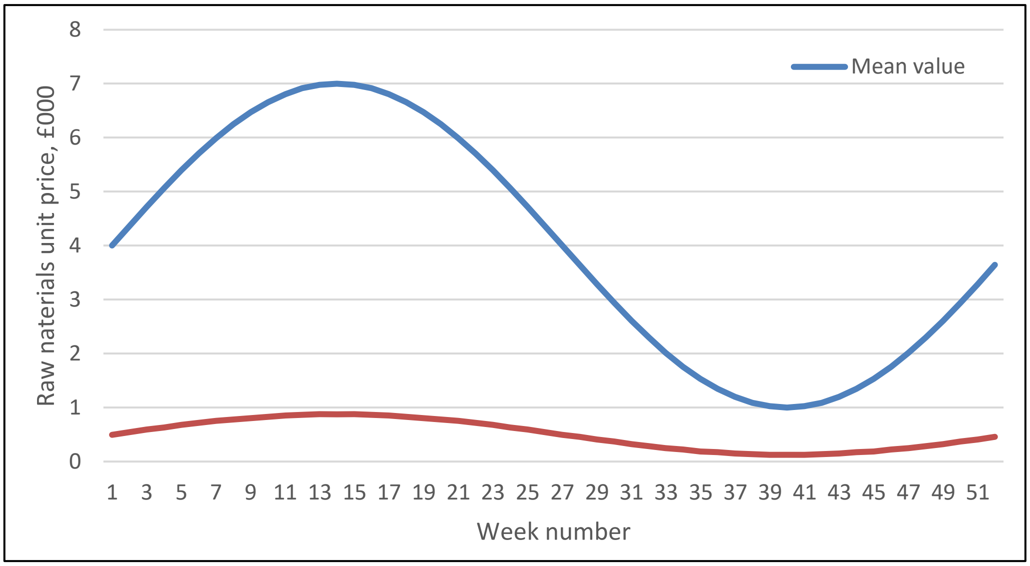

Figure 6-5. Raw materials price means and standard deviation value yearly change pattern.

Again, the effect of introducing seasonality change for the purchasing price of raw materials on the same four parameters the maturity and deterioration rates, money management, final product management, and raw materials management is illustrated in Sections 6.3.1-6.3.4.

6.3.1 Maturity and Deterioration Rates

When applying the neural network closed-loop system after accounting for the seasonality effect on the raw material purchasing price, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 6-6 (a) and (b) for raw materials and final products, respectively.

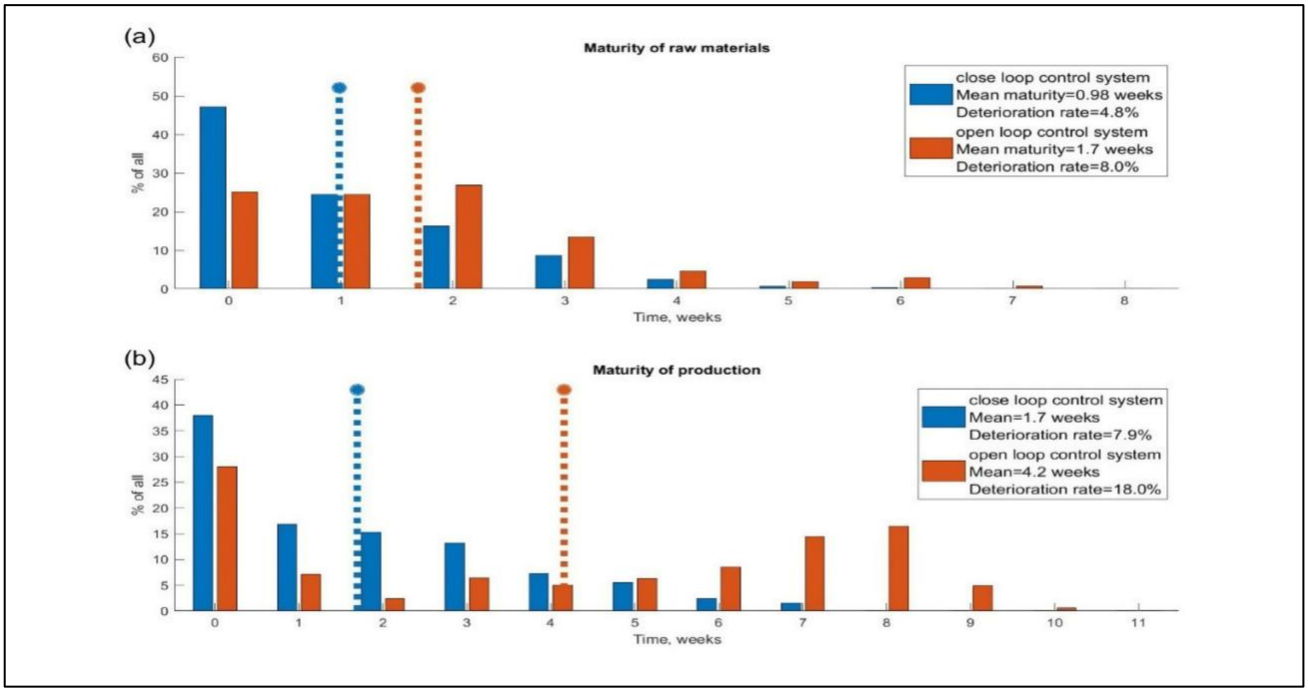

Figure 6-6. Maturity management for the seasonal raw materials price case (fully stochastic scenario): (a) raw materials maturity, and (b) production maturity.

According to Figure 6-6, in the current case, the maturity analysis shows the superiority of the closed-loop control system over the open loop system, as both raw materials and final products are stored for fewer weeks before being used in production or sold. The average maturity for raw materials under the closed-loop system is 1.7 times lower, while it is twice as low for final products. The reason behind this difference is that it is difficult to develop a successful business strategy, regardless of the actual parameters of the stochastic variables, while this parameter can vary greatly. Moreover, each change in the stochastic variable at the beginning of the planning period affects the future levels of raw materials and final products in storage; therefore, it is crucial not to only take into account the average values of the stochastic variables, but also their actual values.

6.3.2 Money Management

Under this parameter, four indicators are used to assess the performance of the developed model in the case of seasonal raw material purchasing price. These indicators are the dynamics of available funds (Figure 6-7 (a)), the dynamics of the amount of money invested in the business (Figure 6-7(b)), the change in the selling price over the planning horizon (Figure 6-7(c)), and the amounts of up credit and down credit (Figure 6-7(d)).

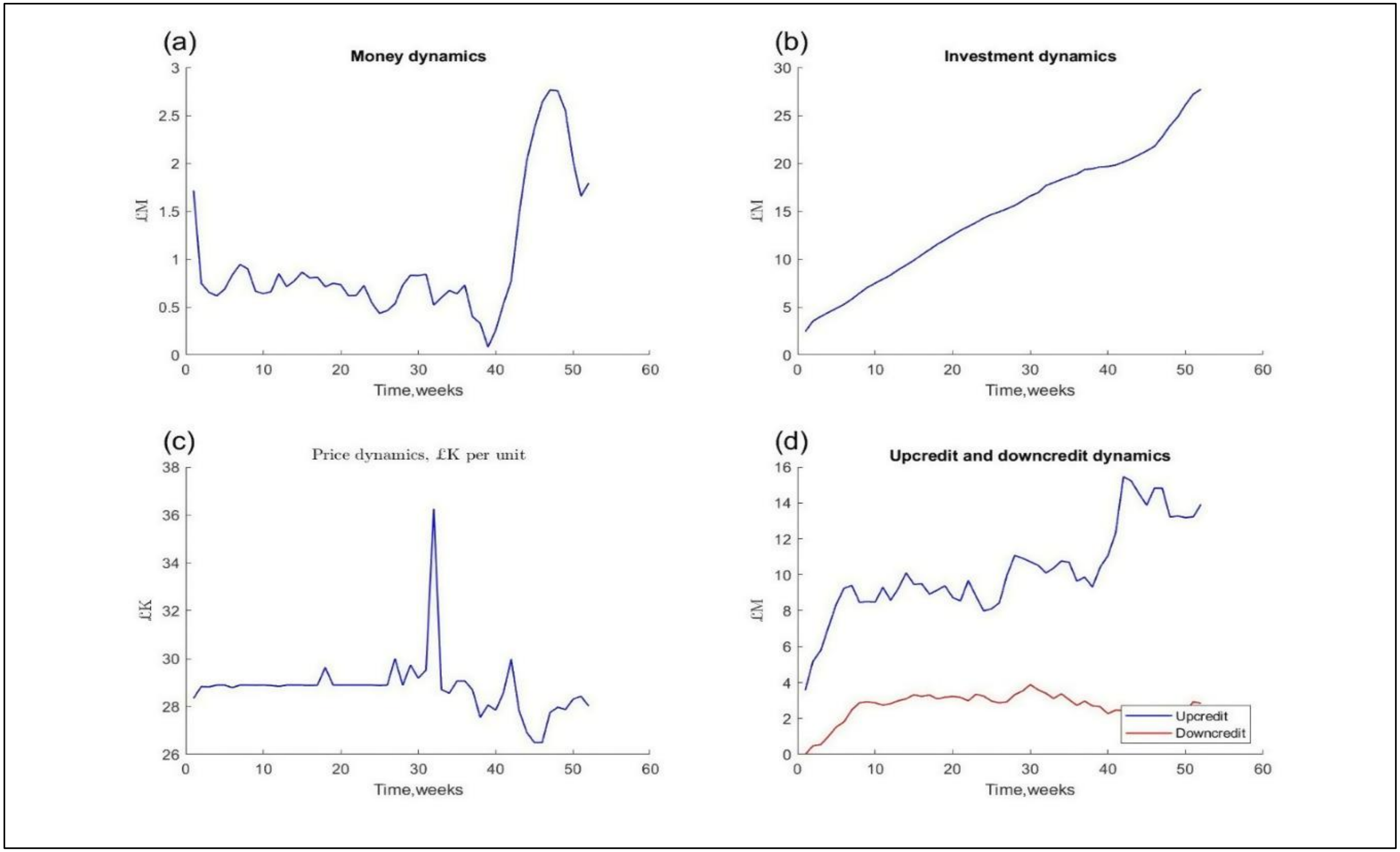

Figure 6-7. Money management for the seasonal raw materials price case (fully stochastic scenario): (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics.

As seen from Figure 6-7(a), the plot starts with the amount of initial funds available before the start of the planning horizon, then it decreases dramatically to almost zero until the 40th week. In the last weeks it experiments a peak, reaching £3M (similar to what it has been shown for the case of increase in storage costs analysed in Section 6.2), to then finalise near £1.5M (similar to the case of the fully stochastic scenario). This indicates that when the purchasing price of the raw materials changes, the model increases the amount of funds available at the end of the planning period to cover any increase in price. On the other hand, although the amount of funds invested (Figure 6-7(b)) steadily increases over the planning horizon, it is still lower than the corresponding investment to the fully stochastic scenario. Figure 6-7(c) shows the dynamics of the final product selling price over the entire planning horizon. As seen from that figure, over most of the planning horizon, the selling price tends to remain relatively constant around the £29K, then it reaches a sudden peak of £36K at week 33, as the price of raw materials reaches its lowest point; consequently, the quantity of final products increases, after which it starts to oscillate. Finally, in Figure 6-7(d), both the up credit and down credit dynamics are observed. Regarding up credit, its amount increased sharply for the first eight weeks to reach £9M as the company starts selling its final products, then it stabilises around the value of £14M, as the quantity of final products that is sold does not increase significantly from one week to another. As for down credit, its curve depicts the shape of the raw materials price, since it reflects the money owed by the company to the suppliers, as it first increases and then starts to decrease in the middle of the planning horizon.

6.3.3 Final Product Management

To assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of final products in storage (Figure 6-8 (a)), the quantity of final products produced (Figure 6-8(b)), the quantity of final products sold (Figure 6-8(c)), and the percentage of produced goods that were sold (Figure 6-8(d)).

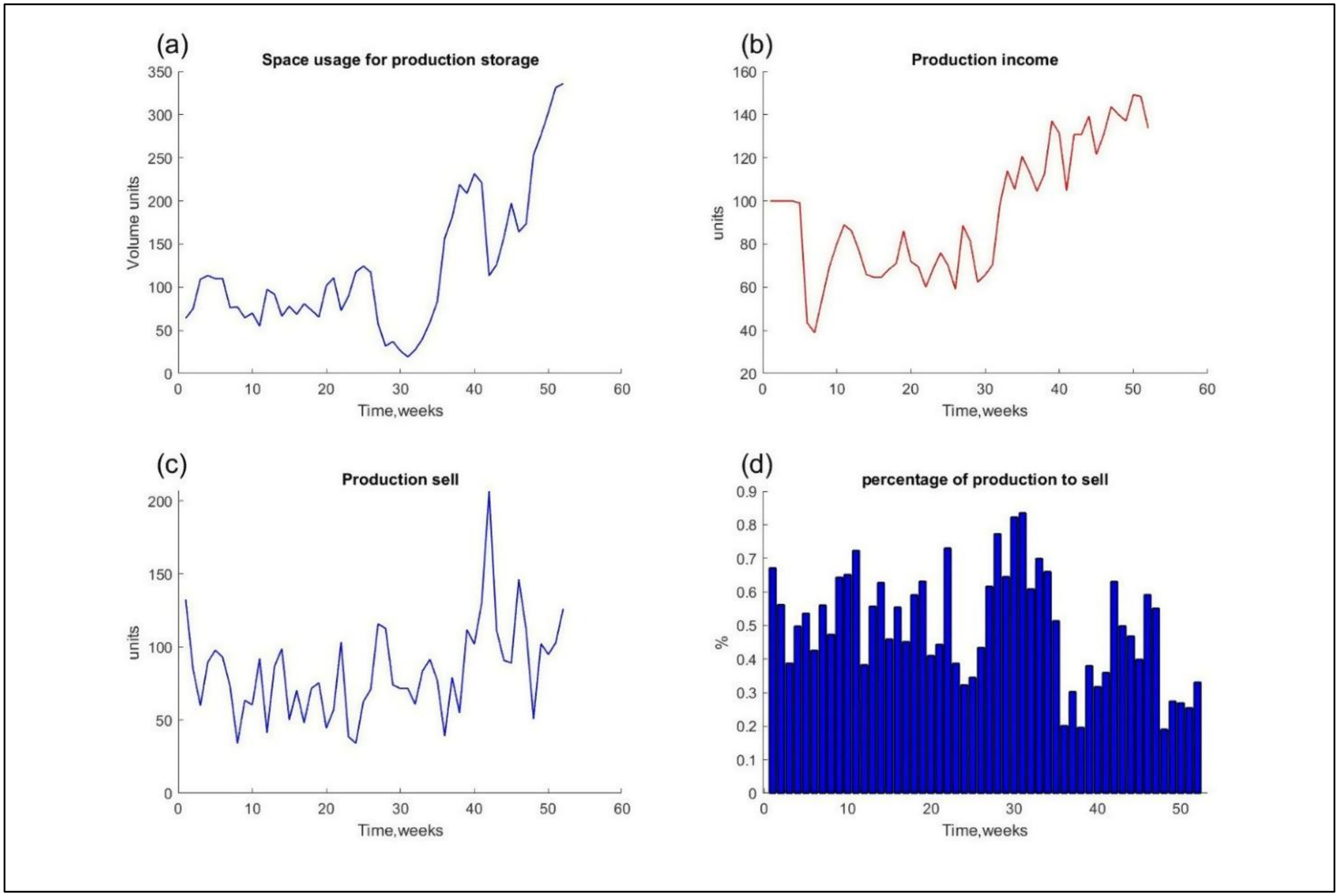

Figure 6-8: Production management for seasonal raw materials price case (fully stochastic scenario): (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold

From Figure 6-8(a), it can be observed that the quantity of final products in storage is majorly boosted in week 39, when the purchasing price of raw materials is at its lowest level. This means that the factory takes advantage of the low price for raw materials and purchases greater quantities; hence, it needs to move the raw materials quickly from storage to free space. Moreover, Figure 6-8(b) depicts the trend of the quantity of final products produced over the planning horizon. As seen from this plot, compared to the fully stochastic scenario, Figure 5-19(b), the production quantity is lower and it only reaches the maximum production power, 150 units per week, at the end of the planning horizon, as the demand never reaches this point, hence there is no need to produce more products. Figure 6-8(c) shows the quantity of final products that were sold. The plot is oscillating around 80 units per week, in spite of the fact that the production reaches 100 units around week 32, and slightly rises over time until the end of the year. This means that the factory was not able to sell all of its produced products in a one-week span. This conclusion is more evident in Figure 6-8(d), as the quantity of final products in storage that were sold is much lower than in the fully stochastic scenario in some weeks it was as low as 20% and never exceeded 80%.

6.3.4 Raw Materials Management

As with final product management, to assess the performance of the developed model under this parameter, in the case of seasonality in the purchasing price of raw materials, four different indicators are used. These indicators are the quantity of raw materials in storage (Figure 6-9(a)), the quantity of raw materials purchased (Figure 6-9(b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 6-9(c)), and the percentage of raw materials that went into production (Figure 6-9(d)).

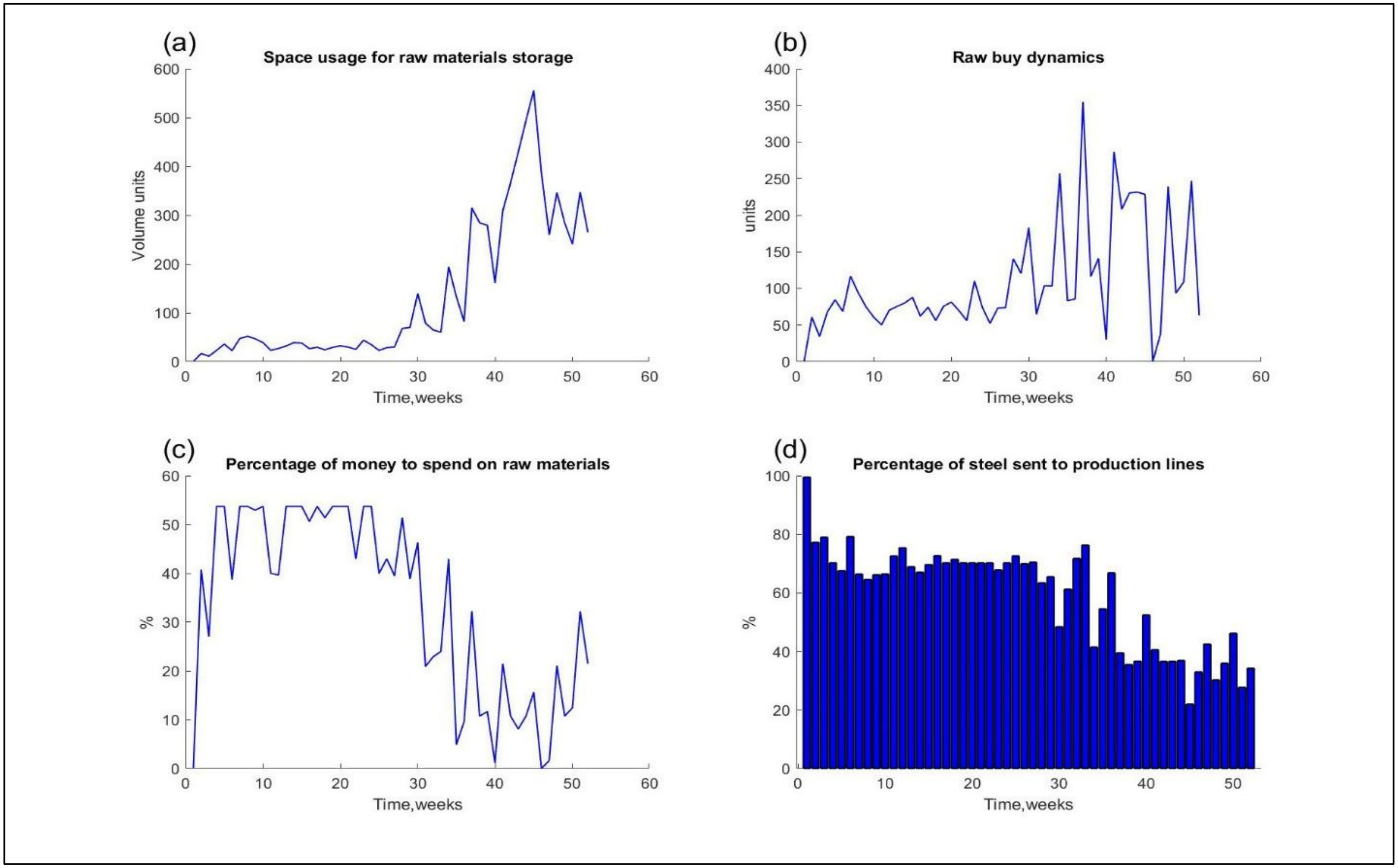

Figure 6-9. Raw materials management for the seasonal raw materials price case: (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) the percentage of raw materials sent to the production lines.

From Figure 6-9(a), as the purchasing price of raw materials becomes lower, the factory purchases a greater quantity, and after accumulating a large quantity of raw materials, it sends a greater quantity to production. Moreover, Figure 6-9(b) shows the quantity of raw materials purchased over the entire planning horizon, which has several peaks during the weeks with the lowest raw material prices. Similarly, Figure 6-9(c) shows the percentage of money that was spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased, as with a reduction in the raw materials' prices, greater quantities are purchased and more money spent. Finally, Figure 6-9(d) shows the percentage of raw materials that moves from storage to the production lines, which is mostly above 60%, reflecting the high efficiency of the model.

6.4 Seasonal Change in Demand

In this case, the demand for final products is assumed to change weekly. Hence, as with the previous case, the simplest periodical function is used to model this seasonal change in demand and its standard deviation according to the following equations:

$$\begin{cases} \overline{D_{prod}(t)} = 70 + 40 \cdot \sin\left(\frac{(t-1)\cdot\pi}{26}\right) \\ \sigma(D_{prod}(t)) = \frac{\overline{D_{prod}(t)}}{7} \end{cases} \tag{6-2}$$

This system of equations is artificially generated to model the simplest seasonal change in demand, which is a sin wave. The second equation makes standard deviation proportional to the mean value, as shown in Figure 6-10.

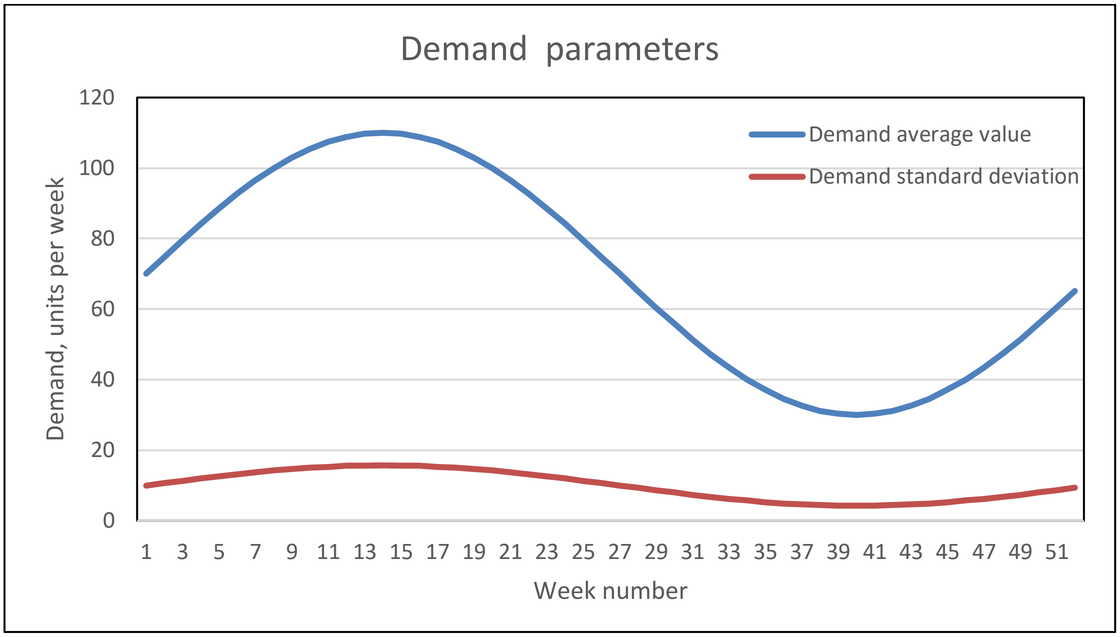

Figure 6-10. Demand mean value and standard deviations yearly change patterns.

6.4.1 Maturity and Deterioration Rates

When applying the neural network closed-loop system after taking into account the seasonality effect of the demand, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 6-11(a) and Figure 6-11 (b) for raw materials and final products, respectively.

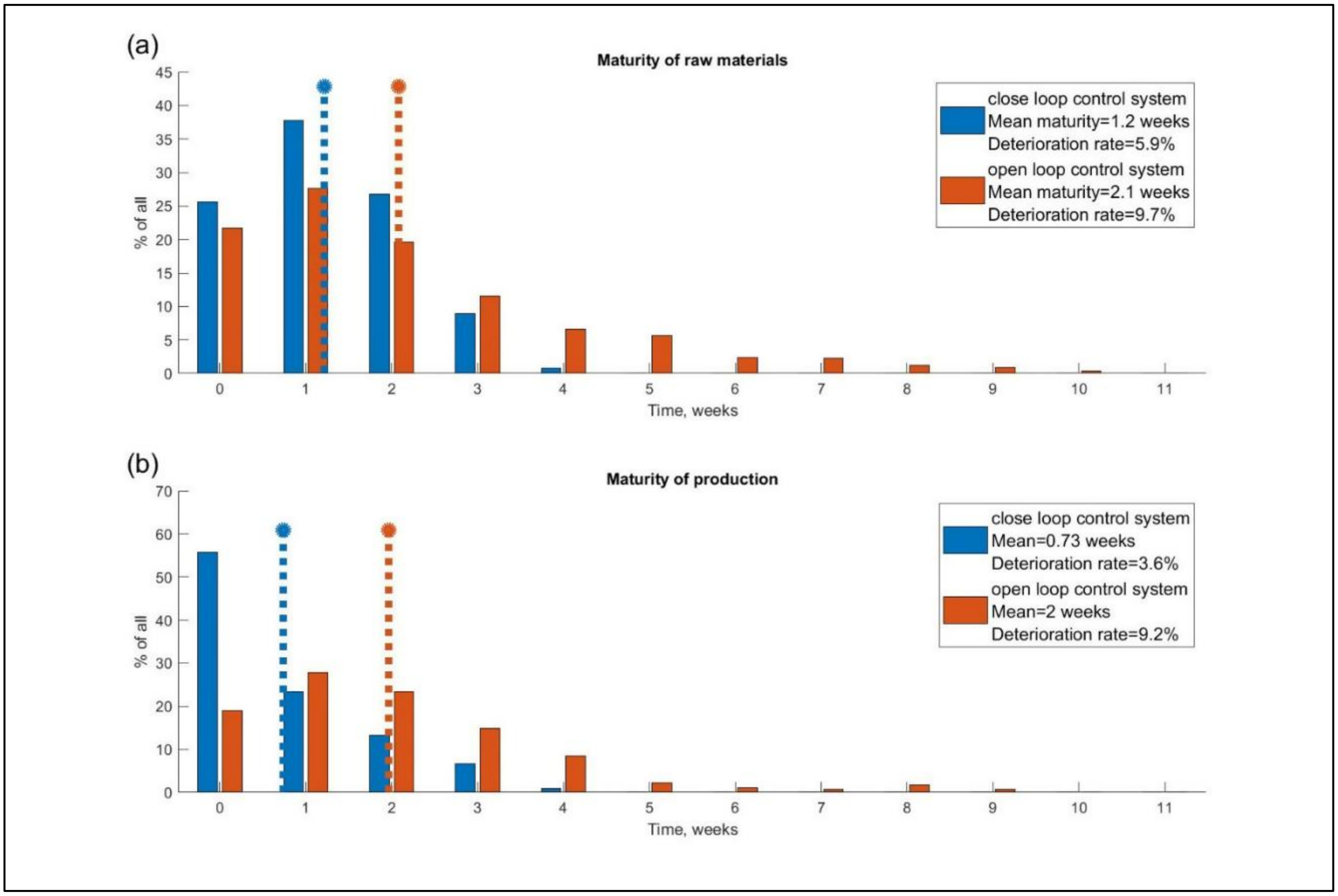

Figure 6-11. Maturity analysis for the seasonal demand case (fully stochastic scenario): (a) raw materials maturity, and (b) production maturity.

As seen from Figure 6-11, the storage of raw materials and final products is much more efficient under the neural network closed-loop control system, as the average storage time is almost twice as low as that of the open loop system for both raw materials and final products. Moreover, when compared to the fully stochastic scenario, the performance is much better in the case of raw materials, and slightly worse in the case of final products.

6.4.2 Money Management

Under this parameter, four indicators are used to assess the performance of the developed model in the case of seasonal demand. These indicators are the dynamics of available funds (Figure 6-12 (a)), the dynamics of the amount of money invested in the business (Figure

6-12(b)), the change in the selling price over the planning horizon (Figure 6-12(c)), and the amounts of up credit and down credit (Figure 6-12(d)).

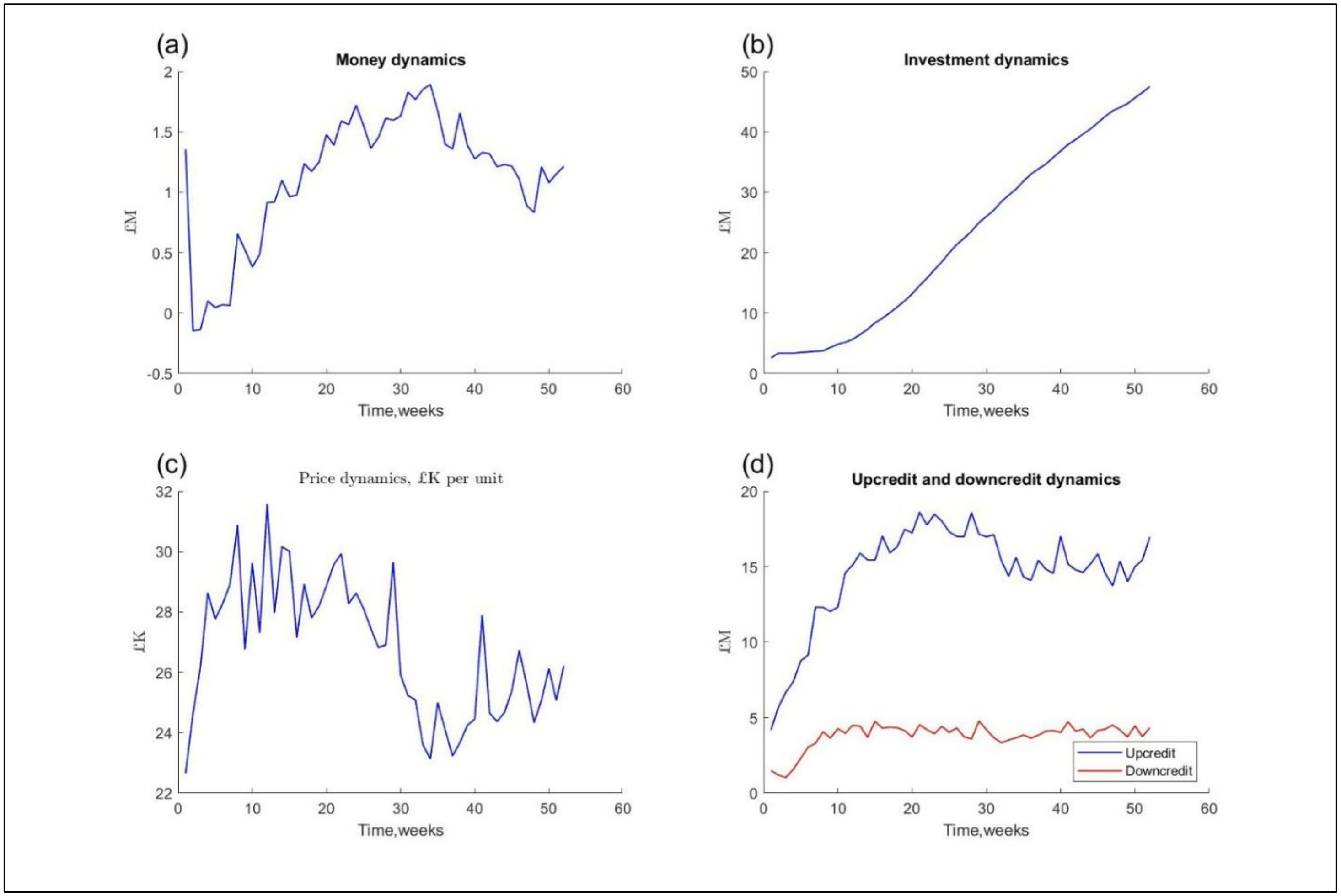

Figure 6-12. Money management for the seasonal raw materials price case (fully stochastic scenario): (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) upcredit and downcredit dynamics.

As seen from Figure 6-12(a), the plot starts with the amount of initial funds available before the start of the planning horizon, then decreases dramatically to almost zero, similar to the fully stochastic scenario, and begins to increase to reach the level of £2M in week 35, before it goes down again to £1M at the end of the planning horizon. This shape of the available funds plot follows the shape of demand, since the factory needs more cash when there is more demand. At the same time, the plot for the amount of funds invested (Figure 6-12(b)) shows almost the same trend as the fully stochastic scenario, as it steadily increases along the planning horizon until it reaches £50M at the end. Figure 6-12(c) shows the dynamics of the final products' selling price over the entire planning horizon. This plot also follows the same trend as the demand plot, since with higher demand, the company can afford to increase the selling price. Finally, in Figure 6-12(d), both the up credit and down credit dynamics are observed. Regarding up credit, its amount first increases as the quantity of final products sold increases, then it suffers from a similar seasonality pattern to that of demand, as the higher the sales, the greater the cash inflow.

6.4.3 Final Product Management

To assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of final products in storage (Figure 6-13 (a)), the quantity of final products produced (Figure 6-13(b)), the quantity of final products sold (Figure 6-13(c)), and the percentage of produced goods that were sold (Figure 6-13(d)).

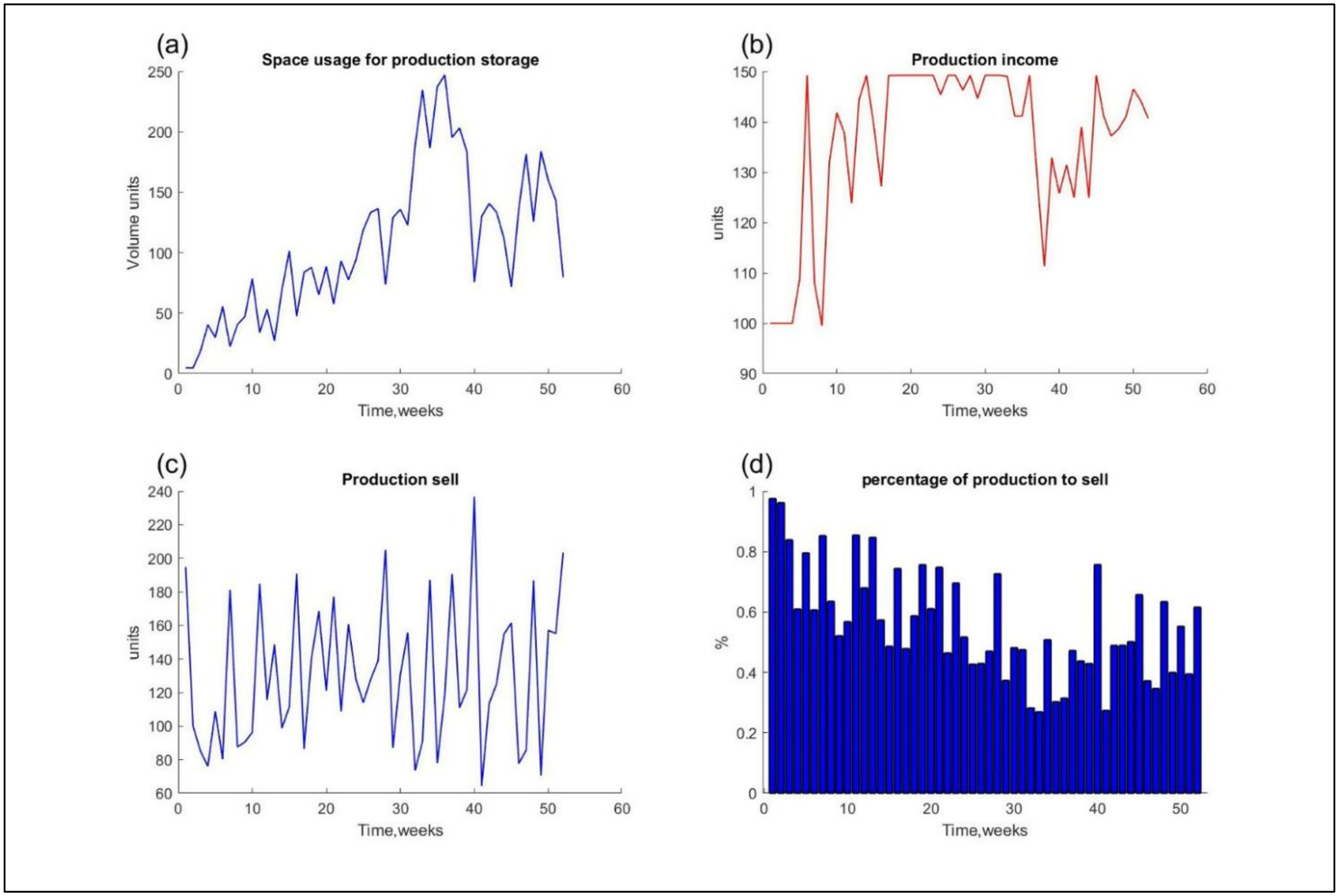

Figure 6-13. Production management for the increased storage cost case (fully stochastic scenario): (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold.

From Figure 6-13(a), it can be observed that the quantity of final products in storage is majorly boosted in week 39 when the demand for final products is at its lowest level. At this point, the low demand led to difficulties in selling the final products produced, thus they are kept in storage. Figure 6-13(b) depicts the trend of the quantity of final products produced over the planning horizon. As seen from this plot, the production quantity reaches its maximum limit several times along the planning horizon, which coincides with periods of high demand, as the factory needs to fulfil this demand by producing more products. Figure 6-13(c) shows the quantity of final products that are sold. The plot is oscillating around 120 units per week with a maximum of 240 units per week. Finally, from Figure 6-13(d), in any given week in the planning horizon, the factory sold more than 30% of the final products in storage, but mostly less than 60%, as the prediction of demand becomes tougher and tougher, so the factory produces more final products than it can sell in a number of weeks over the planning horizon. In this scenario, the model could not manage the production efficiently due to the low accuracy in the demand prediction.

6.4.4 Raw Materials Management

Similar to final product management, to assess the performance of the developed model under this parameter in the case of seasonality in demand, four different indicators are used. These indicators are the quantity of raw materials in storage (Figure 6-14(a)), the quantity of raw materials purchased (Figure 6-14 (b)), the amount of money spent on purchasing the raw materials as a percentage of available funds (Figure 6-14(c)), and the percentage of raw materials that went into production (Figure 6-14(d)).

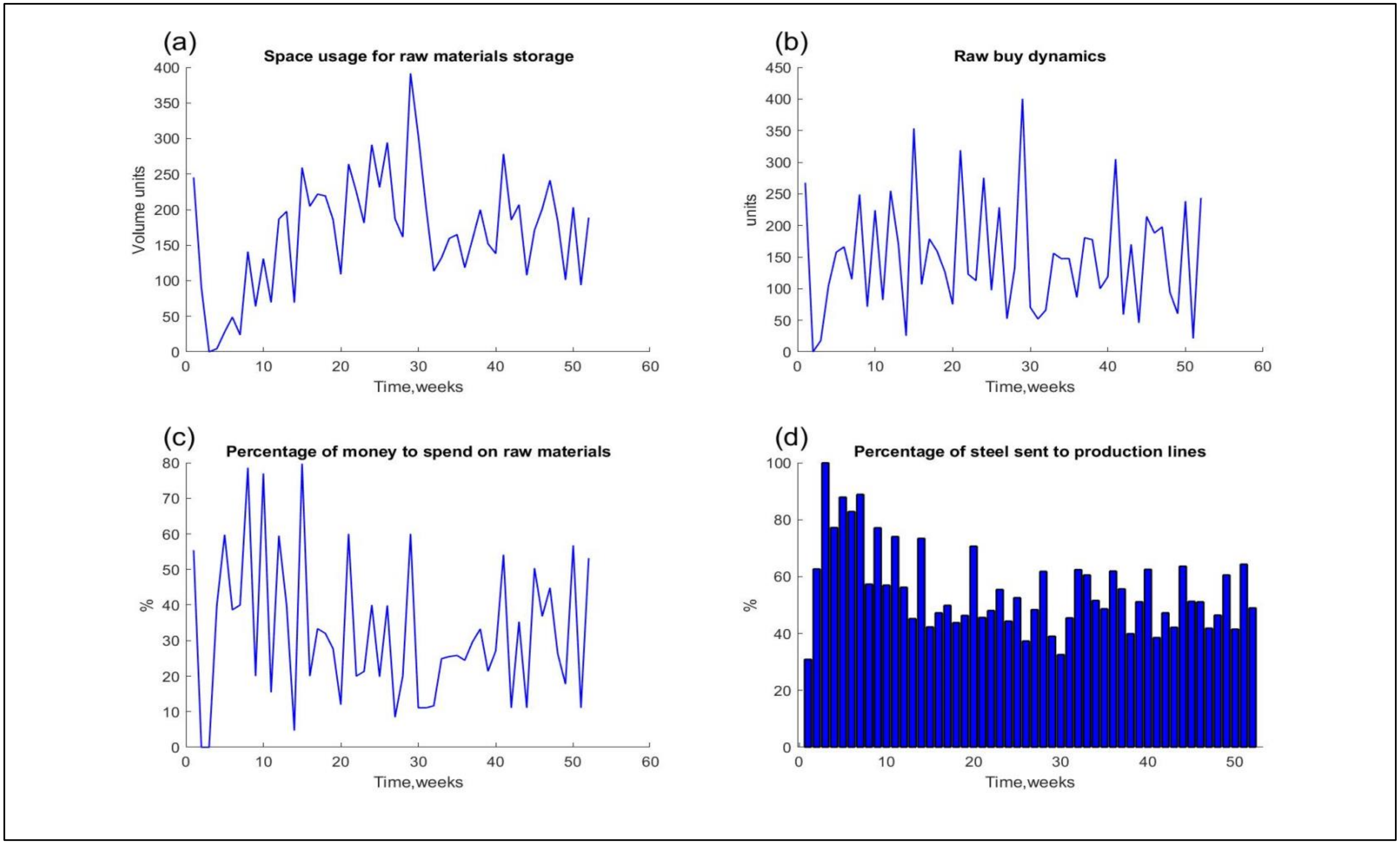

Figure 6-14. Raw materials management for the seasonal demand case: (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) the percentage of raw materials sent to the production lines.

From Figure 6-14(a), after the initial drop in the quantity of raw materials, their level in storage starts to increase gradually over the entire planning horizon to cover any increase in demand, reaching as high as 400 units after 30 weeks. Figure 6-14(b) shows the quantity of raw materials purchased over the entire planning horizon, which has several peaks during the weeks with the lowest raw materials price; on the other hand, at other times, the factory buys almost no raw materials, as there is an excess in storage. Similarly, Figure 6-14(c) shows the percentage of money that was spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased. Finally, Figure 6-14(d) shows the percentage of raw materials that move from storage to the production lines, which, most of the time, oscillates around 40%, which is a low percentage due to the difficulty in predicting demand. In general, all the above sub-plots follow very similar trends to the corresponding sub-plots of the fully stochastic scenario, as all parameters related to the raw materials supply remain the same.

6.5 Sudden and Complete Loss of Demand

In this scenario, for the last 16 weeks, i.e. from week 37 to week 52, demand is set to zero as a result of a severe financial crisis or trade wars; hence, the company will not be able to sell any final products, and the closed-loop neural network model's performance is assessed with respect to the maturity and deterioration rates, money management, final product management, and raw materials management, as illustrated in Sections 6.5.1-6.5.4.

6.5.1 Maturity and Deterioration Rates

When applying the neural network closed-loop system after assuming demand to be zero from week 37 to week 52, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 6-15 (a) and (b) for raw materials and final products, respectively.

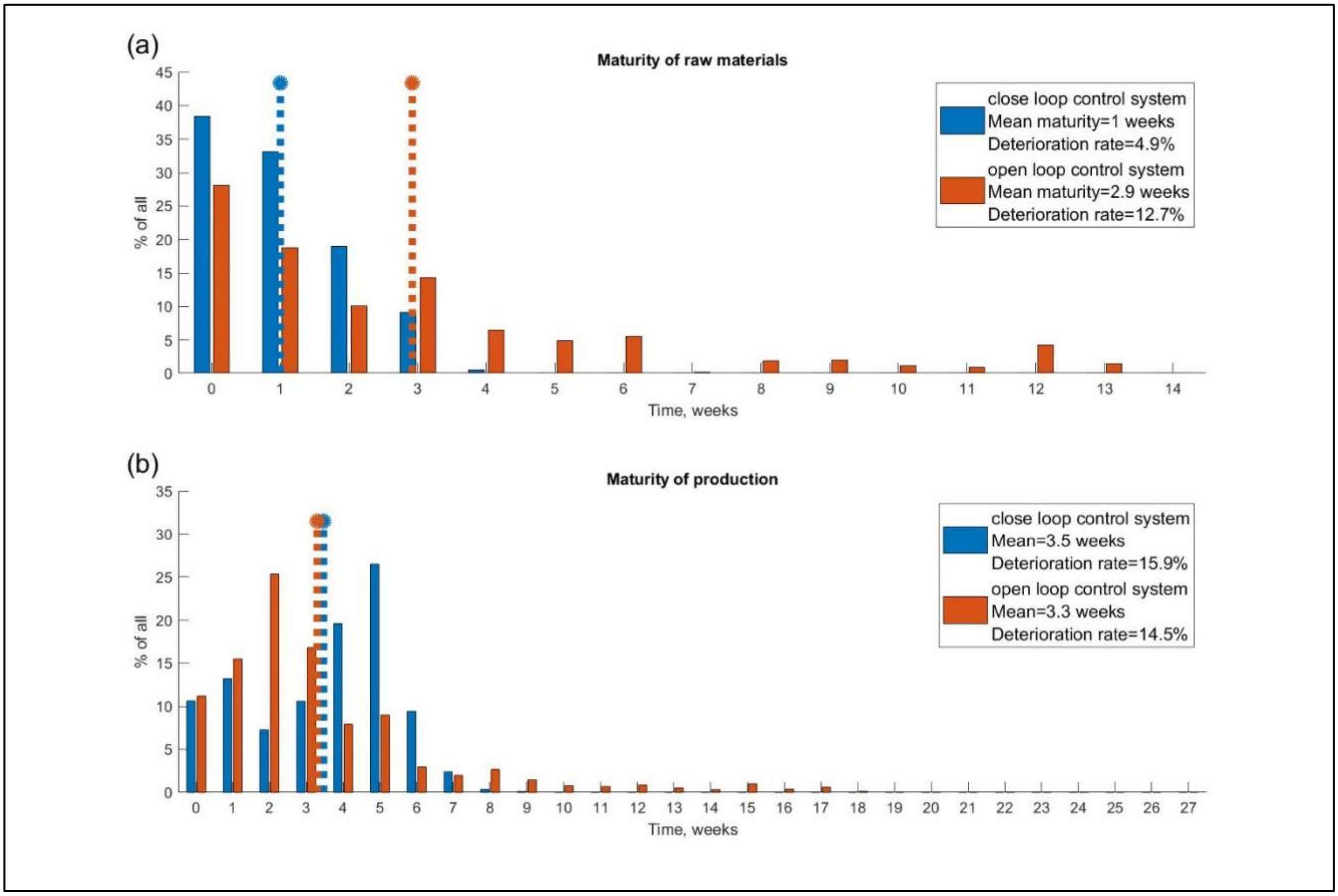

Figure 6-15: Maturity analysis for the demand termination case (fully stochastic scenario): (a) raw materials maturity, and (b) production maturity.

As can be seen from Figure 6-15, the closed-loop neural network system has a much better performance with regard to the storage of raw materials, as the maximum number of weeks in which the raw materials are in storage is lower; however, with regard to final products storage, the performance of both systems is almost the same. Moreover, the open loop control system has a higher maximum maturity but lower median value at week 2. Nevertheless, when compared to the fully stochastic scenario, the performance is almost the same with regard to raw materials, and worse with regard to final products.

6.5.2 Money Management

Under this parameter, four indicators are used to assess the performance of the developed model in the case of sudden and complete loss of demand. These indicators are the dynamics of available funds (Figure 6-16 (a)), the dynamics of the amount of money invested in the business (Figure 6-16 (b)), the change in the selling price over the planning horizon (Figure 6-16 (c)), and the amounts of up credit and down credit (Figure 6-16 (d)).

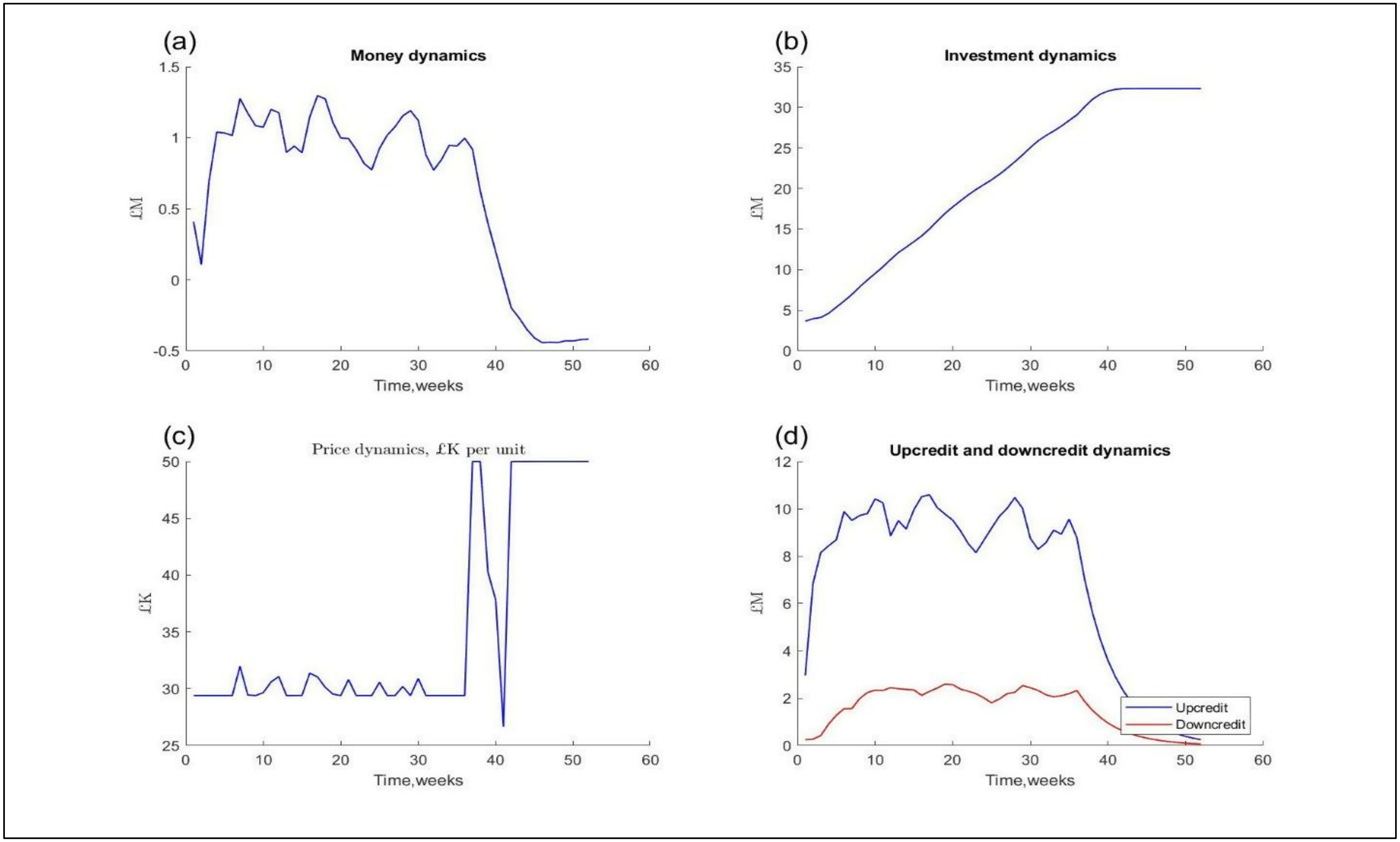

Figure 6-16. Money management for the demand termination case (fully stochastic scenario): (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics.

As seen from Figure 6-16(a), the plot starts with the amount of initial funds available before the start of the planning horizon. As can be observed, the best strategy to adopt in the case of a sudden loss of demand is investing all available funds just before the demand is lost, to protect them against inflation. Similarly, the plot for the amount of funds invested (Figure 6-16(b)) shows that, at the end of the planning horizon, the majority of funds are directed towards investment, and it stabilises after the demand stops; however, the amount of money moved to investments is less than in the fully stochastic scenario. Figure 6-16(c) shows the dynamics of the final products' selling price over the entire planning horizon. From this figure, it can be observed that the selling price of final products falls in the range of £30K until the termination of demand, while after the termination, it becomes useless to measure this parameter, so the algorithm makes it random. Finally, in Figure 6-16(d), both the up credit and down credit dynamics are observed, with both paid back over the time horizon until the end of the 52nd week. Regarding up credit, its amount increases at the start of the planning horizon as the quantity of final products sold increases; however, after week 37, when demand stops and the company no longer sells products, it collects all the up credit owed by customers at the end of the planning horizon. Similarly, all the down credit was paid by the end of the planning horizon, as the company does not purchase any more raw materials.

6.5.3 Final Product Management

To assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of final products in storage (Figure 6-17 (a)), the quantity of final products produced (Figure 6-17(b)), the quantity of final products sold (Figure 6-17(c)), and the percentage of produced goods that were sold (Figure 6-17(d)).

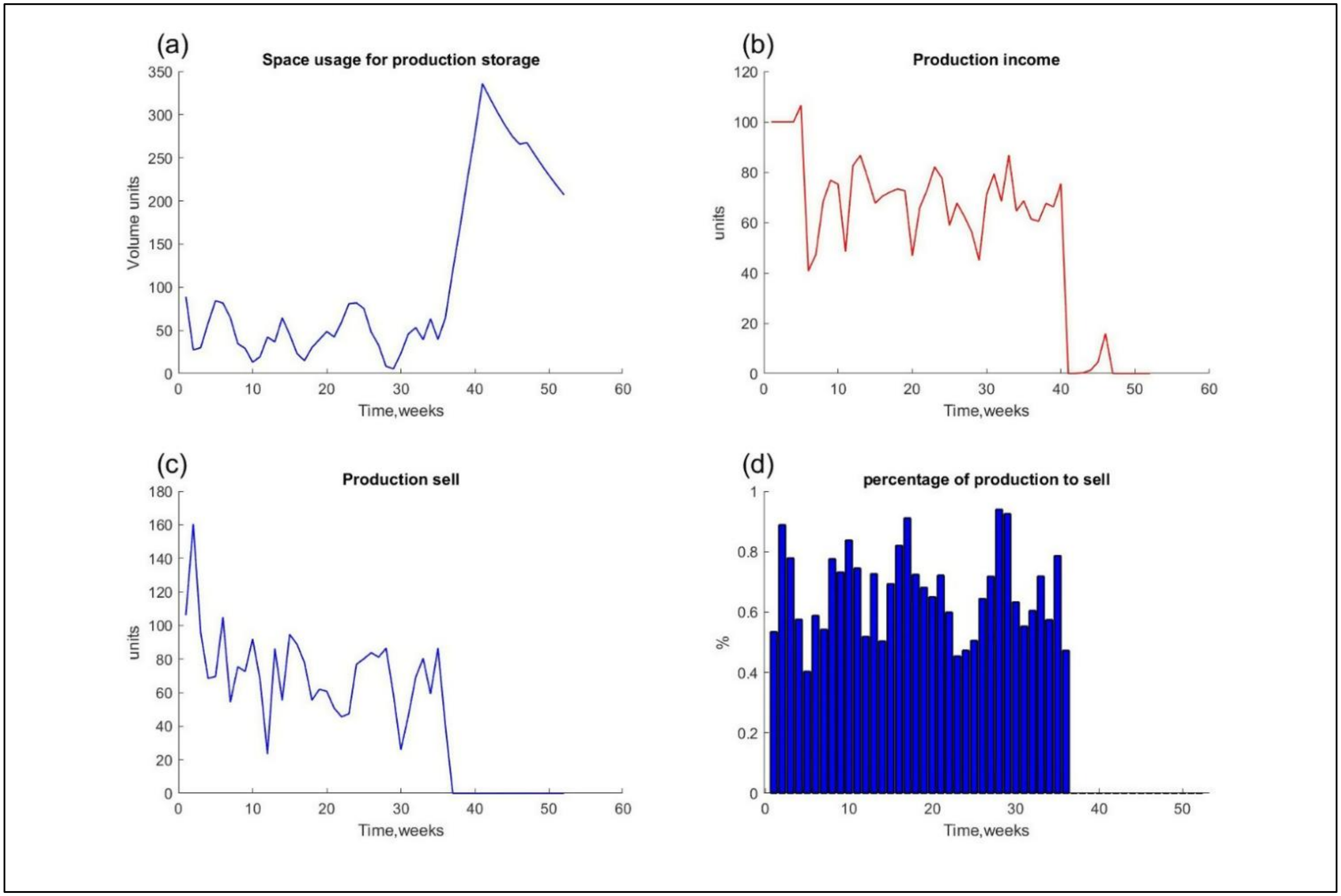

Figure 6-17. Production management for the increased storage cost case (fully stochastic scenario): (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold.

From Figure 6-17(a), it can be observed that the quantity of final products in storage accumulates heavily in week 36, when the demand for final products vanishes. Moreover, Figure 6-17(b) supports the fact that final products are still produced for five weeks after demand stops, since the production time is five weeks and the company cannot cancel the operation once the raw materials enter the production lines. Figure 6-17(c) shows the quantity of final products that were sold. The plot is oscillating around 60 units per week until week 37, after which the company cannot sell any more products because there is no demand. Finally, from Figure 6-17(d), at any given week in the planning horizon, the quantity of final products sold is more than 40% of final products in storage, reaching as high as 90% in some weeks.

6.5.4 Raw Materials Management

As with final product management, to assess the performance of the developed model under this parameter in the case of a sudden and complete loss of demand, four different indicators are used. These indicators are the quantity of raw materials in storage (Figure 6-18(a)), the quantity of raw materials purchased (Figure 6-18 (b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 6-18(c)), and the percentage of raw materials that went into production (Figure 6-18(d)).

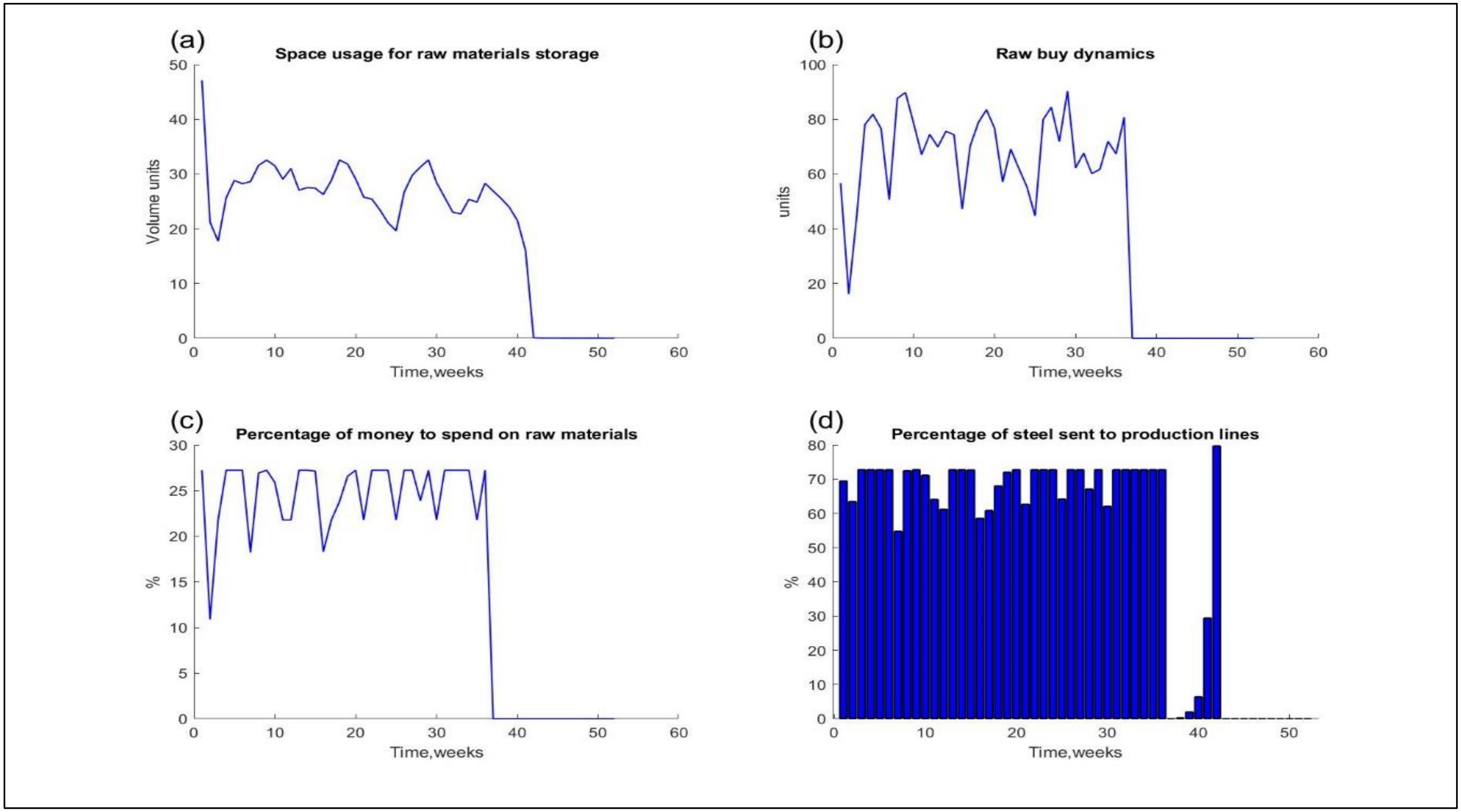

Figure 6-18. Raw materials management for the fixed demand termination case (fully stochastic scenario): (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials, (d) the percentage of raw materials sent to the production lines.

Figure 6-18 (a) shows that the quantity of raw materials follows a zigzag pattern, with each peak lower than its predecessor, as demand fluctuates and diminishes, before stopping completely. Moreover, after week 37, the factory stops purchasing new raw materials as demand stops; by week 41, there are almost no raw materials in storage. Figure 6-18(b) shows that the quantity of raw materials purchased over the entire planning horizon never exceeded 90 units and no raw materials were purchased from the time the demand stopped. Furthermore, Figure 6-18(c) shows the percentage of money that was spent on purchasing raw materials, which logically follows the same trend of the quantity of raw materials purchased, hence no money is spent after week 37. Finally, Figure 6-18(d) shows the percentage of raw materials that moves from storage to the production lines, which shows only one peak after the 37th week. The reason behind this one-time peak is that the internal price of production is higher than the price of raw materials; therefore, if the demand returns to its usual level, the company will have a lot of final products to sell.

6.6 Sudden Supply Termination

In this scenario, it is assumed that the raw materials supply stops after week 37, as a result of supplier bankruptcy or trade embargos and for the last 16 weeks, hence it is set to zero. The effect of sudden supply termination on the maturity and deterioration rates, money management, final product management, and raw materials management is illustrated in Sections 6.6.1-6.6.4.

6.6.1 Maturity and Deterioration Rates

When applying the neural network closed-loop system after assuming that supply is stopped and set to zero from week 37 to week 52, the following results, in terms of maturity and deterioration rates, are obtained, as presented in Figure 6-19 (a) and (b) for raw materials and final products, respectively.

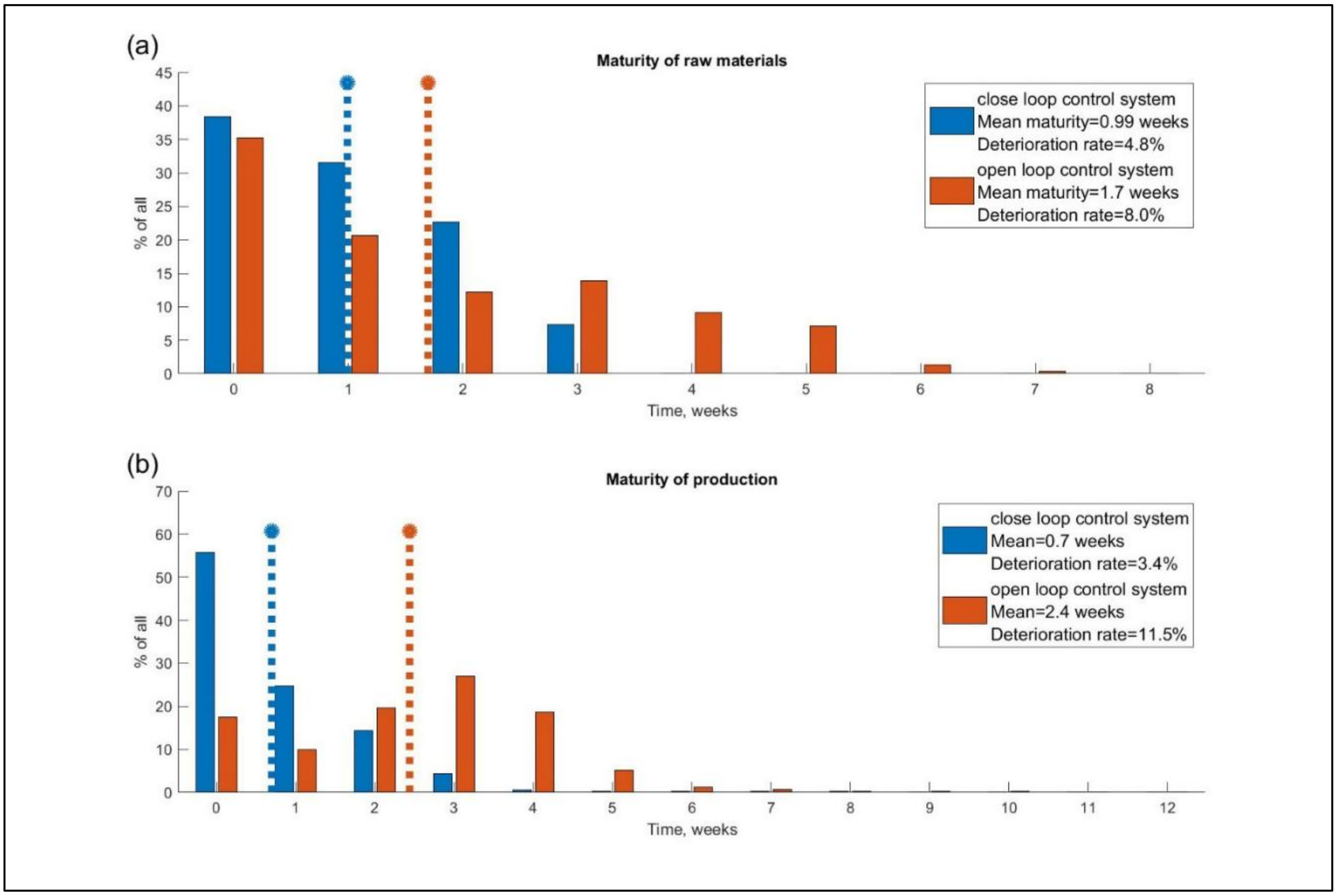

Figure 6-19. Maturity analysis for the supply termination case (fully stochastic scenario): (a) raw materials maturity, and (b) production maturity.

As can be seen from Figure 6-19 (a) and (b), the closed-loop neural network system has much better performance in terms of both raw materials and final product storage, respectively, as the time raw materials and final products spend in storage before being used or sold is much shorter than in the open-loop system. Furthermore, more than 50% of final products are made to order, versus only 18% for the open loop system. The deterioration rate for the closed-loop system is twice as lower as the rate for the open loop system with regard to raw materials storage, and almost three times lower regarding the final product storage. Again, when compared with the fully stochastic scenario, the performance is better in terms of raw materials and almost the same in terms of final products.

6.6.2 Money Management

Under this parameter, four indicators are used to assess the performance of the developed model in the case of sudden supply termination. These indicators are the dynamics of available funds (Figure 6-20 (a)), the dynamics of the amount of money invested in the business (Figure 6-20(b)), the change in the selling price over the planning horizon (Figure 6-20(c)), and the amounts of up credit and down credit (Figure 6-20 (d)).

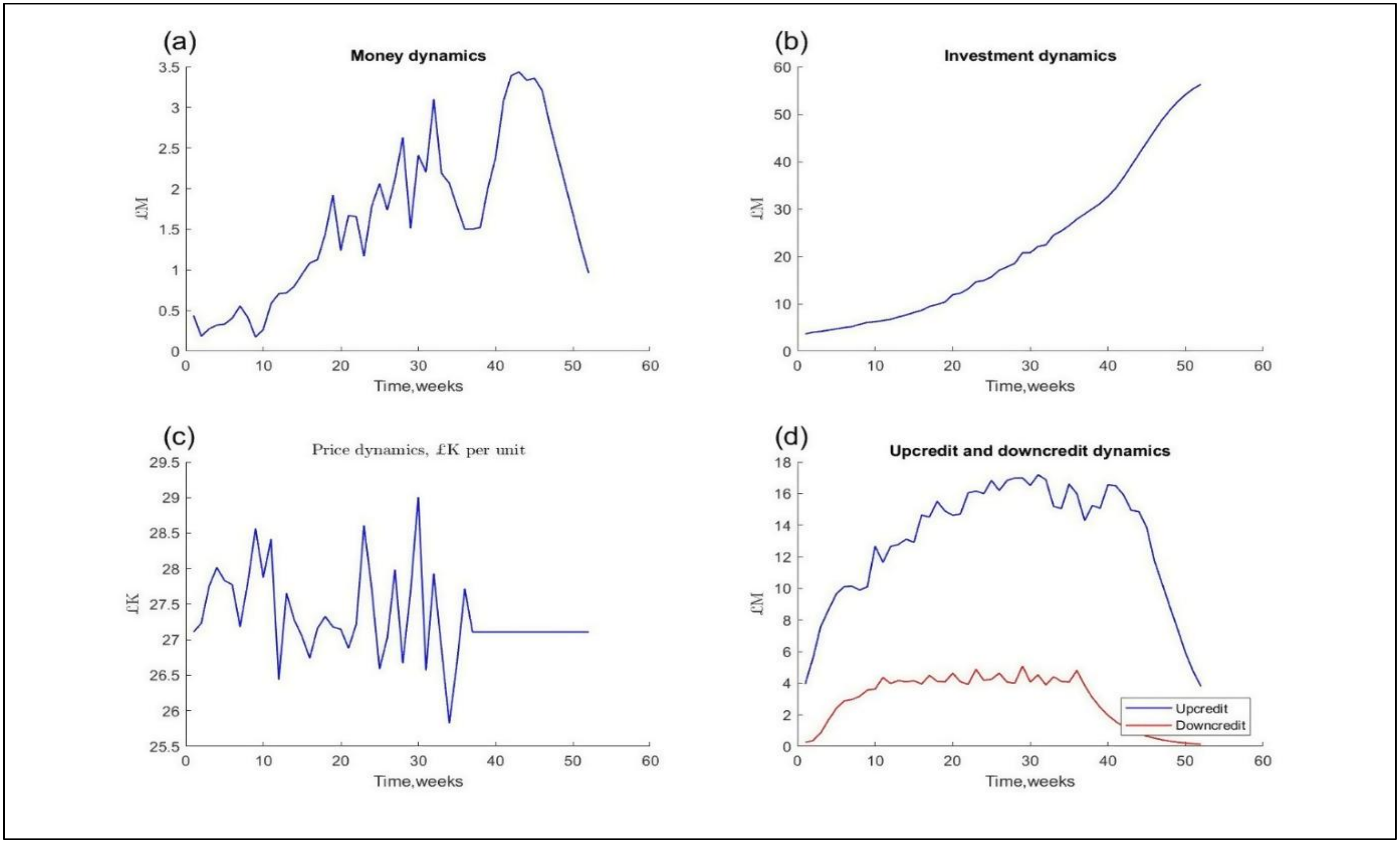

Figure 6-20. Money management for the supply termination case (fully stochastic scenario): (a) money dynamics, (b) investment dynamics), (c) price dynamics, (d) up credit and down credit dynamics.

As seen from Figure 6-20(a), the plot starts with the amount of initial funds available before the start of the planning horizon. As can be observed from this plot, after week 37, the company stops purchasing any new raw materials, which leads to a final increase in the available funds to the up credit interest. However, after week 43, the company moves the available funds to investment in order to protect them against inflation. Similarly, the plot for the amount of funds invested (Figure 6-20(b)) shows that the money directed towards investment is increasing steadily, and at the end of the planning horizon, the majority of funds are directed towards investment, even at a higher amount than in the fully stochastic scenario, as there will be no more production or income from selling final products when the supply stops, thus it is more profitable to invest the money.

Figure 6-20(c) shows the dynamics of the final product selling price over the entire planning horizon. From this figure, it can be observed that the selling price of final products oscillates around the £27K until the termination of supply. After the termination, the company sells the final products at a constant price, as there is no supply and it wants to sell all the final products remaining. Finally, in Figure 6-20(d), both the up credit and down credit dynamics can be observed, where up credit starts to decrease after week 43 when the remaining quantity of final products is sold, as it represents the amount of money owed by customers for products, and down credit starts to decrease from week 37 when the supply stops, as it represents the amount of money owed to suppliers as a result of purchasing raw materials.

6.6.3 Final Products Management

To assess the performance of the developed model under this parameter, four different indicators are used. These indicators are the quantity of final products in storage (Figure 6-21 (a)), the quantity of final products produced (Figure 6-21(b)), the quantity of final products sold (Figure 6-21(c)), and the percentage of produced goods that were sold (Figure 6-21(d))

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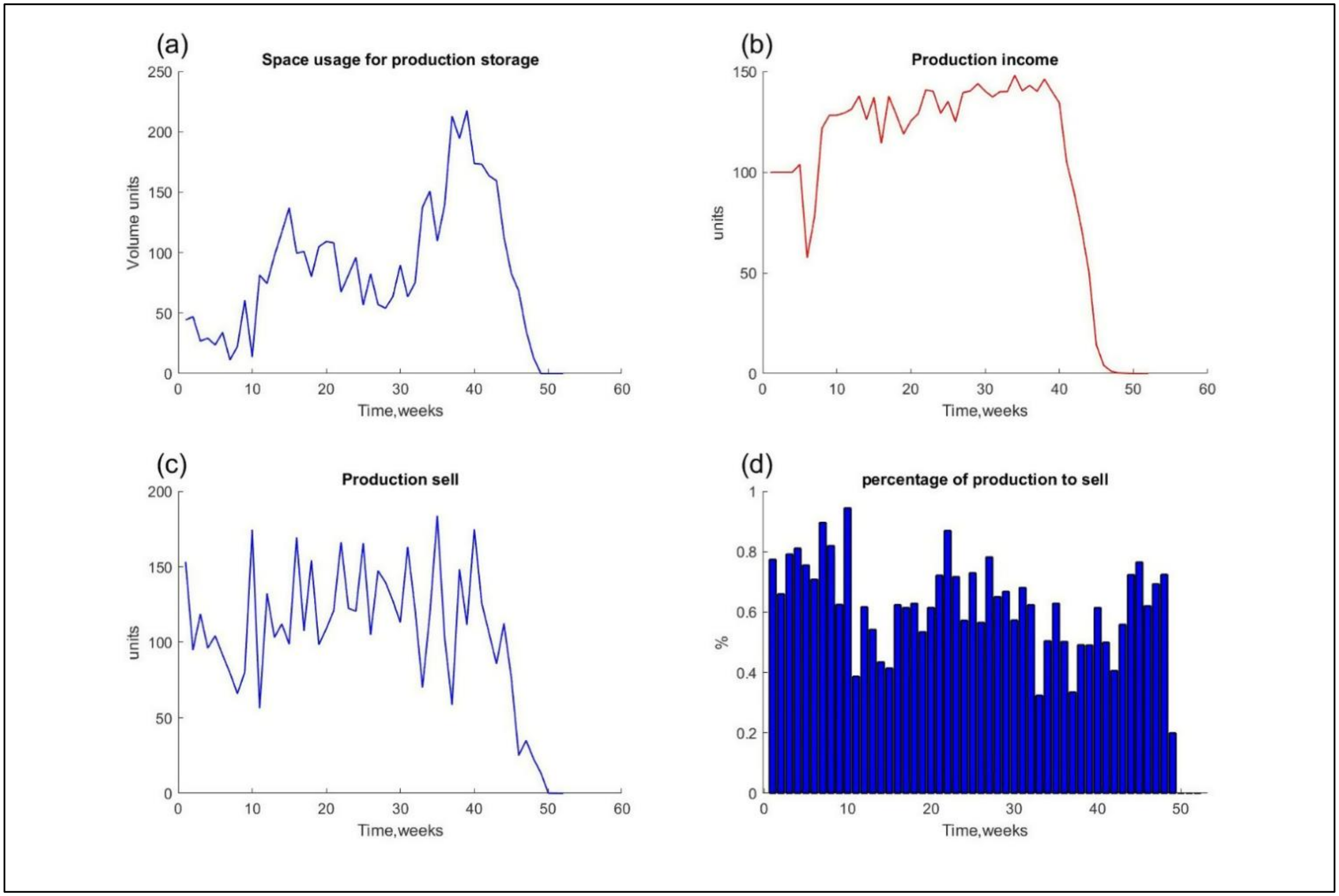

Figure 6-21. Production management for the supply termination case (fully stochastic scenario): (a) storage space usage, (b) quantity of final products produced, (c) quantity of final products sold, (d) percentage of final products produced that were sold.

From Figure 6-21(a), it can be observed that the quantity of final products in storage starts to decrease after week 41, until it reaches zero in week 50. This delay between the termination of both the supply and final products in storage is a result of the production lines' delay, which is five weeks according to the steel factories' common designs, hence at least five weeks are required to sell all the final products. Moreover, Figure 6-21(b) supports the fact that final products are still produced for five weeks after the supply stops, since the production time is five weeks. Similarly, Figure 6-21(c) shows the quantity of final products that were sold, which fluctuates around 130 items and then goes down to zero when there are no more final products to sell. Finally, from Figure 6-21(d), the percentage of final products sold oscillates around 50% along the planning horizon, which suggests that this irregular scenario impacted the robustness of the model; however, it still provides acceptable results.

6.6.4 Raw Materials Management

Similar to final product management, to assess the performance of the developed model under this parameter in the case of a sudden and complete loss of supply, four different indicators are used. These indicators are the quantity of raw materials in storage (Figure 6-22(a)), the quantity of raw materials purchased (Figure 6-22 (b)), the amount of money spent on purchasing raw materials as a percentage of available funds (Figure 6-22(c)), and the percentage of raw materials that went into production (Figure 6-22(d)).

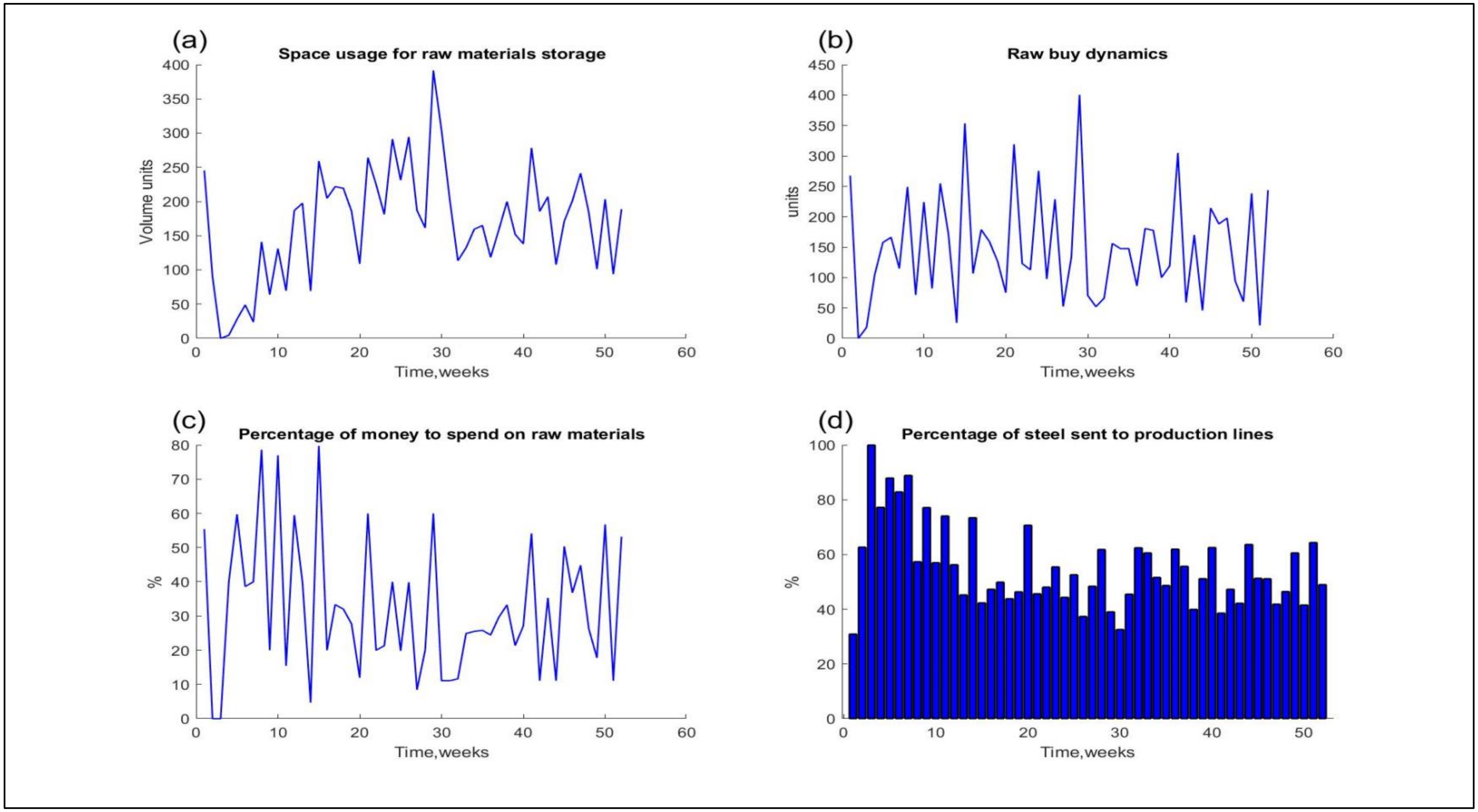

Figure 6-22. Raw materials management for the supply termination case (fully stochastic scenario): (a) raw materials storage usage, (b) quantity of raw materials purchased, (c) percentage of money spent on purchasing raw materials; (d) the percentage of raw materials sent to the production line.

Figure 6-22(a) shows that the quantity of raw materials follows a zigzag pattern, with each peak lower than its predecessor. Moreover, after week 37, the factory cannot purchase any new raw materials as the supply has stopped; thus, after week 41, the quantity of raw materials in storage is almost zero. Figure 6-22(b) shows the quantity of raw materials purchased over the entire planning horizon, which oscillates around 150 units and reaches zero when the supply is interrupted in week 37. Furthermore, Figure 6-22(c) shows the percentage of money that was spent on purchasing raw materials, which logically follows the same trend as the quantity of raw materials purchased, hence no money is spent after week 37. Finally, Figure 6-22(d) shows that the percentage of raw materials that moves from storage to the production lines goes as low as 30% in some weeks. This low percentage might be justified by the fact that, in some instances, it is more effective to temporarily lower production and hold raw materials in storage, in the hope that the production cost declines or the selling price increases.

6.7 Chapter Summary

In the current chapter, a sensitivity analysis has been performed to examine the robustness of the developed model and its ability to handle real-life scenarios that can occur in the steel manufacturing industry. In total, five cases based on the fully stochastic scenario and reflecting either irregular economic patterns or worst-case scenarios were considered and analysed, as follows:

1) An increase in storage costs.
2) Seasonal change in the purchasing price of raw materials.
3) Seasonal change in demand.
4) Sudden and complete loss of demand.
5) Sudden and complete termination to the supply channels.

To assess the closed loop neural network model’s performance in each of these cases, the analysis was performed using the same parameters presented in Sections 5.3.2, 5.3.3 and 5.3.4 of the previous chapter, which are:

1) Storage analysis, which includes maturity distribution, average maturity value and deterioration rate
2) Cash flow analysis
3) Raw materials analysis
4) Production analysis.

From the conducted analysis, the following conclusions can be drawn. In terms of maturity, either for raw materials or final products, the performance was mostly similar in the above five cases to the fully stochastic scenario analysed in Chapter 5. In the case of an increase in the storage cost, the closed loop neural network system was able to adjust its investment level to increase the amount of funds available at the end of the planning horizon, while reducing the amounts invested in order to be able to cover an increase in costs. In addition, in terms of the quantity of final products in storage, the model's performance was similar to the fully stochastic scenario of Chapter 5. On the other hand, it produced lower quantities of produced and sold final products when compared to the fully stochastic scenario. In terms of the raw materials performance indicators, following the impact of an increase in the storage cost, the model produced less accumulation of raw materials compared to the fully stochastic scenario, in order to minimise these costs, which reflects its robustness.

When analysing the case of a seasonal change in the price of raw materials, the results of the closed-loop neural network model show major boosts in the quantities of raw materials purchased and final products produced when the purchasing price of the former is at its lowest level, which proves how effective the model is in adapting to such an irregular event. However, similar to the previous scenario, the quantities of the final product that were produced and sold are still lower than those of the fully stochastic scenario. On the other hand, when the seasonal change occurs in the demand, the model showed high robustness and effectiveness in dealing with this irregular scenario. The model freed funds, making them available to the company to purchase raw materials when demand is high, and, similarly, increased the selling price of final products during periods of high demand. In addition, there was a major boost in the quantities of final products produced and raw materials purchased when demand was high, and the factory reached its maximum production capacity during these periods.

In cases where demand or supply are suddenly lost, the model advises the company to direct most of its funds to investment to protect it from inflation until the normal conditions return, which will protect the company from bankruptcy. In terms of final products and raw materials management, the model was able to adjust to these scenarios and showed trends that depicted the actual real-life scenarios, such as accumulation of final products in storage when demand is lost.

In conclusion, in the sensitivity analysis conducted in this chapter, it was proven that the model has a robust performance, as it was able to both adjust its parameters to different irregular scenarios, and alter inventory and money management strategies in ways that can maximise profit for the company, and protect it from bankruptcy until normal economic and business conditions are resumed.

7 Conclusions and Future Work

7.1 Introduction

In an attempt to help companies with large-volume inventory to effectively manage their inventory and final product storage in a limited space, this research study explored the various aspects of such companies by taking the steel industry as an example. In particular, an important consideration regarding the steel industry is the deterioration of raw materials and final products as a result of environmental factors. In this context, the main questions companies in the steel industry face concern how many raw materials they should purchase as well as how many final products they should produce (and when) in order to optimise their operations in such a way that the cost can be minimised and the profit maximised. This optimisation will also help in improving the sustainability of the steel industry by optimising the use of resources, minimising the energy required to preserve these materials from deterioration, and reducing waste generated by excess ordering of raw materials or production of final products.

As discussed in Section 2.4.4.2.1, in the current business scenario it is evident that the success of companies is not only measured in terms of financial soundness, but also in terms of their environmental friendliness through adopting sustainable practices into their efficiency-based operations. In this context, it becomes crucial to consider not only financial but also sustainable performance measurements when managing the inventory. Nevertheless, according to the analysed studies in the conducted literature review, no tool or model developed by previous research optimised the inventory management of large-volume perishable material while taking into account the sustainability impacts of inventory planning activities. In order to fill this research gap, the overall aim of this research study was to establish a sustainable inventory management model incorporating the different sustainability measures that are applicable to the steel manufacturing industry, such as energy consumption and resource consumption, to manage the environmental impacts of this activity while optimising its economic performance. The developed model was based on the well-known EOQ concept to study and optimise the inventory and order placement decisions over 52-week time horizon periods for high-volume material with limited storage space, such as steel, under stochastic demand, supply and backorders. The proposed model is expected to minimise the high storage and handling costs associated with raw materials and final products of a steel manufacturing company, and to prevent the deterioration of this inventory as a result of different environmental factors, thus maximising the company's profits. In order to do so, the proposed model was developed based on a control system algorithm capable of providing timely recommendations for the storage quantities of both products and raw material. In this way, the decisions regarding the level of investment, steel purchasing strategy, and setting of optimal production levels throughout the planning horizon are facilitated. Two different control system approaches, namely, an open-loop and a closed-loop based on ANNs, were considered. The latter, introduces feedback, allowing the mathematical model to be periodically updated based on the current values of the business parameters. Finally, due to the complexity of the addressed problem and its specific characteristics, a PSO technique was used to solve the developed model.

The objective of this research study was to contribute to the continuing evolution of inventory management models by developing a novel model capable of:

1. Modelling the stochastic nature of the different inventory parameters, such as demand, supply and backorder, for high-volume products with limited storage space when taking the sustainability approach into consideration.

2. Modelling the manner and nature of the deterioration of raw materials and final products of the steel manufacturing factory, and optimising the storage time of the inventory in order to reduce energy costs and, in turn, storage costs.

3. Analysing the cash flow cycle of the steel manufacturing company and incorporating its different parameters and determinants into the inventory management model, in order to ensure the efficiency of the production process and maximise the company's profit.

In this way, this model was intended to assist the managers of steel manufacturing companies in deciding whether and how they can reschedule production and inventory plans to improve the efficiency of their operations and reduce their sustainability impacts, while maximising the company's profits. In this final chapter, the main findings of this research study are summarised. In particular, in Section 7.2 the main conclusions are discussed. In Section 7.3, the main contributions to the state-of-the-art are presented. In Section 7.4 some recommendations are provided and the significance of the research is stated. Finally, in Section 7.5 the limitations of the research study are discussed and future research directions are suggested.

7.2 Main Conclusions

In this research study, an inventory management model was developed to optimise the ordering and storage of large-volume inventory and final products based on developed EOQ concept to incorporate the stochastic nature of demand, supply and backordering, while taking into consideration the deterioration of these items as a result of different environmental factors. In particular, the proposed model was based on extending the well-known EOQ concept to account for steel manufacturing characteristics solved through an ANN based closed-loop which is optimised by a PSO algorithm. The closed-loop architecture introduces feedback, allowing the mathematical model to be periodically updated based on the current values of the business parameters.

In order to develop the mathematical model extending the EOQ model, the business cycle of the steel manufacturing factory was analysed. In this way, the crucial and unique business parameters required for the estimation of the business efficiency of a steel manufacturing factory were determined. According to this analysis, the main components of the steel manufacturing factory's business cycle were defined and classified into stochastic or deterministic, as well as the relationship between them, which act as drivers for generating either profits or losses, and the different transactions that contribute to realising profits or incurring losses were derived and investigated, towards developing the model.

In order to control this mathematical model, an open and closed-loop approaches were introduced. In the case of the open-loop control system, five instances of external stochastic factors were generated using the Monte Carlo method, and then the control for the entire planning horizon of 52 weeks, which would maximise the average profit for all instances, was deduced. In the case of the closed-loop system, a neural network to generate the optimal control parameters is used. The used ANN consists in one hidden layer using as inputs the business parameters from the mathematical model and providing as outputs the control variables that optimise the profit function (objective function). The weights of the ANN are adjusted based on the current business measurements, in order to keep the model updated and provide managers useful information towards helping them in their decision-making process. Finally, a PSO algorithm has been adopted to train both the open and closed-loop systems.

Due to the lack of available benchmark results in the literature as well as the uniqueness of the model developed in this research study, it is not possible to compare the obtained results of the developed model with results already available in the sate-of-the-art. In this context, the developed model was validated by comparing the performances of the open and closed-loop versions against each other. Here it is important to highlight that, in order to make this comparison fair, the same PSO internal parameters were used to train the open and closed-loop models. In particular, experiments applying the developed model within different steel manufacturing scenarios were conducted, comparing both systems in terms of maturity and deterioration rates, money management, final product management, and raw materials management. Experimental results allowed to do the following observations:

1. The closed-loop control system has lower mean maturity and deterioration rate values for both raw materials and final products.
2. The profits generated through the use of one unit of storage are very similar for both control systems.
3. The neural network closed-loop control system results in higher profit than the direct open-loop control system for each Monte Carlo run.
4. The neural network closed-loop control system leads to a more balanced investment strategy, in which a small investment is made at the beginning of the planning horizon, then the amount of this investment increases as we move forward in the planning horizon.
5. The direct open-loop control system has much less money over the entire planning horizon, which hinders the ability of the company to increase the quantity of raw materials purchased as a reaction to any increased demand, which might jeopardise the company's operations and prevent it from maximising its profits.
6. The learning progress of the closed-loop control system was much more efficient than the progress of the open-loop control system.

Based on the above discussion, it can be seen that the ANN closed-loop based model resulted to be the best suited for the inventory management in the steel manufacturing industry, being robust, accurate and efficient. Once the most suitable model was already determined, this model was tested within the complex context of the stochastic demand, supply and backorder. In this line, experiments were conducted under the fully stochastic scenario. In addition, the robustness of the developed model was further tested by a sensitivity analysis where the model was implemented under different irregular economic patterns or worst-case scenarios. In particular, an increase in storage costs, a seasonal change in the purchasing price of raw materials, a seasonal change in demand, a sudden and complete loss of demand, and a sudden and complete termination to the supply channels, were considered. Based on the obtained experimental results the following observations can be done:

1. For some of the tested parameters, the obtained results do not vary significantly through the different extreme business scenarios tested here. Moreover, in these cases, the obtained results are similar to the ones obtained in the fully stochastic scenario. In particular, this has been observed for the maturity, either for raw materials or final products, and the quantity of final products in storage. These results show that the developed model can adapt to these different scenarios in a similar way in terms of the mentioned parameters. Moreover, it

could be said that the model can interpret these scenarios as fully stochastic scenarios and apply the same optimisation strategy towards adjusting to them.

2. When comparing the quantities of produced and sold final products, the produced quantity is lower in these scenarios than in the fully stochastic one. This shows that the extreme scenarios' conditions have a greater (negative) impact on these quantities than the fully stochastic scenario. In this sense, the developed model performs better when optimising these quantities in the fully stochastic scenario, allowing the company to sell more final products.
3. In the case of an increase in the storage cost, the closed loop neural network system was able to adjust its investment level to increase the amount of funds available at the end of the planning horizon, while reducing the amounts invested in order to be able to cover an increase in costs. This shows the robustness of the developed model with respect to changes in the storage costs.
4. In terms of the raw materials performance indicators, following the impact of an increase in the storage cost, the model produced less accumulation of raw materials compared to the fully stochastic scenario, in order to minimise these costs. This result is promising since it shows that the developed model is capable of adapting to an increase in the storage cost, producing less accumulation which means, on one hand using less space, and on the other, reducing the storage cost.
5. In the case of a seasonal change in the price of raw materials, the results showed major boosts in the quantities of raw materials purchased and final products produced when the purchasing price of the former is at its lowest level. This proves the effectiveness of the proposed model in adapting to such an irregular event.
6. The results obtained for the case of a seasonal change in the demand show that the model has a high robustness and effectiveness when dealing with this irregular scenario. The model freed funds, making them available to the company to purchase raw materials when demand is high and, similarly, increased the selling price of final products during periods of high demand. In addition, there was a major boost in the quantities of final products produced and raw materials purchased when demand was high, and the factory reached its maximum production capacity during these periods.
7. In cases where demand or supply are suddenly lost, the model advises the company to direct most of its funds to investment to protect it from inflation until the normal conditions return, which will protect the company from bankruptcy. In terms of final products and raw materials management, the model was able

to adjust to these scenarios and showed trends that depicted the actual real-life scenarios, such as accumulation of final products in storage when demand is lost.

8. When there is a sudden decrease or interruption in the supply of raw materials, the curve of down credit depicts the shape of the raw materials price, since it reflects the money owed by the company to suppliers.

Based on the observations done as results of the sensitivity analysis, it can be concluded that the developed model based on the ANN closed-loop is robust against different extreme business scenarios. In particular, it has shown to be able to adjust its parameters to the different irregular scenarios, and alter inventory and money management strategies towards maximising the company's profit even in such complex environment. In this way, the developed model can prevent the company from losing money or even from bankruptcy, until normal economic and business conditions are resumed. Here, it is important to highlight that the robustness of the developed model and its capability of adapting to such extreme scenarios relies on 1) the extension of the EOQ model by identifying, modelling and including the main steel business parameters; 2) the feedback provided by the closed-loop control system; 3) the high learning capability and of the ANNs; and 4) the accurate and fast parameter optimisation strategy based on the PSO technique.

Based on the above discussion, it can be concluded that the presented research study is significant in the field of inventory models developed for steel manufacturing applications by developing a sustainable space-dependant model which not only accounts for the most concerning characteristics in this industry, such as the limited storage space or the stochastic demand, but it also accounts for sustainability measures, such as energy consumption and resource consumption, to manage the environmental impacts of this activity while optimising its economic performance. This has been shown in the conducted experiments, where the model has proven to be capable of maximising profit while minimising adverse environmental impacts, even for extreme business scenarios.

7.3 Contribution to the Body of Knowledge

The developed model and the conducted research study make significant contributions to the body of knowledge in the inventory management field. These contributions include:

1. Developing an inventory management model that accounts for the specific characteristics of the steel manufacturing industry, such as stochastic demand and space limitations. In this way, the developed space-dependent model fills the gap

regarding modelling the inventory management of the steel manufacturing company taking into account the stochastic nature of demand, supply and backordering.

2. Developing an inventory management model that accounts for sustainable aspects of the business. In particular, since the developed model is focused on minimising storage cost and time, this, in turn, will assist in minimising the negative environmental impacts of ordering and holding inventory. In this way, the model makes a significant contribution towards the scarce literature regarding the sustainability aspects of the inventory management in the steel manufacturing industry.
3. Developing an inventory management model that optimises a multidimensional objective function. On the contrary of the majority of the previously available models in the literature which minimises costs, the developed model maximises a net profit function that includes different parameters, such as investments, current cash, up credit, down credit and backorder loss. In this way, the optimisation of the proposed objective function allows maximising the profit while minimising the storage costs.
4. Developing an accurate and robust inventory management model for large-volume materials solved using the PSO technique. As discussed throughout this research study, the inclusion of several parameters accounting for steel manufacturing characteristics in the mathematical model extending the EOQ model, makes it not possible to use the traditionally used techniques to solve the model. Moreover, some previous works in the literature had to limit the model's complexity in order to be able to solve it, being not able to fully reflect the real-life characteristics. In this line, there has always been a trade-off between the model's complexity and its solvability. In this research study, this limitation is overcome by using a PSO algorithm to solve the model, constituting one of the most important contributions to the field in the sense of proving the suitability of such technique to the addressed application, even in extreme conditions as the ones of the conducted experiments.
5. Developing an accurate and robust inventory management model for large-volume materials based on ANNs. The developed model takes advantages of the ANNs high learning capability to ensure the parameters of the model can be updated on a weekly basis based on the current values of the business indicators. In this way, the mathematical model can be adjusted towards adapting to the economic and business environment giving managers useful and updated information that can help them in their decision-making process.

7.4 Recommendations and Significance

This research study is aimed at supporting the steel manufacturing companies in their inventory management decisions, specifically speaking, when they have limited storage space. The recommended adoption of this model by the steel manufacturing companies will help in:

1. Reducing the costs of holding and ordering inventory for the steel manufacturing companies.
2. Reducing the probability of deterioration of the raw materials and final products of a steel manufacturing company as a result of environmental factors.
3. Maximising the profits of such companies.
4. Improving the sustainability of the steel manufacturing industry as a whole, and the supply chain of this industry, in particular.

Consequently, the above impacts are expected to provide major benefits to the environment, the steel manufacturing companies, and the overall economy. The significance of this model in protecting the environment is three-fold. First, by optimising the storage duration of raw materials and final products, the energy required to preserve these materials, in terms of lighting and cooling, will be reduced, which will lower the energy consumption of the steel manufacturing industry. This, in turn, will help to reduce pollution resulting from the consumption of high quantities of energy. Second, optimising the duration of the storage of these materials while considering their deterioration will ensure that these materials are not ruined, thus optimising the use of natural resources and preventing their depletion. Third, when materials deteriorate, companies tend to dispose of them as waste, which pollutes soil and water resources. Therefore, preventing this deterioration will reduce the amount of waste and mitigate the need to dispose of it, hence preventing the contamination of soil and water resources.

With regard to steel manufacturing companies, the positive impacts of implementing the developed model have been heavily emphasised in different parts of this research study. Regarding the significance of this model to the overall economy, the steel manufacturing industry is a critical industry for the health of any economy. Therefore, if companies of the industry are able to manage their inventory and resources, this will help to avoid any unnecessary increase in the price of this critical commodity, which in turn will benefit other sectors of the economy, such as the construction industry, which is a major driver for a booming economy. Moreover, there will be additional freed-up cash in the economy by both the manufacturers and consumers, which will boost investments in other sectors of the economy. Finally, preventing any unnecessary increase in the price of steel will benefit the entire society, as housing will become more affordable, and building new infrastructure projects will become more feasible.

7.5 Limitations and Suggestions for Future Research

This research study introduced a new inventory management model for large-volume materials that deteriorate over time, while taking into consideration the sustainability impacts of managing such inventory. Despite the proven accuracy and reliability of the model's results, future research can be beneficial in investigating new solution methods to obtain new optimal solutions for the multi-objective problem and improve the effectiveness of the solution methodology. Additionally, new models that consider the uncertainty of costs and demands can be developed through fuzzy models.

Although this model presents a breakthrough in inventory management research, and is an effective tool in enhancing the inventory management decision-making process for companies that deal with such types of inventory, a number of future research opportunities are available to further enhance and improve this process. These opportunities include, but are not limited to:

1) Extending the model to include more than one market policy for providing raw material to depict the competition present.
2) Accounting for the variable nature of the transportation cost of raw materials and final products.
3) Accounting for the variable nature of the inflation rate, which impacts the costs of raw materials and the selling prices of final products.
4) Using actual steel manufacturing data to improve the accuracy of the developed models.

7.5.1 Extending the Model to Include More than One Market Policy for providing the raw material

The developed model assumed a scenario in which the market is based on one market policy for providing the necessary raw materials. However, more scenarios will lead to more accurate results, and especially in strategic industries such as the steel industry, whereby more than one market policy could push the manufacturers to use more than one supplier price policy to avoid any disruption in operations. This scenario will create competition among the suppliers and manufacturers, which will impact both the costs of purchasing raw materials and the prices of selling final products. Hence, a model that depicts this scenario will help in reflecting the business case of the steel manufacturing industry in a more accurate way.

7.5.2 Accounting for the Variable Nature of the Transportation Cost

Transportation cost is one of the major cost components of purchasing raw materials or selling final products. These costs can vary according to the relative locations of the supplier and the manufacturer, the mode of transportation, the change of fuel costs, and the quantity of purchased raw materials or sold products. However, in developing the model in this research study, this cost was assumed to be fixed over the entire planning horizon, which is rarely the case in real life. Therefore, a model that depicts the change in the transportation cost of raw materials and final products, due to one or more of the above reasons, will help to improve the accuracy of the inventory management of the steel manufacturing companies.

7.5.3 Accounting for the Variable Nature of the Inflation Rate

Similar to transportation cost, the inflation rate is rarely constant over the planning horizon, as it normally changes on a monthly basis. These rates impact the cost of purchasing raw materials or selling final products, as suppliers/manufacturers increase their profit margins to account for an increase in the inflation rate. Nonetheless, to reduce the complexity of the developed model, this rate was assumed constant over the entire planning horizon. A model that depicts the change in the inflation rate, and its impact on the purchasing cost of raw materials and the selling price of final products, would accurately depict the real-life business logic of the steel manufacturing industry.

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Appendix A: List of papers in the literature reviews

Table A 1. EOQ model studies by parameter.

Type	Number of Studies	Studies
Price	25	Ladany and Sternlieb (1974); Ray, Gerchak and Jewkes (2005); Fibich et al. (2003); Chou and Parlar (2006); Anderson et al. (1992); Singh and Vives (1984); Vives (1999); Jeuland and Shugan (1988); Agrawal and Ferguson (2007); Hanssens and Parsons (1993); Song et al. (2008); Chen et al. (2006); Chen and Simchi-Levi (2012); Federgruen and Heching (1999); Petruzzi and Dada (1999); Chen and Simchi-Levi (2004); Kocabıyıkoglu and Popescu (2011); Phillips (2005); Agrawal and Ferguson (2007); Maddah and Noueihed (2017); Kalish (1985); Zhao and Zheng (2000); Bitran and Mondschein (1997); Xu and Hopp (2009)
Time	9	Resh, Friedman and Barbosa (1976); Donaldson (1977); Bose, Goswami and Chaudhuri (1995); Giri, Goswami and Chaudhuri (1996); Dave and Patel (1981); Elsayed and Teresi (1983); Hariga and Benkherouf (1994); Mirzazadeh (2010); Elsayed and Teresi (1983)
Lead Time	6	Palaka, Erlebacher and Kropp (1998); Albana, Frein and Hammami (2017); Pekgun, Griffin and Keskinocak (2017); Ho and Zheng (2004); Zhao, Stecke and Prasad (2012); Liberatore (1979)
Space	15	Huang, Lai and Shyu (2007); Yen, Chung and Chen (2012); Ghosh, Sarkar and Chaudhari (2015); Mondal, Garai and Roy (2018); Singh, Khurana and Tayal (2016); Giri and Bardhan (2015); Dordevic et al. (2017); Farhangi and Mehdizadeh (2016); Ouyang et al. (2015); Mohanty, Kumar and Goswami (2016); Sekar, Uthayakunar and Mythuradevi (2017); Tiwari et al. (2017); Tiwari et al. (2018); Sana (2015); Singha, Buddhakulsomsiri and Parthanadee (2017)
Promotion	14	Pattnaik and Gahan (2018); Gahan and Pattnaik (2017); Pattnaik (2015); Avinadav et al. (2017); Rajan and Uthayakumar (2017); Hertini et al. (2018); De and Sana (2015); Yang, Liao and Shi (2015); Tsao (2015); Noori-daryan and Taleizadeh (2018); Soni and Suthar (2018); Roy, Sana and Chaudhuri (2015); Maihami and Karimi (2014); Chen, Chen and Bidanda (2016)
Advertising	10	Bhunia et al. (2015); Geetha and Udayakumar (2015); Shah and Vaghela (2017); Shah, Chaudhari and Jani (2018); Rathore (2019); Chanda and Kumar (2016); Manna, Dey and Mondal (2017); Hazari et al. (2015);

Type	Number of Studies	Studies
		Kumar and Chanda (2017); Gupta, Biswas and Kumar (2018)
Product Quality	6	Maiti and Giri (2015); Modak, Panda and Sana (2015); Seifbarghy, Nouhi and Mahmoudi (2015); Liu, Ahang and Tang (2015); Feng (2019); Rabbani, Zia and Rafiei (2016)
Service Quality	3	Hou, Koster and Yu (2018); Xia, Xiao and Zhang (2016); Xiao and Qi (2012)

Table A 2. EOQ model studies by demand function.

Type of Demand Function	Number of Studies	Studies
Deterministic	60	Ladany and Sternlieb (1974); Ray, Gerchak and Jewkes (2005); Fibich et al. (2003); Chou and Parlar (2006); Anderson et al. (1992); Singh and Vives (1984); Vives (1999); Jeuland and Shugan (1988); Agrawal and Ferguson (2007); Hanssens and Parsons (1993); Song et al. (2008); Chen et al. (2006); Chen and Simchi-Levi (2012); Resh, Friedman and Barbosa (1976); Donaldson (1977); Bose, Goswami and Chaudhuri (1995); Giri, Goswami and Chaudhuri (1996); Dave and Patel (1981); Elsayed and Teresi (1983); Hariga and Benkherouf (1994); Palaka, Erlebacher and Kropp (1998); Albana, Frein and Hammami (2017); Pekgun, Griffin and Keskinocak (2017); Huang, Lai and Shyu (2007); Yen, Chung and Chen (2012); Ghosh, Sarkar and Chaudhari (2015); Mondal, Garai and Roy (2018); Singh, Khurana and Tayal (2016); Giri and Bardhan (2015); Dordevic et al., (2017); Farhangi and Mehdizadeh (2016); Ouyang et al., (2015); Mohanty, Kumar and Goswami (2016); Sekar, Uthayakunar and Mythuradevi (2017); Tiwari et al. (2017); Tiwari et al. (2018); Pattnaik and Gahan (2018); Gahan and Pattnaik (2017); Pattnaik (2015); Avinadav et al. (2017); Rajan and Uthayakumar (2017); Hertini et al. (2018); De and Sana (2015); Yang, Liao and Shi (2015); Tsao (2015); Noori-daryan and Taleizadeh (2018); Maiti and Giri (2015); Modak, Panda and Sana (2015); Seifbarghy, Nouhi and Mahmoudi (2015); Liu, Ahang and Tang (2015); Feng (2019); Hou, Koster and Yu (2018); Xia, Xiao and Zhang (2016); Xiao and Qi (2012); Bhunia et al. (2015); Geetha and

Type of Demand Function	Number of Studies	Studies
		Udayakumar (2015); Shah and Vaghela (2017); Shah, Chaudhari and Jani (2018); Rathore (2019)
Stochastic	28	Federgruen and Heching (1999); Petruzzi and Dada (1999); Chen and Simchi-Levi (2004); Kocabıyıkoglu and Popescu (2011); Phillips (2005); Agrawal and Ferguson (2007); Maddah and Noueihed (2017); Kalish (1985); Zhao and Zheng (2000); Bitran and Mondschein (1997); Xu and Hopp (2009); Mirzazadeh (2010); Elsayed and Teresi (1983); Zhao, Stecke and Prasad (2012); Liberatore (1979); Ho and Zheng (2004); Sana (2015); Singha, Buddhakulsomsiri and Parthanadee (2017); Soni and Suthar (2018); Roy, Sana and Chaudhuri (2015); Maihami and Karimi (2014); Chen, Chen and Bidanda (2016); Rabbani, Zia and Rafiei (2016); Chanda and Kumar (2016); Manna, Dey and Mondal (2017); Hazari et al. (2015); Kumar and Chanda (2017); Gupta, Biswas and Kumar (2018)

Table A 3 EOQ application studies by field.

Type	Number of Studies	Studies
Transportation	14	Lippman (1971); Iwaniec (1979); Abdelwahab and Sargious (1992); Swenseth and Godfrey (2002); Lee (1986); Chan et al. (2002a; 2002b); Lee et al. (2003); Mendoza and Ventura (2008); Rieksts and Ventura (2010); Konur and Toptal (2012); Hu and Munson (2002); Munson and Hu (2010); Krichen et al. (2011)
Supply Chain	21	Cachon (2001); Lim et al. (2003); Miranda and Garrido (2004); Lau et al. (2008); Battini et al. (2010); Chan et al. (2010); Snyder (2014); Chen and Chen (2005); Xia et al. (2008); Wahab et al. (2011); Khan and Jaber (2011); Chan and Lee (2012); Mutlu and Cetinkaya (2011); Duan et al. (2012); Li et al. (2007); Nagarajan and Rajagopalan (2008); Hajiaghaei-Keshteli and Fard (2018); Rezaeiahari and Sharifyazdi (2016); Xu, Yin and Dong (2016); Saxana, Singh and Sangal (2016); Yadav, Pareek and Mittal (2018)
Manufacturing	14	Ukil et al. (2015); Taleizadeh, Najafi and Niaki (2010); Taleizadeh, Sadjadi and Niaki (2011); Taleizadeh, Shavandi and Haji (2011); Taleizadeh et al. (2012); Taleizadeh, Cárdenas-Barrón and Mohammadi (2014); Pasandideh et al. (2015); Nobil, Sedigh and Cárdenas-

Type	Number of Studies	Studies
		Barrón (2016a); Nobil et al. (2019); Nobil and Taleizadeh (2016); Nobil and Sedigh (2017); Ramezanian and Saidi-Mehrabad (2012); Nobil, Sedigh and Cárdenas-Barrón (2016b); Nobil, Sedigh and Cárdenas-Barrón (2018)
Sustainability	45	Teunter (2001); Dobos and Richter (2000; 2003; 2004; 2006); Teunter (2004); Gou (2008); Alinovi et al. (2012); Jonrinaldi and Zhang (2017); Jain et al. (2018); Benkherouf, Skouri and Konstantaras (2016); Kozlovskaya, Pakhomova and Richter (2019); Mawandiya, Jha and Thakkar (2018); Singh, Sharma and Kumar (2016); Turki et al. (2017); Demirel, Demirel and Gokcen (2016); Kozlovskaya, Pakhomova and Richter (2015); Shekarian et al. (2016); Liao and Deng (2018); Hovalaque and Bironneau (2015); Shu et al. (2017); Soleymanfar et al. (2015); Hua et al. (2016); Bozorgi (2016); Konur et al. (2016); Cheng et al. (2017); Lee, Yoo and Cheong (2017); Wangsa (2017); Alinaghian and Zamani (2019); Tiwari, Daryanto and Wee (2018); Bazan, Jaber and El Saadany (2015); Nozick and Turnquist (2001); Rahimi, Baboli and Rekik (2017); Malladi and Sowlati (2018); Elbek and Wohlk (2016); Habibi et al. (2017); Habibi et al. (2018); Hiassat et al. (2017); Soysal et al. (2015); Soysal et al. (2018); Azadeh et al. (2017); Janssen et al. (2018); Iassinovskaia, Limbourg and Riane (2017); Kazemi et al. (2017); Timajchi et al. (2018)
Steel	9	Shardeo (2015); Singh and Mondal (2016); Xiong and Petri (2005); Liu, Tang and Song (2006); Tang, Liu and Liu (2008); Zhang et al. (2011); Zhang et al. (2015); Rabieh et al. (2016); Bula, Medina and Sierra (2018)

Table A 4. Sub-classification of sustainability studies

Type	Number of Studies	Studies
Emissions	14	Hovalaque and Bironneau (2015); Shu et al. (2017); Soleymanfar et al. (2015); Hua et al. (2016); Bozorgi (2016); Konur et al. (2016); Cheng et al. (2017); Lee, Yoo and Cheong (2017); Wangsa (2017); Alinaghian and Zamani (2019); Tiwari, Daryanto and Wee (2018); Bazan, Jaber and El Saadany (2015); Nozick and Turnquist (2001); Rahimi, Baboli and Rekik (2017)

Type	Number of Studies	Studies
Remanufacturing / Recycling	19	Teunter (2001); Dobos and Richter (2000; 2003; 2004; 2006); Teunter (2004); Gou (2008); Alinovi et al. (2012); Jonrinaldi and Zhang (2017); Jain et al. (2018); Benkherouf, Skouri and Konstantaras (2016); Kozlovskaya, Pakhomova and Richter (2019); Mawandiya, Jha and Thakkar (2018); Singh, Sharma and Kumar (2016); Turki et al. (2017); Demirel, Demirel and Gokcen (2016); Kozlovskaya, Pakhomova and Richter (2015); Shekarian et al. (2016); Liao and Deng (2018)
Waste Reduction and Management	12	Malladi and Sowlati (2018); Elbek and Wohlk (2016); Habibi et al. (2017); Habibi et al. (2018); Hiassat et al. (2017); Soysal et al. (2015); Soysal et al. (2018); Azadeh et al. (2017); Janssen et al. (2018); Iassinovskaia, Limbourg and Riane (2017); Kazemi et al. (2017); Timajchi et al. (2018)
Environmental Impacts	7	Fichtinger et al. (2015); Hariga, As'ad and Shamayleh (2017); Blass, Chebach and Ashkenazy (2017); Fercoq, Lamouri and Carbone (2016); Uzturk and Büyüközkan (2016); Ries, Grosse and Fichtinger (2016); Marklund and Berling (2017)

Table A 5. EOQ application studies by demand function.

Type of Demand Function	Number of Studies	Studies
Deterministic	26	Pakhomova and Richter (2019); Turki et al. (2017); Singh, Sharma and Kumar (2016); Demirel, Demirel and Gokcen (2016); Kozlovskaya, Pakhomova and Richter (2015); Elbek and Wohlk (2016); Habibi et al. (2018); Soysal et al. (2015); Iassinovskaia, Limbourg and Riane (2017); Kazemi et al. (2017); Timajchi et al. (2018); Hovalaque and Bironneau (2015); Shu et al. (2017); Soleymanfar et al. (2015); Bozorgi (2016); Lee, Yoo and Cheong (2017); Alinaghian and Zamani (2019); Tiwari, Daryanto and Wee (2018); Bazan, Jaber and El Saadany (2015); Liu, Tang and Song (2006); Tang, Liu and Liu (2008); Zhang et al. (2011); Zhang et al. (2015); Rabieh et al. (2016); Bula, Medina and Sierra (2018)

Type of Demand Function	**Number of Studies**	**Studies**
Stochastic	15	Jain et al. (2018); Benkherouf, Skouri and Konstantaras (2016); Mawandiya, Jha and Thakkar (2018); Shekarian et al. (2016); Habibi et al. (2017); Hiassat et al. (2017); Soysal et al. (2018); Azadeh et al. (2017); Janssen et al. (2018); Liao and Deng (2018); Hua et al. (2016); Konur et al. (2016); Wangsa (2017); Rahimi, Baboli and Rekik (2017); Xiong and Petri (2005)

Appendix B: Example of input data for programming implementation of the model

Name	**Description**					
MC_samples	Number of samples in Monte Carlo method. This variable should exist, be positive and integer. No stochastic variables should appear before it	5				
Horizon	How many periods ahead we do plan	52				
Leadtime	Lead time	5				
leadprod	Production during Lead_Time periods before start	100	100	100	100	100
	Initial data					
prodinit	Initial production in inventory	100				
moneyinit	Initial money	5000				
rawinit	Initial raw materials	100				
	ORDERING RAW MATERIALS					
raw_value	Value of raw materials in inventory by end of time horizon	5				
prod_value	Value of production in inventory by end of time horizon	10				
discount_val	Value of discount that will be added if order quantity exceed discount_size with such number	0.05				
discount_size	Size of raw materials order that have to be placed to add discount	100				
min_order	Minimum order	30				
fixcost_smooth	Artifitial parameter. If company buy less than this value, fixed cost reduces by quadratic dependence	50				
N_SupplyFailProb	Probability of delivery fail average	0.1	0.1	0.1	0.1	0.1
	Standard deviation	0	0	0	0	0
SupplyFail_Duration	Number of planning periods when company would have to pay more for raw materials in case of delivery fail (It should be integer, otherwise one have adjust 'RiskDeliveryFail' parameter)	2				
N_BackorderCost	extra charge per unit of raw material of delivery fail average	5	5	5	5	5
	Standard deviation	0	0	0	0	0
N_fixedcost	Cost of raw materials order (cost of order not related to ordered quantity) average	575	575	575	575	575
	Standard deviation	0	0	0	0	0
N_cost	basic cost of one unit of materials average	4	4	4	4	4
	Standard deviation	0	0	0	0	0

Name	Description					
	STORAGE					
storageAboveThreshold	**Additional price for storage above threshold**	0.5				
storageThreshold		200				
N_RawStorage	Storage expences per period average	0.5	0.5	0.5	0.5	0.5
	Standard deviation	0.05	0.05	0.05	0.05	0.05
N_ProductStorage	Storage expences per period average	0.5	0.5	0.5	0.5	0.5
	Standard deviation	0.05	0.05	0.05	0.05	0.05
RawDeterioration	Fraction of raw materials that will be deteriorated during time period	0.05				
ProduceDeterioration	Fraction of products ---	0.05				
	PRODUCTION					
Basic_power	how many units can be produced daily (1unit of raw materials used to produce every unit of production)	100				
Production_cost	cost of unit production	5				
Overtime_power	how many units can be produced during overtime	50				
Overtime_extra	overtime extra cost price per unit	2.5				
DefectCost	How much cost to fix moderate defect	5				
Defectprob	Probability of moderate defect	0.02				
HighDefectProb	Probability of critical defect (production go to waste)	0.005				
	SELLING					
N_Price0	Average	30	30	30	30	30
	Standard deviation	5	5	5	5	5
N_Demand0	Average	70	70	70	70	70
	Standard deviation	2.5	2.5	2.5	2.5	2.5
	MONEY					
Fixedcosts	Fixed costs per period (unconditional)	300				
Inflation	inflation per period forecast (can include lose from not investing those money)	0.002				
OverdraftRate	Rate of overdraft per period	0.008				
TaxRate	Base of tax is revenue of all goods	0.05				
UPcreditPay	fraction of upcredit to repay per period	0.2				
DOWNcreditPay	fraction of downcredit to repay per period	0.2				
Fail_penalty	Additional penalty for negative outcome (If company end period in loss in some cases, then additional penalty adds Loss*Fail_Penalty. It shows how many dollars of average profit we	1				

	can sacrifice to reduce expectation of losses by 1 dollars					
	EMPIRICAL PARAMETERS					
Pmax	Maximum price for good	50				
el	Demand elasticity by price	5				

Appendix C: Significant parts of the MATLAB code

```
%Entry point, lauch of swarm optimization
filename='seasonal supply price';
m=EOQ_inventory(filename);
%%
hlc=10;
control_length=13*hlc+hlc*4+hlc+4;
options = optimoptions('particleswarm', 'Display', 'iter',
'MaxIterations',1000,'SwarmSize',1000,...
'UseVectorized', true,'MaxStallIterations',100);
[x,~,exitflag,output] =
particleswarm_mine(@(x)profit_NN(x,m,@make_NN_control),control_length,-
ones(1,control_length),ones(1,control_length),options);
[fval,bparam]=profit_NN(x,m,@make_NN_control);
save(strcat('results\',filename),'m','x','bparam','output','options')
%%
control_length=52*4;
options = optimoptions('particleswarm', 'Display', 'iter',
'MaxIterations',2000,'SwarmSize',1000,...
'UseVectorized', true,'MaxStallIterations',100);
[x,~,exitflag,output] =
particleswarm_mine(@(x)profit_NN(x,m,@make_identity_control),control_length,zeros(1,
control_length),ones(1,control_length),options);

[fval,bparam]=profit_NN(x,m,@make_identity_control);
save(strcat('results\',filename,' (static)'),'m','x','bparam','output','options')

%%%%


%Economic model-Profit
function [pr0,res] = profit_NN(X,m,fun)
s1=size(X,1);
if (s1==1)
   res=struct;
   res.raw=zeros(m.MC_samples,m.horizon);
   res.raw_buy=zeros(m.MC_samples,m.horizon);
   res.prod=zeros(m.MC_samples,m.horizon);
   res.prod_order=zeros(m.MC_samples,m.horizon);
   res.money=zeros(m.MC_samples,m.horizon);
   res.invested=zeros(m.MC_samples,m.horizon);
   res.downcredit=zeros(m.MC_samples,m.horizon);
   res.upcredit=zeros(m.MC_samples,m.horizon);
   res.price=zeros(m.MC_samples,m.horizon);
   res.prod_income=zeros(m.MC_samples,m.horizon);
   res.prod_sell=zeros(m.MC_samples,m.horizon);
   res.profit=zeros(m.MC_samples,1);
   res.control=zeros(m.MC_samples,m.horizon,4);
end
pr0=0;
```

```
for it=1:m.MC_samples
   m=generate_rand_instance(m,s1);
   order=zeros(s1,m.horizon,'gpuArray');
   raw=ones(s1,1,'gpuArray')*m.rawinit;
   prod_available=ones(s1,1,'gpuArray')*m.prodinit;
   money=ones(s1,1,'gpuArray')*m.moneyinit;
   invested=zeros(s1,1,'gpuArray');
   downcredit=zeros(s1,1,'gpuArray');
   upcredit=zeros(s1,1,'gpuArray');
   pr_init=max(-1e+9,-(invested+money-
downcredit+upcredit+m.raw_value*raw+m.prod_value*prod_available));
   leadprod=gpuArray(repmat(m.leadprod,s1,1));
   for i=1:m.horizon

data=[money*0.001,downcredit*0.001,upcredit*0.001,raw*0.1,prod_available*0.1,leadpro
d(:,1:2)*0.1,...
         m.cost(:,i),...
         (m.fixedcost(:,i)-m.fixedcost_av(1))/m.fixedcost_av(2),...
         (m.Price0(:,i)-m.Price0_av(1))/m.Price0_av(2),...
         (m.Demand0(:,i)-m.Demand0_av(1))/m.Demand0_av(2),...
         (m.RawStorage(:,i)-m.RawStorage_av(1))/m.RawStorage_av(2),...
         (m.ProductStorage(:,i)-m.ProductStorage_av(1))/m.ProductStorage_av(2)]';
      control=fun(X,data,i);
      % MONEY MANAGEMENT
      inv=money.*control(:,1); % invest planned %% of all free money
      inv(inv<0)=0;
      money=money-inv;
      invested=invested+inv;

      money=money.*(1-m.Inflation); % money losing due to inflation (negative sum would
depreciate too,
      % but overdraft rate usually more )

      money=money+upcredit*m.UPcreditPay-downcredit*m.DOWNcreditPay;
      % upcredit and downcredit duration is at least one period,
      % then given percent of credits have to be repayed
      upcredit=upcredit*(1-m.UPcreditPay);
      downcredit=downcredit*(1-m.DOWNcreditPay);
      money(money<0)=money(money<0).*m.OverdraftRate;

      % BUY RAW MATERIALS
      max_buy=(money-m.fixedcost(:,i))./m.cost(:,i);
      raw_buy=max_buy.*control(:,2);
      raw_buy(raw_buy<m.min_order)=0;
      downcredit=downcredit+raw_buy.*m.cost(:,i).*(raw_buy>=m.min_order).*...
      (1-sum((raw_buy>=m.discount_size).*m.discount_val,2))...
      +m.fixedcost(:,i).*(raw_buy>=m.min_order).*min((raw_buy./m.fixcost_smooth-
1).^2,1);

      raw=raw+raw_buy;
      % PRODUCE GOODS
      max_produce=min(raw,money/m.Production_cost);
      max_produce(max_produce<0)=0;
```

```
max_produce(max_produce>m.Basic_power+m.Overtime_power)=m.Basic_power+m.Ov
ertime_power;
        order(:,i)=max_produce.*control(:,3);
        raw=raw-order(:,i); % we can't spend raw more then we have
        money=money-order(:,i)*m.Production_cost; % but we spend money for whole plan
        prod_available=prod_available+leadprod(:,1);
        leadprod(:,1:m.leadtime-1)=leadprod(:,2:m.leadtime);
        leadprod(:,m.leadtime)=order(:,i)*(1-m.HighDefectProb);
        money=money-(order(:,i)>m.Basic_power).*(order(:,i)-
m.Basic_power)*m.Overtime_extra;
        %SELL PRODUCTION
        price=m.Pmax*control(:,4);
        max_qty=m.Demand0(:,i).*(m.Price0(:,i)./(price+1e-8)).^m.el;
        sell=min(max_qty,prod_available);
        % trying to sell more then we have should be interpreted
        % as selling all we have, however in order to prevent solution to
        % increase selling amount when it is not feasible, we assume that
        % price is decreasing
        % It will work correct only if demand is decreasing function of
        % price.

        upcredit=upcredit+(1-m.TaxRate)*sell.*price;
        prod_available=prod_available-sell;
        money=money-sell*m.Defectprob*m.DefectCost/(1-m.HighDefectProb);


        %fixed cost, storage and deterioration
        money=money-raw.*m.RawStorage(:,i)-prod_available.*m.ProductStorage(:,i)-
m.Fixedcosts;
        money=money-max((raw-m.storageThreshold),0).*m.storageAboveThreshold;
        raw=raw*(1-m.RawDeterioration);
        prod_available=prod_available*(1-m.ProduceDeterioration);
        reserve(:,i)=raw; % reserve of materials

    if (s1==1)
    res.raw(it,i)=gather(raw);
    res.raw_buy(it,i)=gather(raw_buy);
    res.prod(it,i)=gather(prod_available);
    res.prod_order(it,i)=gather(order(:,i));
    res.money(it,i)=gather(money);
    res.invested(it,i)=gather(inv);
    res.downcredit(it,i)=gather(downcredit);
    res.upcredit(it,i)=gather(upcredit);
    res.price(it,i)=gather(price);
    res.prod_income(it,i)=gather(leadprod(:,1));
    res.prod_sell(it,i)=gather(sell);
    res.control(it,i,:)=gather(control);
    end
    end
    %BACKORDER analysis
    Backorderrisk=zeros(s1,1);
    for i=1:m.horizon-m.SupplyFail_Duration+1
        % shortage is how much would raw material would we short
        shortage=-reserve(:,i)+sum(order(:,i:i-1+m.SupplyFail_Duration),2);
```

```
        shortage=shortage.*(shortage>0);
Backorderrisk=Backorderrisk+shortage.*m.BackorderCost(:,i).*m.SupplyFailProb(:,i);
    end
    pr=-(invested+money-
downcredit+upcredit+m.raw_value*raw+m.prod_value*(prod_available+sum(leadprod,2))-
pr_init-Backorderrisk);
    pr=pr.*(1+(pr>0)*m.Fail_penalty);
    if (exist('res','var'))
        res.profit(it)=gather(pr);
    end
    pr0=pr0+gather(pr)/m.MC_samples;
end
end



%%Economic model
function Out = make_NN_control(X,data,i)
data(isnan(data))=0;
hldim=round((size(X,2)-4)/18);
outlim=4;
%%transfer input to hidden
W1=X(:,1:size(data,1)*hldim);
X=X(:,size(data,1)*hldim+1:end);
W1=reshape(W1,[],size(data,1),hldim);
W1 = permute(W1,[3 2 1]);
data=gpuArray(reshape(data,[size(data,1),1,size(data,2)]));
H1 = pagefun(@mtimes,W1,data);
%add bias
B1=X(:,1:hldim)';
X=X(:,hldim+1:end);
B1=reshape(B1,size(B1,1),1,size(B1,2));
H1=H1+B1;
H1=max(H1,0);
%transfer hidden to output
W2=X(:,1:hldim*outlim);
X=X(:,hldim*outlim+1:end);
W2=reshape(W2,[],hldim,outlim);
W2 = permute(W2,[3 2 1]);
Out=pagefun(@mtimes,W2,H1);
%add second bias
B2=X';
B2=reshape(B2,size(B2,1),1,size(B2,2));
Out=Out+B2;
Out=sigmf(Out,[1,0]);
Out=squeeze(Out)';
end
```

Appendix D: Matlab User Interface

D.1 User Interface

The Matlab software, specifically, the Matlab Graphics User Interface (GUI) application, provides different functional characteristics and simplifies the process of using the algorithm. This application allows end users to load data into the application, edit this data, train the closed-loop control system, view and save the results, and analyse the model's performance. Figure D 1 shows the main window of the interface, which includes the following buttons:

1) "Load data", which allows the user to load input data from an Excel file.
2) "Fixed parameters", which allows the user to make final adjustments to the fixed parameters before the start of the training process.
3) "Stochastic parameters", which allows the user to make final adjustments to the stochastic parameters before the start of the training process.
4) "Train the model", which launches the PSO algorithm in order to train the neural network closed-loop control system.
5) "Save results", which saves the parameters of the trained control system and the expected business indicators to a *.mat file.

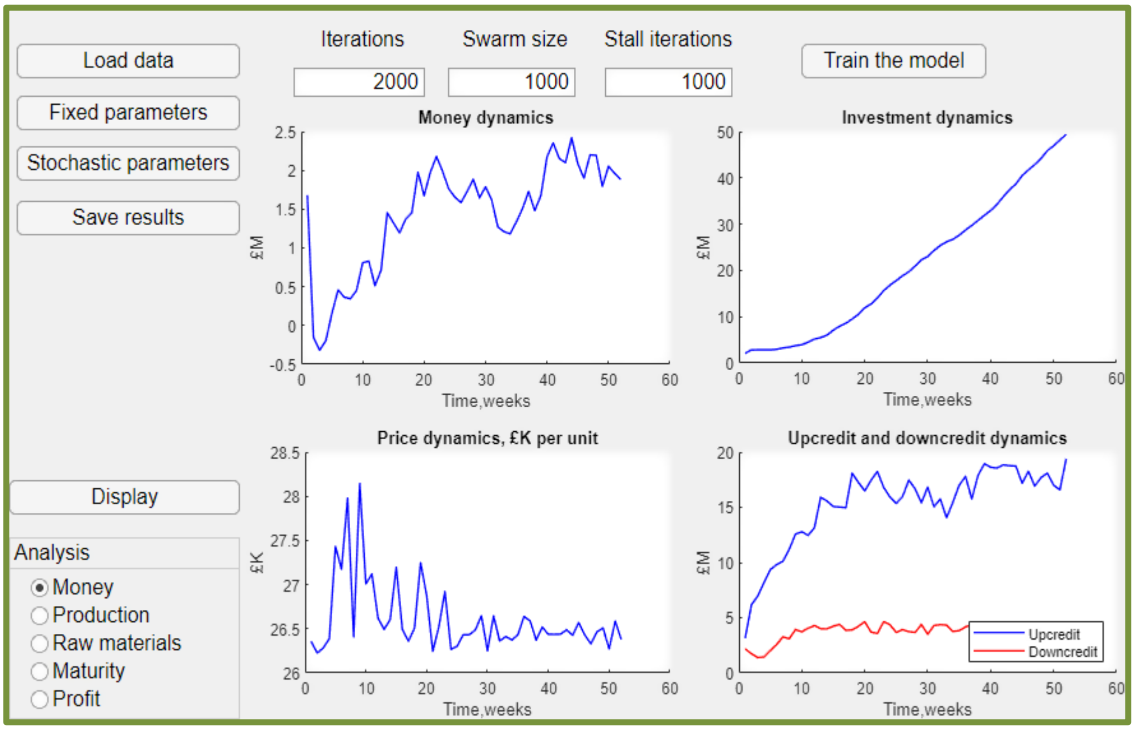

Figure D 1. Program GUI.

In addition, as seen from the above figure, the software application provides the user with the option to display any of the following performance indicators, which are saved from the five Monte Carlo runs:

1) Level of money
2) Level of investments
3) Amount of purchased raw materials
4) Amount of raw materials in storage
5) Quantity of ordered final products
6) Quantity of final products in storage
7) Amount of up credit
8) Amount of down credit
9) Current selling price
10) Quantity of final products that were sold.

There is also a block of radio buttons at the bottom-left corner of the main window of the application, which are used to select which category of business indicators and parameters to display. This block consists of the following buttons:

1) Money: which displays the level of funds, level of investment, current price, and up credit and down credit amounts.
2) Production: which displays the quantity of final products produced, the quantity of final products in storage, the quantity of final products sold, and the percentage of final products stored and then sold.
3) Raw materials: which displays the quantity of raw material that was purchased, the quantity of raw materials used in production, and the percentage of raw materials that entered production after being stored.
4) Maturity: which shows the distribution of raw materials and final product maturity.
5) Profit: which shows the annual net profit for each Monte Carlo run, displayed in the form of a bar chart.

Finally, at the top of the window shown in Figure D 1, there are three edit fields that allow the user to edit the PSO algorithm options, as follows:

1) Maximum number of training iterations
2) Swarm size
3) Maximum number of stall iterations (without enhancing the objective function)

Figure D 2 shows the window that appears after pressing the “Fixed parameters” button in the main window. This window allows the end user to fine tune the static parameters related to production, defect and deterioration, and global economic parameters. The full list of the editable parameters corresponds to the list described in Section 4.3.

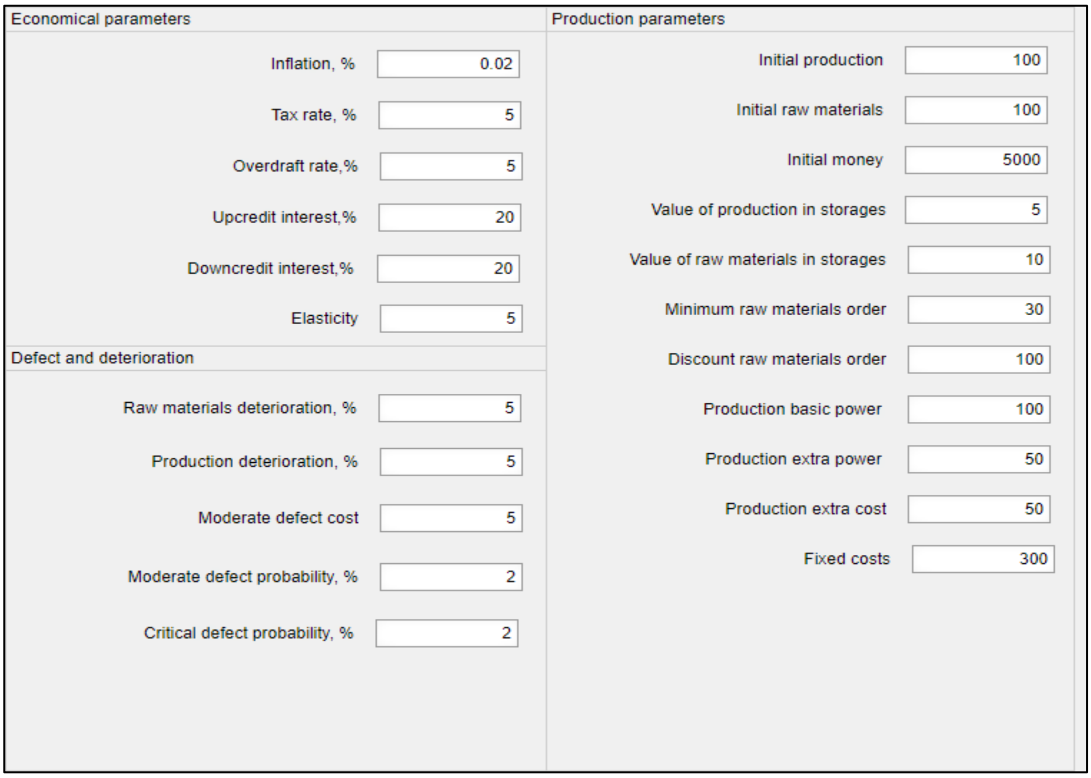

Figure D 2. Program GUI (fixed variables window).

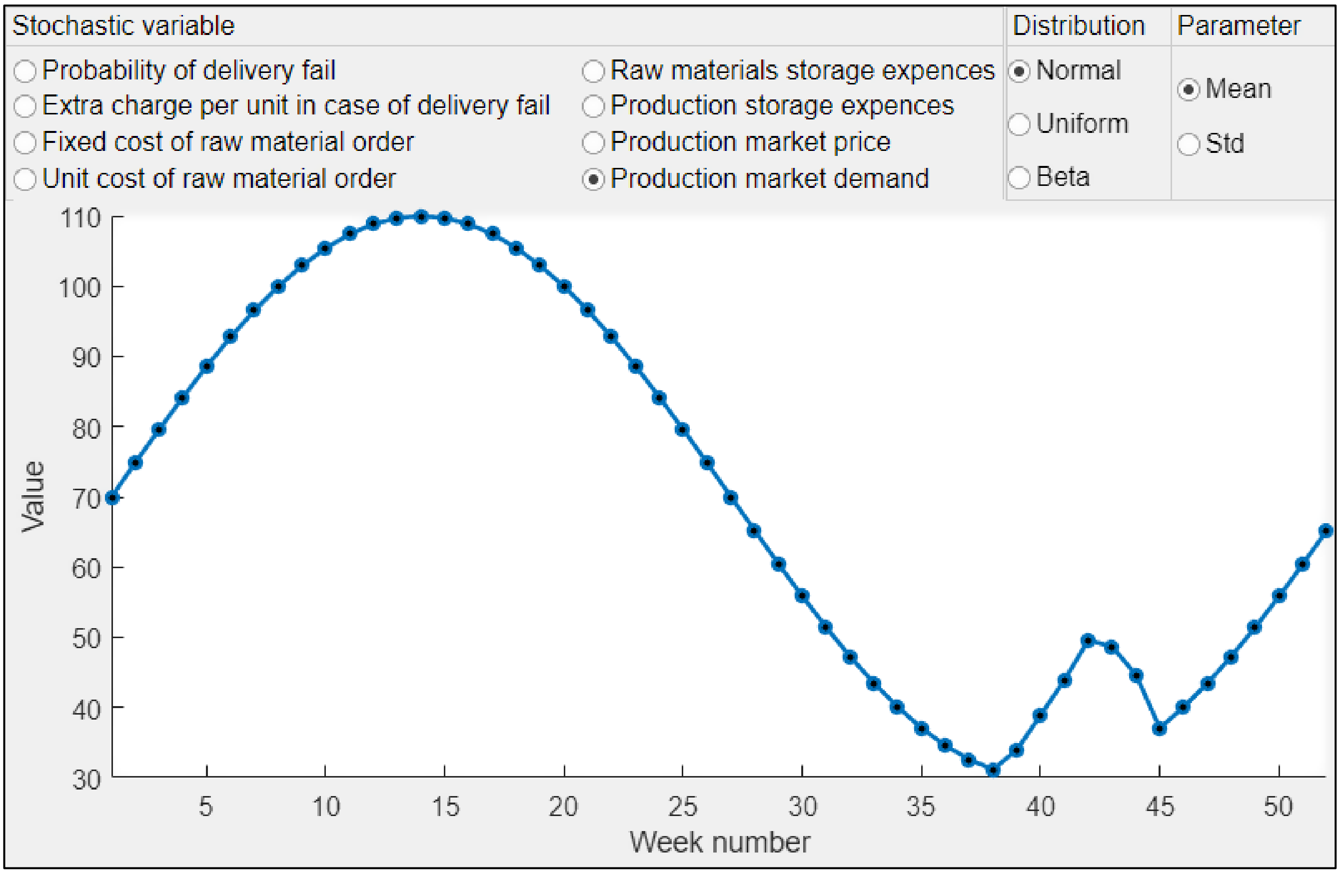

Figure D 3. Program GUI (stochastic variables window).

Figure D 3 shows the window that appears after pressing the "Stochastic parameters" button in the main window. This window allows the user to edit the stochastic variables of the model, and select the variable name and type of variable distribution from the following three distributions:

1) Normal distribution
2) Uniform distribution
3) Beta distribution

The outputs of the last two distributions lie between zero and one; hence, they are especially convenient to be used to model the stochastic variables that are represented through probabilities, such as backorder risk. The third tab allows the user to select whether to edit the mean value or standard deviation of the variable. Furthermore, the application allows the user to drag points in the plot and save the edited variable to an Excel file, and, at the end, save all the stochastic variables to a *.mat file.

The final application window is designed to set up the architecture of the closed-loop neural network control system (Figure D 4). At the top of the window, two edit fields allow the end user to set up the number of neurons in the input and hidden layers, while the number of neurons in the output layer is fixed and equal to the number of controls. Furthermore, the two drop-boxes above the charts are needed to select the activation function of the hidden and output layers, respectively. For each layer, the user can choose one activation function from the following functions:

1) Binary step function
2) Sigmoid function
3) Logistic function
4) Hyperbolic tangent
5) Rectifier linear unit

For the output layer, it is advisable to select one of the functions that takes values from zero to one, as all controls take values in this interval.

The developed interface can be easily used by the management of the steel manufacturing factory in order to obtain the optimal controls that maximise profit. As discussed above, the interface allows the user to edit both stochastic and fixed input data, perform the training process of the closed-loop neural network system, and analyse the state of business over the planning horizon. In addition, as the application interface is created with Matlab, it allows the management to automate their business decisions based on the input data, and assists owners and shareholders to analyse the performance of the company before making any investment decisions.

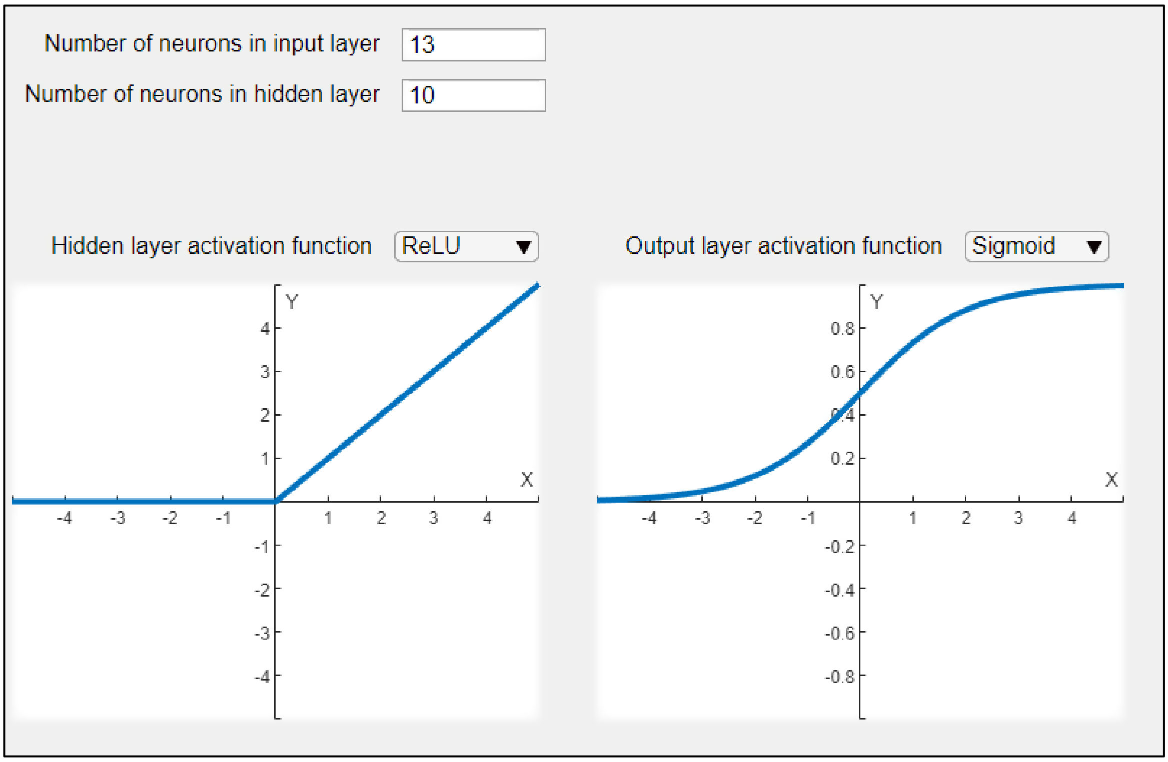

Figure D 4. Program GUI (controls system options window).

Finally, Figure D 5 displays the manager's decision tool. In this tool, the manager inputs the current actual values of all stochastic parameters on a weekly basis. After inputting all the values, the "Generate decisions" button is used to display the current values for investment, raw materials purchase, production rate and current price in the blue window at the bottom-right corner. Furthermore, the manager can adjust decisions generated by the control system and apply them to the business of his factory.

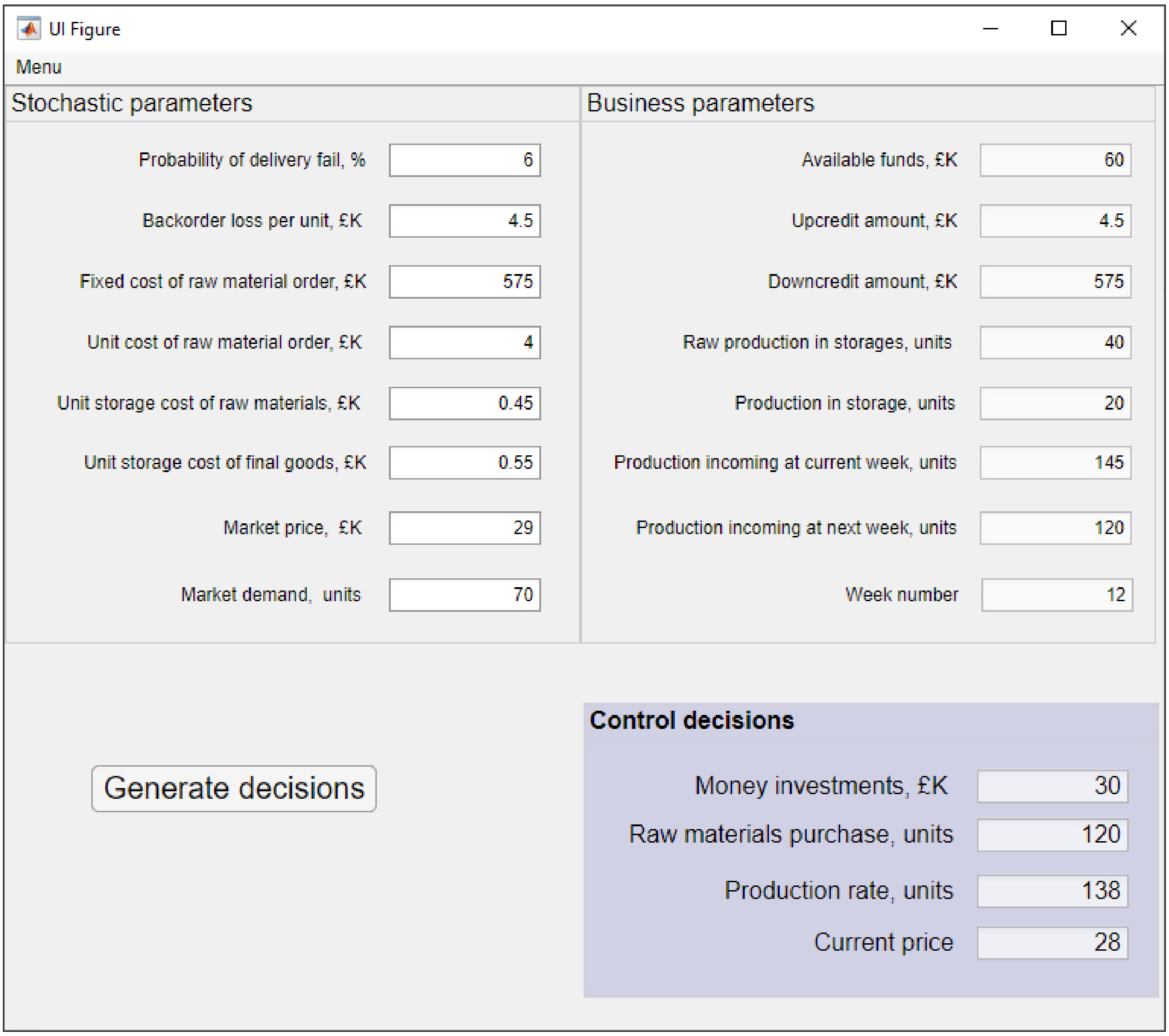

Figure D 5. Manager's decision tool.